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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 14

Math of Space: Surface Area and Volume

Complete Exercise Solutions

Exercise Sets 14.1 – 14.4 · End-of-Chapter Exercises

Contents

Class 9 Mathematics · Ganita Manjari · Chapter 14 — Math of Space: Surface Area and Volume

IT
In-Text Questions, Examples & Activities — 3
12 items · T1 T2 E1 T3 E2 E3 T4 E4 A1 E5 E6 T5

14.1
Exercise Set 14.1 — 12
8 questions · Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8

14.2
Exercise Set 14.2 — 18
5 questions · Q1 Q2 Q3 Q4 Q5

14.3
Exercise Set 14.3 — 22
9 questions · Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9

14.4
Exercise Set 14.4 — 27
8 questions · Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8

EC
End-of-Chapter Exercises — 32
22 questions · Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9 Q10 Q11 Q12 Q13 Q14 Q15 Q16 Q17 Q18 Q19 Q20 Q21 Q22

Every question is followed by its answer. Question numbers above are clickable and the PDF bookmarks panel lets you jump to any exercise set or question.

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IT

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 14

In-Text Questions, Examples & Activities

Solutions · 12 items

IN-TEXT 1

Compare the formula for the volume of a cuboid with the formula for the area of a rectangle (length × width).

ANSWER

The area of a rectangle counts unit squares in two dimensions: **A = l × w**. The volume of a cuboid counts unit cubes in three dimensions: **V = l × w × h**.

$$V = l \times w \times h = (l \times w) \times h = \text{area of base} \times \text{height}$$

So the volume formula is the area formula multiplied by the third dimension (the height). This is why a cuboid is called the ‘three-dimensional version’ of a rectangle.

IN-TEXT 2

Think and Reflect
Try to work out for yourself why this model explains the formula for the volume of a cuboid, i.e., volume = area of base × height.

Write the formula for the surface area and the volume of a cube.



Fig. 14.3

ANSWER

1 **Why the stacking model works**

Think of the cuboid as a pack of n thin rectangular sheets, each of length l, width w and a very small thickness t.

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Volume of one sheet = l × w × t

Volume of n sheets = n × (l × w × t) = (l × w) × (n × t)

The sheets are stacked on each other, so their total thickness $n\,t$ is the height h of the cuboid. Hence

V = (l × w) × h = area of base × height

2 Cube of side a

A cube is a cuboid with $l = w = h = a$. Substituting in the formulas of a cuboid:

Total surface area = 2(wl + hl + hw) = 2(a^2 + a^2 + a^2) = 6a^2

Volume = lwh = a × a × a = a^3

Therefore, for a cube of side a: TSA = 6a^2 and V = a^3.

EXAMPLE 1

Two solid objects are made from the same material: Cube A has side 6 cm and Cuboid B has dimensions 9 cm × 6 cm × 4 cm. Compare their volumes and surface areas and determine which object has a greater surface area. How is this useful in a real-life situation?

ANSWER

A Cube A (side 6 cm)

Volume = a^3 = 6^3 = 216 cm^3

Surface area = 6a^2 = 6 × 6^2 = 216 cm^2

B Cuboid B (9 cm × 6 cm × 4 cm)

Volume = l × w × h = 9 × 6 × 4 = 216 cm^3

Surface area = 2(lw + wh + lh) = 2[(9 × 6) + (6 × 4) + (4 × 9)]

= 2(54 + 24 + 36) = 2 × 114 = 228 cm^2

Both objects have the **same volume** (216 cm^3), but the cuboid has the **greater surface area** (228 cm^2 > 216 cm^2).

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Real-life use: for the same storage capacity, a cube needs less covering (packaging) material, while a cuboid exposes more area to its surroundings. This matters in packaging, heat transfer and engineering.

Therefore, Cuboid B has the greater surface area (228 cm² against 216 cm²).

IN-TEXT 3

Think and Reflect

Suppose a swimming pool is being built in your school. Two choices are available for the dimensions of the pool: (A) 8 ft (depth) × 50 ft × 20 ft and (B) 6 ft (depth) × 100 ft × 15 ft. Which option would be suitable/appropriate for your school? Why? What parameters/aspects did you consider to make the choice? [1 cubic foot ≈ 28.3 litres]

ANSWER

This question is open-ended; the choice depends on the parameters considered. We compare the two options with numbers first.

1 Water needed (volume)

A: $8 \times 50 \times 20 = 8000$ ft³ $\approx 8000 \times 28.3 = 2{,}26{,}400$ litres

B: $6 \times 100 \times 15 = 9000$ ft³ $\approx 9000 \times 28.3 = 2{,}54{,}700$ litres

2 Surface to be tiled/painted (floor + four walls)

A: floor $50 \times 20 = 1000$ ft²; walls $2(50 \times 8) + 2(20 \times 8) = 1120$ ft²; total $= 2120$ ft²

B: floor $100 \times 15 = 1500$ ft²; walls $2(100 \times 6) + 2(15 \times 6) = 1380$ ft²; total $= 2880$ ft²

3 Other parameters

Safety (depth of water for students), length available for lap swimming, water-surface area (number of swimmers at a time), cost of construction, tiling and water, filtration and maintenance, and space available in the school.

Option B holds about 12% more water (28,300 litres more) and needs about 36% more tiling, but its water surface is larger (1500 ft² against 1000 ft²), so more students can swim at once, its 100 ft length suits lap training, and its depth is 2 ft less. Option A is cheaper but uniformly 8 ft deep, which is less safe for school students.

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Therefore, Option B is the more suitable choice for a school (safer depth, larger swimming area), provided the extra cost is acceptable; Option A is the economical choice.

EXAMPLE 2

Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope (see Fig. 14.6). What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius? You may take $\pi = \frac{22}{7}$.

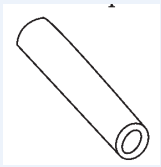

Fig. 14.6

ANSWER

Radius r = 3.5 cm and height (length) h = 25 cm. The chart paper covers only the curved surface.

$$\text{Area of chart paper} = \text{CSA} = 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 3.5 \times 25$$
$$= 2 \times 22 \times 0.5 \times 25 = 550 \text{ cm}^2$$

Therefore, the area of chart paper required is 550 cm².

EXAMPLE 3

A right triangle ABC with sides 5 cm, 12 cm and 13 cm is rotated through 360° about the side with length 12 cm. Find the volume of the solid so obtained.

ANSWER

Check: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$, so the triangle is right-angled with perpendicular sides 5 cm and 12 cm and hypotenuse 13 cm.

Rotating a right triangle about one of its perpendicular sides gives a right circular cone. Here the axis is the 12 cm side, so

$$\text{height } h = 12 \text{ cm}, \text{ radius } r = 5 \text{ cm}$$

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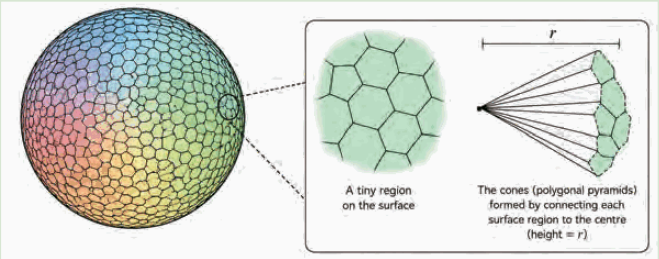
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 5^2 \times 12$

$= \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi \text{ cm}^3$

Therefore, the volume of the solid is 100π cm³ (≈ 314.3 cm³ using π = 22/7).

IN-TEXT 4

Volume of a sphere = $\frac{1}{3}$ × surface area of sphere × radius of sphere. Try to work out the connecting links on your own.



ANSWER

Imagine the whole surface of the sphere divided into a very large number of tiny regions. Join every tiny region to the centre O. We get a large number of thin cones (pyramids), each of height equal to the radius *r* of the sphere.

Volume of one thin cone = $\frac{1}{3}$ × (its base area) × r

The volume of the sphere is the sum of the volumes of all these cones. Since all have the same height *r*:

$V = \frac{1}{3} \times r \times$ (sum of all base areas)

The sum of the base areas = surface area of the sphere = $4\pi r^2$

$V = \frac{1}{3} \times r \times 4\pi r^2 = \frac{4}{3}\pi r^3$

Therefore, $V = \frac{1}{3} \times (4\pi r^2) \times r = \frac{4}{3}\pi r^3$.

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EXAMPLE 4

A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

ANSWER

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$$
$$= \frac{2}{3} \times \frac{22}{7} \times 42.875 = \frac{2}{3} \times 134.75$$
$$\approx 89.8 \text{ cm}^3$$

Therefore, the bowl can contain about 89.8 cm³ of water.

ACTIVITY

Identify and list objects or structures around you that are cubical, cuboidal, cylindrical, conical, spherical, or pyramidal in shape, or closely resemble these shapes.

Activity: Find the approximate volumes of a gas cylinder, a bucket, any cylindrical vessel, a cupboard, a water tank that you can find near you (use 1000 cm³ = 1 litre).

ANSWER

1 Objects around us

Cubical: dice, Rubik's cube, sugar cube. **Cuboidal:** brick, matchbox, book, cupboard, classroom. **Cylindrical:** pipe, tin, glass, gas cylinder. **Conical:** ice-cream cone, traffic cone, joker's cap. **Spherical:** football, marble, globe. **Pyramidal:** pyramids of Egypt, Shivling Peak, a tetrahedral puzzle.

2 Approximate volumes (sample measurements)

The measurements below are only an illustration; use the measurements of the objects near you in the same way.

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Gas cylinder (r ≈ 15 cm, h ≈ 53 cm): $V = \pi r^2 h = 3.14 \times 15 \times 15 \times 53 \approx 37{,}400\ cm^3 \approx 37$ litres

Bucket, taken as a cylinder (r ≈ 14 cm, h ≈ 28 cm): $V = \frac{22}{7} \times 14 \times 14 \times 28 = 17{,}248\ cm^3 \approx 17$ litres

Cylindrical vessel (r = 10 cm, h = 20 cm): $V = 3.14 \times 10 \times 10 \times 20 = 6280\ cm^3 \approx 6.3$ litres

Cupboard (90 cm × 45 cm × 180 cm): $V = lwh = 7{,}29{,}000\ cm^3 = 729$ litres

Water tank (100 cm × 100 cm × 100 cm): $V = 10{,}00{,}000\ cm^3 = 1000$ litres

Method: identify the shape, measure the needed lengths, apply the volume formula, and convert using 1000 cm^3 = 1 litre.

EXAMPLE 5

Lallan's golgappa (a small hollow ball made of wheat) stall has a cylindrical vessel with pani (spiced water) in it. The vessel is of radius 10 cm and it has pani up to a height of 50 cm. Estimate the number of golgappas he can serve assuming all golgappas are spheres. Before calculating, make a guess.



ANSWER

Guess: the answer is probably in the hundreds or low thousands. Now calculate.

Volume of pani $= \pi r^2 h = \pi \times 10 \times 10 \times 50 = 5000\pi\ cm^3 \approx 15{,}708\ cm^3$

Number of golgappas = (total pani in the vessel) ÷ (pani in each golgappa). The answer depends on the assumptions:

1 Assumption 1: radius 1.5 cm, 60% filled with pani

Volume of a golgappa $= \frac{4}{3} \times \pi \times (1.5)^3 \approx 14.14\ cm^3$

Pani in one $= 14.14 \times 0.6 \approx 8.5\ cm^3$

Number $= 5000\pi \div 8.5 \approx 1848$

2 Assumption 2: radius 1.75 cm, 80% filled with pani

Volume of a golgappa $= \frac{4}{3} \times \pi \times (1.75)^3 \approx 22.4\ cm^3$

Pani in one $= 22.4 \times 0.8 \approx 18\ cm^3$

Number $= 5000\pi \div 18 \approx 873$

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Therefore, Lallan can serve about 1850 golgappas (Assumption 1) or about 870 (Assumption 2); the answer depends on the assumptions made.

EXAMPLE 6

Anirban saw a few tennis balls lying around. He wondered, "How many tennis balls can fit in an empty classroom?". The classroom is of dimensions 30 ft × 30 ft × 15 ft. Estimate how many tennis balls of radius 3 cm fit in this classroom. Before calculating, make a guess.



ANSWER

Guess: some lakhs to a few crores. Now calculate (taking 1 ft ≈ 30 cm, so 1 ft³ ≈ 28,000 cm³).

Volume of a ball = $\frac{4}{3} \times \pi \times 3^3 = 36\pi \approx 113$ cm³

Volume of classroom = 30 × 30 × 15 = 13,500 ft³ ≈ 37,80,00,000 cm³

1 Upper bound

37,80,00,000 ÷ 113 ≈ 33,45,132

Balls cannot fill the room without gaps, so this is only an upper bound: no more balls than this can fit.

2 First configuration (balls directly above one another)

Room: 900 cm × 900 cm × 450 cm; diameter of a ball = 6 cm

Along length: 900 ÷ 6 = 150; along width: 150; along height: 450 ÷ 6 = 75

Number = 150 × 150 × 75 = 16,87,500

Therefore, about 16,87,500 balls fit in the first configuration (and never more than about 33,45,132).

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IN-TEXT 5

Make an estimate based on the second and third configurations. (Fig. 14.17)

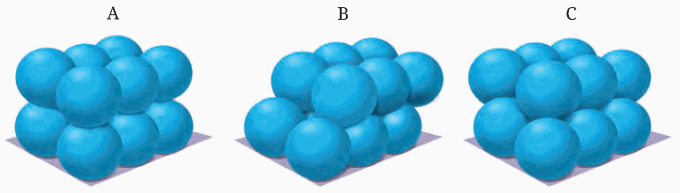

Fig. 14.17

ANSWER

In the denser configurations, the balls of each layer sit in the hollows of the layer below, so the layers come closer together. For two touching balls of diameter 6 cm in neighbouring layers, the layer spacing is $6 \div \sqrt{2} \approx 4.24$ cm instead of 6 cm.

Number of layers in 450 cm: $1 + (450 - 6) \div 4.24 \approx 1 + 104 = 105$

Layers alternate: $150 \times 150 = 22{,}500$ balls and $149 \times 149 = 22{,}201$ balls

Number $\approx 53 \times 22{,}500 + 52 \times 22{,}201 = 11{,}92{,}500 + 11{,}54{,}452 = 23{,}46{,}952$

A close packing of equal spheres fills about 74% of the space: $0.74 \times 33{,}45{,}132 \approx 24{,}77{,}000$, which agrees with this estimate.

Therefore, the denser configurations hold about 23 to 25 lakh balls, more than the 16,87,500 of the first configuration but less than the upper bound of 33,45,132.

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14.1

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Exercise Set 14.1

Solutions · 8 questions

QUESTION 1

The volume of a cube is 64 cm³. What is its total surface area?

ANSWER

Let the side of the cube be a.

$a^3 = 64 \Rightarrow a = 4$ cm

$\text{TSA} = 6a^2 = 6 \times 4^2 = 6 \times 16 = 96$ cm²

Therefore, the total surface area is 96 cm².

QUESTION 2

How many small cubes with side 20 cm can be packed tight in a cubical box with side 2 m?

ANSWER

Side of the box = 2 m = 200 cm. Number of small cubes along each edge = 200 ÷ 20 = 10.

Number of cubes = $10 \times 10 \times 10 = 1000$

(Check: $200^3 \div 20^3 = 80{,}00{,}000 \div 8000 = 1000$)

Therefore, 1000 small cubes can be packed.

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QUESTION 3

The dimensions of a godown are 40 m × 25 m × 10 m. If it is filled with cuboidal boxes, each of dimensions 2 m × 1.25 m × 1 m, then find the number of boxes.

ANSWER

Volume of godown = $40 \times 25 \times 10 = 10{,}000\ \text{m}^3$

Volume of one box = $2 \times 1.25 \times 1 = 2.5\ \text{m}^3$

Number of boxes = $10{,}000 \div 2.5 = 4000$

Check by edges: $40 \div 2 = 20$, $25 \div 1.25 = 20$, $10 \div 1 = 10$, and $20 \times 20 \times 10 = 4000$, so the boxes fit exactly.

Therefore, the number of boxes is 4000.

QUESTION 4

Two cubes each of volume 125 cm³ are joined end to end. Find the surface area of the resulting cuboid.

ANSWER

Edge of each cube: $a^3 = 125$, so $a = 5$ cm. Joining two cubes end to end gives a cuboid of length 10 cm, width 5 cm and height 5 cm.

$\text{TSA} = 2(lw + wh + lh) = 2(10 \times 5 + 5 \times 5 + 10 \times 5)$

$= 2(50 + 25 + 50) = 2 \times 125 = 250\ \text{cm}^2$

Therefore, the surface area of the cuboid is 250 cm².

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QUESTION 5

A cube of side 4 cm is cut into cubes of side 1 cm. What is the ratio of the surface areas of the original cube and all the cut-out cubes? (Note that there is no change in volume but a big change in the surface area. This property has major consequences in the biological world.)

ANSWER

Surface area of original cube = $6 \times 4^2 = 96$ cm²

Number of small cubes = $4^3 \div 1^3 = 64$

Surface area of one small cube = $6 \times 1^2 = 6$ cm²

Surface area of all small cubes = $64 \times 6 = 384$ cm²

Ratio = 96 : 384 = 1 : 4

Therefore, the ratio of the surface areas (original : all cut-out cubes) is 1 : 4.

QUESTION 6

The surface areas of the three faces of a cuboid that meet at one of the corners of the cuboid are 6 cm², 15 cm², and 10 cm² respectively. What is the volume of the cuboid?

ANSWER

Let the edges be *l, w, h*. The three faces meeting at a corner have areas *lw, wh, lh*.

$lw = 6$, $wh = 15$, $lh = 10$

$(lw)(wh)(lh) = (lwh)^2 = 6 \times 15 \times 10 = 900$

$lwh = \sqrt{900} = 30$

(For example, the edges are 2 cm, 3 cm and 5 cm: $2 \times 3 = 6$, $3 \times 5 = 15$, $2 \times 5 = 10$.)

Therefore, the volume of the cuboid is 30 cm³.

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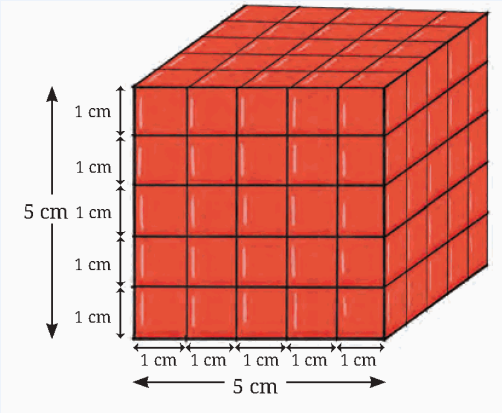
QUESTION 7 *

A cube of side 5 cm is painted on all its faces. If it is sliced into 1 cm³ cubes, how many of these 1 cm³ cubes have



(i) exactly three faces painted?

(ii) exactly two faces painted?

(iii) exactly one face painted?

(iv) no face painted?

ANSWER

Slicing gives $5 \times 5 \times 5 = 125$ small cubes. Classify them by position:

(i) Exactly three faces painted

These are the corner cubes. A cube has 8 corners.

Answer: 8

(ii) Exactly two faces painted

These lie along the edges, excluding the corners. Each of the 12 edges has $5 - 2 = 3$ such cubes.

$$12 \times 3 = 36$$

Answer: 36

(iii) Exactly one face painted

These lie in the middle of each face. Each face has $(5 - 2)^2 = 9$ such cubes, and there are 6 faces.

$$6 \times 9 = 54$$

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Answer: 54

(iv) No face painted

These form the inner cube of side $5 - 2 = 3$ cm.

$$3 \times 3 \times 3 = 27$$

Answer: 27

Check: 8 + 36 + 54 + 27 = 125, the total number of small cubes.

QUESTION 8 *

Find a cuboid with edges whose lengths are integers (in cm), given that it has a total surface area of exactly 100 cm².

(i) Is there more than one such cuboid?

(ii) Can you find them all?

(iii) Show that you have found them all.

ANSWER

Let the edges be *l, w, h* (integers) with $l \geq w \geq h \geq 1$. Then

$$2(lw + wh + lh) = 100 \Rightarrow lw + wh + lh = 50$$

A cuboid with surface area 100 cm²

One such cuboid is **2 cm × 4 cm × 7 cm**, and another is **1 cm × 2 cm × 16 cm**.

$2 \times 4 \times 7$: $2(8 + 28 + 14) = 2 \times 50 = 100$ cm²

$1 \times 2 \times 16$: $2(2 + 32 + 16) = 2 \times 50 = 100$ cm²

(i) Is there more than one such cuboid?

Yes, there is more than one: the two cuboids above are different.

(ii) Can we find them all?

Yes. Exactly two cuboids exist: 1 × 2 × 16 and 2 × 4 × 7 (in cm).

(iii) Showing that these are all

Since $l \geq w \geq h$, each of lw, wh, lh is at least h^2, so $3h^2 \leq 50$, which gives $h \leq 4$. Check each case:

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h = 1: l + w + lw = 50 ⇒ (l + 1)(w + 1) = 51 = 3 × 17 (the factor pair 1 × 51 would give w = 0). So w = 2, l = 16.

h = 2: 2l + 2w + lw = 50 ⇒ (l + 2)(w + 2) = 54 with w + 2 ≥ 4. Only 6 × 9 works: w = 4, l = 7.

h = 3: (l + 3)(w + 3) = 59, a prime number, with w + 3 ≥ 6. No solution.

h = 4: (l + 4)(w + 4) = 66 with w + 4 ≥ 8. 66 has no factor pair with both factors ≥ 8 (66 = 6 × 11 = 3 × 22 = 2 × 33 = 1 × 66). No solution.

Therefore, there are exactly two such cuboids: 1 × 2 × 16 and 2 × 4 × 7 (in cm).

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14.2

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Exercise Set 14.2

Solutions · 5 questions

QUESTION 1

Two cylinders, A and B, are given. The radius of cylinder B is twice that of cylinder A, and the height of cylinder B is half that of cylinder A. Find the ratio of the curved surface area of A to the curved surface area of B. Also find the ratio of the volume of A to the volume of B.

ANSWER

Let cylinder A have radius r and height h. Then cylinder B has radius $2r$ and height $h/2$.

1 Curved surface area

CSA of A = $2\pi rh$

CSA of B = $2\pi(2r)(h/2) = 2\pi rh$

CSA of A : CSA of B = 1 : 1

2 Volume

V of A = $\pi r^2 h$

V of B = $\pi(2r)^2(h/2) = \pi \times 4r^2 \times h/2 = 2\pi r^2 h$

V of A : V of B = 1 : 2

Therefore, the ratio of curved surface areas is 1 : 1 and the ratio of volumes is 1 : 2.

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QUESTION 2

The radii of two cylinders are in the ratio 2:3, and their heights are in the ratio 3:2. Find (a) the ratio of their volumes, and (b) the ratio of their curved surface areas.

ANSWER

Let the radii be $2k$ and $3k$, and the heights $3m$ and $2m$.

(a) Ratio of volumes

$$V_1 : V_2 = \pi(2k)^2(3m) : \pi(3k)^2(2m) = 12k^2m : 18k^2m = 2 : 3$$

(b) Ratio of curved surface areas

$$CSA_1 : CSA_2 = 2\pi(2k)(3m) : 2\pi(3k)(2m) = 12km : 12km = 1 : 1$$

Therefore, (a) the ratio of volumes is 2 : 3 and (b) the ratio of curved surface areas is 1 : 1.

QUESTION 3

The edge of a cube measures r cm. The largest possible right circular cylinder is cut out of the cube. What do you think is the volume of the cylinder (in cm³)?

ANSWER

The largest cylinder has its base circle touching the sides of a face of the cube, so its diameter is r cm (radius $r/2$ cm), and its height equals the edge of the cube, r cm.

$$V = \pi(r/2)^2 \times r = \pi \times r^2/4 \times r = \pi r^3/4$$

Therefore, the volume of the cylinder is $\frac{\pi r^3}{4}$ cm³ (about 0.785 r³ cm³).

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QUESTION 4 *

The radius of the base of a cylinder is increased by 10%. At the same time, the height of the cylinder is decreased by x%. Given that the volume of the cylinder remains unchanged, find the value of x.

ANSWER

Let the original radius be r and height h. The new radius is $1.1r$ and the new height is $h(1 - x/100)$.

$$\pi(1.1r)^2 \times h(1 - x/100) = \pi r^2 h$$

$$1.21\,(1 - x/100) = 1$$

$$1 - x/100 = 1/1.21 = 100/121$$

$$x/100 = 21/121$$

$$x = 2100/121 \approx 17.36$$

Therefore, x = 2100/121 ≈ 17.36, i.e., the height is decreased by about 17.36%.

QUESTION 5

A solid metallic cube of side 12 cm is melted and recast into solid cylindrical rods, each having radius 2 cm and height 12 cm. Find:

(i) the volume of the cube,

(ii) the volume of one cylindrical rod,

(iii) the approximate number of complete cylindrical rods that can be formed.

($\pi \approx \frac{22}{7}$)

ANSWER

(i) Volume of the cube

$$V = 12^3 = 1728 \text{ cm}^3$$

(ii) Volume of one rod

$$V = \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 12 = 22 \times 48 \div 7 = 1056/7 \approx 150.86 \text{ cm}^3$$

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(iii) Number of rods

Number = $1728 \div (1056/7) = 1728 \times 7 \div 1056 = 12096 \div 1056 \approx 11.45$

Only complete rods are counted, so we take the whole-number part, 11.

Answers: (i) 1728 cm³; (ii) ≈ 150.86 cm³; (iii) 11 complete rods.

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14.3

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 14

Exercise Set 14.3

Solutions · 9 questions

Directions: For π, use one of the following approximations: $\pi \approx \frac{22}{7}$ or $\pi \approx 3.14$

QUESTION 1

Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

ANSWER

Slant height $l = 21$ m; radius $r = 24 \div 2 = 12$ m. Take $\pi = 22/7$.

$$\text{TSA} = \pi r(l + r) = \frac{22}{7} \times 12 \times (21 + 12)$$
$$= \frac{22}{7} \times 12 \times 33 = 8712 \div 7 \approx 1244.57 \text{ m}^2$$

Therefore, the total surface area of the cone is about 1244.57 m².

QUESTION 2

Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.

ANSWER

$$\text{CSA} = \pi r l = \frac{22}{7} \times 7 \times 10 = 220 \text{ cm}^2$$

Therefore, the curved surface area is 220 cm².

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QUESTION 3

The height of a cone is 16 cm, and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone.

ANSWER

Slant height: $l^2 = h^2 + r^2 = 16^2 + 12^2 = 256 + 144 = 400$, so $l = 20$ cm. Take $\pi = 22/7$.

$$\text{CSA} = \pi r l = \frac{22}{7} \times 12 \times 20 = 5280 \div 7 \approx 754.29 \text{ cm}^2$$

$$\text{TSA} = \pi r(l + r) = \frac{22}{7} \times 12 \times 32 = 8448 \div 7 \approx 1206.86 \text{ cm}^2$$

Therefore, CSA ≈ 754.29 cm² and TSA ≈ 1206.86 cm².

QUESTION 4

A cone has a height of 15 cm. If its volume is 1570 cm³, find the radius of the base.

ANSWER

Take $\pi = 3.14$.

$$V = \frac{1}{3}\pi r^2 h \Rightarrow 1570 = \frac{1}{3} \times 3.14 \times r^2 \times 15 = 15.7\, r^2$$

$$r^2 = 1570 \div 15.7 = 100$$

$$r = 10 \text{ cm}$$

Therefore, the radius of the base is 10 cm.

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QUESTION 5

The curved surface area of a cone is 308 cm² and its slant height is 14 cm. Find (i) the radius of the base, (ii) the total surface area of the cone.

ANSWER

(i) Radius of the base

$\pi rl = 308 \Rightarrow \frac{22}{7} \times r \times 14 = 308 \Rightarrow 44r = 308$

$r = 7$ cm

(ii) Total surface area

$TSA = \pi r(l + r) = \frac{22}{7} \times 7 \times (14 + 7) = 22 \times 21 = 462$ cm²

Answers: (i) 7 cm; (ii) 462 cm².

QUESTION 6

A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

ANSWER

Slant height: $l = \sqrt{(7^2 + 24^2)} = \sqrt{(49 + 576)} = \sqrt{625} = 25$ cm. The cap has no base, so only the curved surface is needed.

CSA of one cap $= \pi rl = \frac{22}{7} \times 7 \times 25 = 550$ cm²

Sheet for 10 caps $= 10 \times 550 = 5500$ cm²

Therefore, the area of sheet required is 5500 cm².

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QUESTION 7 *

What length of tarpaulin 3 m wide is required to make a conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that is required for stitching margins and wastage in cutting is 20 cm.

ANSWER

Slant height: $l = \sqrt{(8^2 + 6^2)} = \sqrt{100} = 10$ m. Take $\pi = 22/7$.

Area of tarpaulin (CSA) = $\pi rl = \frac{22}{7} \times 6 \times 10 = 1320/7 \approx 188.57$ m²

Length = area ÷ width = 188.57 ÷ 3 ≈ 62.86 m

Extra material = 20 cm = 0.2 m

Total length ≈ 62.86 + 0.2 = 63.06 m

Therefore, about 63.06 m of tarpaulin is required.

QUESTION 8

A right triangle with sides 6 cm, 8 cm and 10 cm is rotated through 360° about the side of 8 cm. Find the volume and the curved surface area of the solid so formed.

ANSWER

Rotating about the side of 8 cm gives a cone with height $h = 8$ cm, base radius $r = 6$ cm and slant height $l = 10$ cm (the hypotenuse). Take $\pi = 22/7$.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8 = 2112/7 \approx 301.71$ cm³

$CSA = \pi rl = \frac{22}{7} \times 6 \times 10 = 1320/7 \approx 188.57$ cm²

Therefore, the volume is about 301.71 cm³ (= 96π cm³) and the curved surface area is about 188.57 cm² (= 60π cm²).

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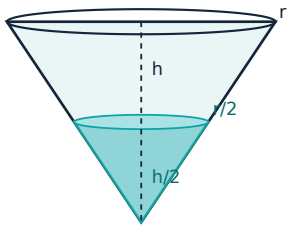
QUESTION 9 *

Suppose you have a cup in the shape of a right circular cone. Fill it with water to half the depth of the cone. What fraction of the volume of the cup is occupied by the water?



ANSWER

Let the cup have height h and radius r. The water forms a smaller cone of height $h/2$. By similar triangles, its radius is $r/2$.



$$\text{Volume of water} = \frac{1}{3}\pi(r/2)^2(h/2) = \frac{1}{8} \times \frac{1}{3}\pi r^2 h$$

$$\text{Volume of cup} = \frac{1}{3}\pi r^2 h$$

$$\text{Fraction} = \frac{1}{8}$$

Therefore, the water occupies $\frac{1}{8}$ of the volume of the cup.

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14.4

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 14

Exercise Set 14.4

Solutions · 8 questions

QUESTION 1

A ball bearing has a radius of 0.7 cm. Find its surface area.

ANSWER

Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times 0.7 \times 0.7$

$= 4 \times 22 \times 0.1 \times 0.7 = 6.16 \text{ cm}^2$

Therefore, the surface area is 6.16 cm².

QUESTION 2 *

Two solid spheres made of the same metal have weights 5920 g and 740 g. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

ANSWER

For the same metal, weight is proportional to volume, and volume is proportional to the cube of the radius.

$V_1 : V_2 = 5920 : 740 = 8 : 1$

$(r_1/r_2)^3 = 8 \Rightarrow r_1/r_2 = 2$

Radius of smaller sphere $= 5 \div 2 = 2.5$ cm

Radius of larger sphere $= 2 \times 2.5 = 5$ cm

Therefore, the radius of the larger sphere is 5 cm.

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QUESTION 3

The diameter of the moon is approximately one fourth the diameter of the earth. Given that the moon and earth are both roughly spherical, find the ratio of their surface areas.

ANSWER

Let the radius of the earth be R. The radius of the moon is $R/4$.

Surface area of moon : surface area of earth $= 4\pi(R/4)^2 : 4\pi R^2$

$= R^2/16 : R^2 = 1 : 16$

Therefore, the ratio of the surface areas (moon : earth) is 1 : 16.

QUESTION 4

Find (i) the curved surface area and (ii) the total surface area of a hemisphere of radius 21 cm.

ANSWER

Take $\pi = 22/7$.

(i) Curved surface area

$$\text{CSA} = 2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 = 2 \times 22 \times 3 \times 21 = 2772 \text{ cm}^2$$

(ii) Total surface area

$$\text{TSA} = 3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 = 3 \times 22 \times 3 \times 21 = 4158 \text{ cm}^2$$

Answers: (i) 2772 cm²; (ii) 4158 cm².

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QUESTION 5

Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed, the box is filled with preservative liquid. Find the volume of this liquid. Round your answer to the nearest integer.

ANSWER

Volume of box = $16 \times 8 \times 8 = 1024$ cm³

Volume of one sphere = $\frac{4}{3}\pi \times 2^3 = 32\pi/3$ cm³

Volume of 16 spheres = $16 \times 32\pi/3 = 512\pi/3 \approx 536.17$ cm³

Volume of liquid = $1024 - 536.17 \approx 487.83$ cm³ ≈ 488 cm³

(Using π = 22/7 gives 487.62, which also rounds to 488.)

Therefore, the volume of the liquid is about 488 cm³.

QUESTION 6

The radius of a sphere is increased by 10%. Show that the volume increases by approximately 33.1%.

ANSWER

Let the original radius be r. The new radius is $1.1r$.

Original volume $V = \frac{4}{3}\pi r^3$

New volume $V' = \frac{4}{3}\pi(1.1r)^3 = 1.331 \times \frac{4}{3}\pi r^3 = 1.331\ V$

Increase = $1.331V - V = 0.331V = 33.1\%$ of V

Therefore, the volume increases by 33.1% (exactly 33.1%, since $1.1^3 = 1.331$).

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QUESTION 7 *

The radius of a sphere is increased by x%. The volume of the sphere increases by 72.8%. Find the value of x.

ANSWER

New volume = 172.8% of original = 1.728 V

$(1 + x/100)^3 = 1.728$

$1 + x/100 = \sqrt[3]{1.728} = 1.2$ (since $1.2^3 = 1.728$)

$x/100 = 0.2 \Rightarrow x = 20$

Therefore, x = 20.

QUESTION 8

The hemispherical dome of a building needs to be painted (see Fig. 14.16). If the circumference of the base of the dome is 35.2 m, find the cost of painting it, given the cost of painting is ₹ 10 per 100 cm².

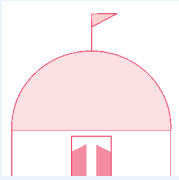

Fig. 14.16

ANSWER

Only the curved surface of the dome is painted. Take π = 22/7.

1 Radius

$2\pi r = 35.2 \Rightarrow r = 35.2 \times 7 \div (2 \times 22) = 246.4 \div 44 = 5.6$ m

2 Area to be painted

$\text{CSA} = 2\pi r^2 = 2 \times \frac{22}{7} \times 5.6 \times 5.6 = 197.12$ m²

197.12 m² = 197.12 × 10,000 cm² = 19,71,200 cm²

3 Cost

Number of 100 cm² units = 19,71,200 ÷ 100 = 19,712

Cost = 19,712 × ₹ 10 = ₹ 1,97,120

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Therefore, the cost of painting the dome is ₹ 1,97,120.

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CLASS 9 MATHS · GANITA MANJARI · CHAPTER 14

End-of-Chapter Exercises

Solutions · 22 questions

Directions: For guesstimate problems, make a guess first before solving.

QUESTION 1

Estimate how many scoops of ice cream can be obtained from a cuboidal container of dimensions 10 cm × 15 cm × 20 cm.



ANSWER

Guess: about 40 to 50 scoops. Now estimate. Assume a scoop is a ball of diameter 5 cm (radius 2.5 cm).

Volume of container $= 10 \times 15 \times 20 = 3000\ \text{cm}^3$

Volume of a scoop $= \frac{4}{3}\pi (2.5)^3 \approx 65.4\ \text{cm}^3$

Number of scoops $\approx 3000 \div 65.4 \approx 45.9$

Therefore, about 45 scoops can be obtained (the answer depends on the size of the scoop).

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QUESTION 2

A cube of integer side length a is made of unit cubes. Write an expression giving the number of unit cubes to be added to make a cube of side length a + 1.

ANSWER

Unit cubes in the cube of side a = a^3

Unit cubes in the cube of side a + 1 = $(a + 1)^3 = a^3 + 3a^2 + 3a + 1$

Cubes to be added = $(a + 1)^3 - a^3 = 3a^2 + 3a + 1$

Therefore, $3a^2 + 3a + 1$ unit cubes are to be added.

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QUESTION 3 *

Solve the following:

(i) Could a person drink enough water in a lifetime to fill an entire room the size of your classroom?

(ii) Estimate the number of bricks used to build the walls of your classroom.



ANSWER

These are estimation questions; the answer depends on the assumptions made, so we state them.

(i) Water in a lifetime

Assumed classroom: 8 m × 6 m × 3 m = 144 m^3 = 1,44,000 litres

A person drinks about 3 litres a day for about 80 years: 3 × 365 × 80 = 87,600 litres

87,600 litres is less than 1,44,000 litres (even 4 litres a day gives only 1,16,800 litres).

Answer: No, a person would not drink enough water in a lifetime to fill such a classroom.

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(ii) Number of bricks

Perimeter of walls = 2(8 + 6) = 28 m; area = 28 × 3 = 84 m^2

Subtract door and windows (about 10 m^2): 74 m^2

Wall thickness 23 cm = 0.23 m: volume of walls = 74 × 0.23 ≈ 17 m^3

Brick with mortar ≈ 20 cm × 10 cm × 10 cm = 2000 cm^3 = 0.002 m^3

Number of bricks ≈ 17 ÷ 0.002 ≈ 8500

Answer: about 8500 bricks (for the assumed classroom).

QUESTION 4

You are given two options: (A) one chocolate cube of side 50 mm, (B) hundred chocolate cubes of side 10 mm. Which option gives you more chocolate?

ANSWER

Option A: 50^3 = 1,25,000 mm^3

Option B: 100 × 10^3 = 100 × 1000 = 1,00,000 mm^3

Therefore, Option A gives more chocolate (1,25,000 mm^3 against 1,00,000 mm^3, i.e., 25% more).

QUESTION 5

If all the people in the world are crowded together in one location, how much area would it cover?



ANSWER

Assume the world population is about 8 billion (8 × 10^9) and that a tightly packed crowd has 4 people per square metre.

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Area = $8 \times 10^9 \div 4 = 2 \times 10^9$ m²

1 km² = 10^6 m², so area = 2000 km²

This is a square of side about $\sqrt{2000} \approx 45$ km

Therefore, the whole world population would cover roughly 2000 km² (an estimate).

QUESTION 6

A school provides milk to its students in cylindrical glasses, each with a diameter of 7 cm. If each glass is filled with milk to a height of 12 cm, how many litres of milk are needed to serve 1600 students? Use 1000 cm³ = 1 litre.

ANSWER

Radius = 7 ÷ 2 = 3.5 cm. Take π = 22/7.

Milk in one glass = $\pi r^2 h = \frac{22}{7} \times 3.5 \times 3.5 \times 12 = 462$ cm³

Milk for 1600 students = 1600 × 462 = 7,39,200 cm³

= 7,39,200 ÷ 1000 = 739.2 litres

Therefore, 739.2 litres of milk are needed.

QUESTION 7

The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and the volume of the cone.

ANSWER

Surface area of sphere = $4\pi \times 5^2 = 100\pi$ cm²

CSA of cone = $\pi r l = \pi \times 4 \times l = 4\pi l$

$100\pi = 5 \times 4\pi l \Rightarrow l = 5$ cm

Height: $h = \sqrt{(l^2 - r^2)} = \sqrt{(25 - 16)} = 3$ cm.

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$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 16 \times 3 = 16\pi \text{ cm}^3 \approx 50.3 \text{ cm}^3$$

Therefore, the height of the cone is 3 cm and its volume is 16π cm³ (≈ 50.3 cm³).

QUESTION 8

Take Earth to be a perfect sphere with radius 6370 km. Take Jupiter to be a perfect sphere with radius 69,900 km. Take the Sun to be a perfect sphere with radius 6,95,700 km. Compute approximately:

(i) the ratio of the volume of Jupiter to the volume of the Earth;

(ii) the ratio of the volume of the Sun to the volume of the Earth.
(Calculator can be used.)

ANSWER

Volume of a sphere is proportional to the cube of its radius, so the common factor $\frac{4}{3}\pi$ cancels.

(i) Jupiter : Earth

$$V(\text{Jupiter}) \div V(\text{Earth}) = (69{,}900 \div 6370)^3 \approx (10.973)^3 \approx 1321$$

(ii) Sun : Earth

$$V(\text{Sun}) \div V(\text{Earth}) = (6{,}95{,}700 \div 6370)^3 \approx (109.2)^3 \approx 13{,}03{,}000$$

Answers: (i) about 1321; (ii) about 13 lakh (≈ 1.3 × 10⁶).

QUESTION 9

Show that the volume of a sphere is equal to $\frac{2}{3}$ of the volume of the smallest cylinder which encloses it.

ANSWER

A sphere of radius r fits tightly in a cylinder of radius r and height $2r$; this is the smallest enclosing cylinder.

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Volume of cylinder = $\pi r^2 \times 2r = 2\pi r^3$

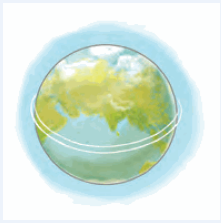
Volume of sphere = $\frac{4}{3}\pi r^3$

Sphere ÷ cylinder = $\frac{4}{3} \div 2 = \frac{2}{3}$

Therefore, the volume of the sphere is $\frac{2}{3}$ of the volume of the smallest enclosing cylinder.

QUESTION 10 *

Suppose the Earth is perfectly spherical. A string is wrapped tightly around the equator of the Earth. Another string, 1 metre longer, is placed around the Earth so that it forms a larger circle, staying the same distance above the ground everywhere.
How high above the ground is the second string?
Repeat this for: (i) the Moon (ii) Jupiter (iii) a volleyball. Are you surprised by the three answers? Why or why not?



ANSWER

Let the radius of the body be R and the height of the second string above the ground be h.

First string: $2\pi R$

Second string: $2\pi(R + h) = 2\pi R + 1$

$2\pi h = 1 \Rightarrow h = 1 \div (2\pi) \approx 0.159$ m

The radius R cancels, so the answer does not depend on the size of the body.

Earth: h ≈ 0.159 m ≈ 15.9 cm

(i) Moon: h ≈ 15.9 cm

(ii) Jupiter: h ≈ 15.9 cm

(iii) Volleyball: h ≈ 15.9 cm

Surprise? It is surprising that adding just 1 metre to a string around the whole Earth lifts it about 16 cm everywhere, and that a volleyball gives the same height. But the reason is simple: the circumference is 2π times the radius, so a change of 1 metre in circumference always changes the radius by 1 ÷ 2π, whatever the radius.

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Therefore, the second string is about 15.9 cm above the ground in all four cases.

QUESTION 11 *

We have a cylinder with a base radius of r cm and height h cm. A square pyramid is fitted inside it. The square base of the pyramid lies on the base of the cylinder, its corners on the boundary of the cylinder. The apex of the pyramid lies on the top of the cylinder. Find the ratio of the volume of this pyramid to the volume of the cylinder.

ANSWER

The square base is inscribed in the circular base, so its diagonal equals the diameter $2r$. If the side is s, then $s\sqrt{2} = 2r$, so

$$\text{Area of square base} = s^2 = (2r)^2 \div 2 = 2r^2$$

The height of the pyramid is h (the apex is on the top of the cylinder).

$$\text{Volume of pyramid} = \frac{1}{3} \times 2r^2 \times h = \frac{2r^2h}{3}$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Ratio} = \frac{2r^2h}{3} \div \pi r^2 h = \frac{2}{3\pi}$$

Therefore, the ratio of the volume of the pyramid to that of the cylinder is 2 : 3π.

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QUESTION 12

What is the change in volume when:

(i) the length of a cuboid with dimensions l, w, h is increased by 1 unit?
(a) 1 cubic unit (b) (lwh + 1) cubic unit (c) wh cubic units (d) lw cubic units (e) lh cubic units

(ii) the radius of a cylinder with dimensions r, h is increased by 1 unit?
(a) 1 cubic unit (b) $\pi r^2 h$ cubic units (c) πr^2 cubic units (d) $2\pi rh + 2\pi h$ cubic units (e) $2\pi rh + \pi h$ cubic units

***(iii) the radius of a sphere is decreased by 1 unit?**

ANSWER

(i) Cuboid: length increased by 1

New volume − old volume = $(l + 1)wh - lwh = wh$

Answer: (c) wh cubic units.

(ii) Cylinder: radius increased by 1

$\pi(r + 1)^2 h - \pi r^2 h = \pi h[(r + 1)^2 - r^2] = \pi h(2r + 1)$
$= 2\pi rh + \pi h$

Answer: (e) $2\pi rh + \pi h$ cubic units.

***(iii) Sphere: radius decreased by 1**

$$\text{Change} = \frac{4}{3}\pi(r - 1)^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi[(r - 1)^3 - r^3]$$
$$(r - 1)^3 - r^3 = -3r^2 + 3r - 1$$
$$\text{Change} = -\frac{4}{3}\pi(3r^2 - 3r + 1)$$

Answer: the volume decreases by $\frac{4}{3}\pi(3r^2 - 3r + 1)$ cubic units (equivalently, $4\pi r^2 - 4\pi r + \frac{4\pi}{3}$ cubic units).

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QUESTION 13 *

Given a cube with volume V, express its total surface area S in terms of V.

ANSWER

Let the side be a, so $V = a^3$ and $a = V^{1/3}$.

$$S = 6a^2 = 6\,(V\text{^}(1/3))^2 = 6V\text{^}(2/3)$$

Therefore, $S = 6V^{2/3}$ (that is, $6 \times \sqrt[3]{(V^2)}$).

QUESTION 14 *

Given a cube with total surface area S, express its volume V in terms of S.

ANSWER

Let the side be a. Then $S = 6a^2$, so $a = \sqrt{(S/6)}$.

$$V = a^3 = (S/6)\text{^}(3/2) = (S/6)\sqrt{(S/6)} = S\sqrt{S} \div (6\sqrt{6})$$

Therefore, $V = (S/6)^{3/2}$.

QUESTION 15

A cylindrical glass of height 25 cm and radius 4 cm has water up to a height of 16 cm. A crow wants to drink water from this glass. The water must be at a height of 20 cm for the crow to reach it. There are some marbles lying around. How many marbles of radius 1 cm should the crow drop into the glass to make the water reach the required height?



ANSWER

The water must rise by $20 - 16 = 4$ cm. The volume of the marbles dropped in must equal the volume of this extra water layer.

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Volume of extra water = $\pi r^2 h = \pi \times 4^2 \times 4 = 64\pi$ cm³

Volume of one marble = $\frac{4}{3} \pi \times 1^3 = \frac{4\pi}{3}$ cm³

Number of marbles = $64\pi \div (4\pi/3) = 64 \times 3 \div 4 = 48$

The glass is 25 cm high, so the water (20 cm) does not overflow.

Therefore, the crow should drop 48 marbles.

QUESTION 16

Find the volume of ink in a new ball point pen. Take the necessary measurements and make approximations, as needed.

ANSWER

The ink is stored in a thin cylindrical tube (the refill). Sample measurements (measure your own pen in the same way):

Length of the ink column in the refill: h ≈ 10 cm

Inner diameter of the tube ≈ 2 mm, so radius r = 1 mm = 0.1 cm

$V = \pi r^2 h = 3.14 \times 0.1 \times 0.1 \times 10 \approx 0.31$ cm³

Therefore, a new ball point pen contains about 0.3 cm³ (0.3 mL) of ink, for these measurements.

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QUESTION 17

Solve:

(i) A ball of chapati dough of radius 6 cm is prepared. Estimate how many chapatis can be made from it?

***(ii) Cut a coconut/muskmelon in half and find out the approximate volume of edible coconut flesh/fruit by taking the necessary measurements.**



ANSWER

(i) Chapatis from the dough ball

Volume of dough $= \frac{4}{3}\pi \times 6^3 = 288\pi \approx 904.8$ cm³

Assume a chapati is a thin disc of radius 8 cm and thickness 2 mm = 0.2 cm

Volume of a chapati $= \pi \times 8^2 \times 0.2 = 12.8\pi \approx 40.2$ cm³

Number of chapatis $\approx 288\pi \div 12.8\pi = 22.5$

Answer: about 22 chapatis (for these assumptions).

*(ii) Edible part of half a coconut

Each half is a hemispherical shell. Sample measurements (measure your own): outer radius R = 7 cm, flesh thickness 1 cm, so inner radius r = 6 cm.

Volume of flesh $= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3}\pi(343 - 216) = \frac{2}{3}\pi \times 127$

≈ 266 cm³ for each half

Answer: about 266 cm³ of flesh in one half (about 532 cm³ in the whole coconut), for these measurements.

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QUESTION 18

Looking at the Ganita Manjari, Grade 9, Part 2 textbook, Sheela wonders:

(i) If all the pages of this textbook were laid out side-by-side on the floor would they cover the entire classroom floor?

(ii) What is the maximum number of textbooks that can fit in an empty storeroom of dimensions 15 ft × 20 ft × 30 ft?

ANSWER

Measure your own book. Assumptions: each page is about 20 cm × 27 cm; the book has about 200 pages and is about 1 cm thick; the classroom floor is 8 m × 6 m; 1 ft ≈ 30 cm.

(i) Do the pages cover the floor?

Area of one page = $20 \times 27 = 540$ cm²

Area of 200 pages = $200 \times 540 = 1{,}08{,}000$ cm² = 10.8 m²

Classroom floor = $8 \times 6 = 48$ m²

Answer: No. The pages cover only about 10.8 m², less than a quarter of the floor.

(ii) Textbooks in the storeroom

Storeroom = 15 ft × 20 ft × 30 ft ≈ 450 cm × 600 cm × 900 cm = 24,30,00,000 cm³

Volume of one book = $20 \times 27 \times 1 = 540$ cm³ (volume check: $24{,}30{,}00{,}000 \div 540 = 4{,}50{,}000$ is an upper bound)

Packing along the edges:

$450 \div 20 \rightarrow 22$ books; $600 \div 27 \rightarrow 22$ books; $900 \div 1 \rightarrow 900$ books

Number = $22 \times 22 \times 900 = 4{,}35{,}600$

Answer: about 4.4 lakh textbooks (4,35,600), for these measurements.

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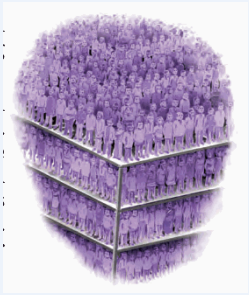
QUESTION 19 *

If the entire human population decided to climb into one giant cube, how long would the side have to be?



ANSWER

Assume the population is about 8×10^9. People stand on floors 2 m apart, 4 per m² (as in Question 5), so each person needs 0.25 m² × 2 m = 0.5 m³.

Volume of the cube = $8 \times 10^9 \times 0.5 = 4 \times 10^9$ m³

Side = $\sqrt[3]{(4 \times 10^9)} \approx 1587$ m ≈ 1.6 km

Therefore, the side of the cube would have to be about 1.6 km (an estimate).

QUESTION 20 *

The Earth's surface has an estimated volume of 1.38 billion km³ of water. Suppose the Earth is a perfect sphere and all this water forms a uniform layer completely covering the Earth's surface, like a thin water bubble. Estimate the thickness of this water layer. The Earth's radius is ~6371 km.

(i) Write an expression that gives the thickness of this water layer.

(ii) Simplify the expression in (i) using a calculator.

ANSWER

Let R = 6371 km, $V = 1.38 \times 10^9$ km³ and let the thickness be t.

(i) Expression for the thickness

The water forms a spherical shell between radius R and R + t:

$$\frac{4}{3}\pi(R + t)^3 - \frac{4}{3}\pi R^3 = V$$

$$(R + t)^3 = R^3 + 3V \div 4\pi$$

$$t = \sqrt[3]{(R^3 + 3V \div 4\pi)} - R$$

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(ii) Simplified value

$$t = \sqrt[3]{(6371^3 + 3 \times 1.38 \times 10^9 \div 4\pi)} - 6371 \approx 2.70 \text{ km}$$

(Since t is very small compared with R, the approximation $t \approx V \div 4\pi R^2 = 1.38 \times 10^9 \div (5.10 \times 10^8) \approx 2.71$ km gives almost the same answer.)

Answers: (i) $t = \sqrt[3]{(R^3 + 3V/4\pi)} - R$; (ii) $t \approx 2.7$ km.

QUESTION 21 *

Give the dimension of a cuboid whose volume is halved when its surface area is doubled.

ANSWER

The question asks for a cuboid that can be changed into another cuboid having half the volume and double the surface area. Many such pairs exist; one pair is found below.

Original cuboid: 8 cm × 12 cm × 16 cm

Volume = $8 \times 12 \times 16 = 1536$ cm³

Surface area = $2(8 \times 12 + 12 \times 16 + 8 \times 16) = 2(96 + 192 + 128) = 832$ cm²

New cuboid: 1 cm × 16 cm × 48 cm

Volume = $1 \times 16 \times 48 = 768$ cm³ = $1536 \div 2$

Surface area = $2(1 \times 16 + 16 \times 48 + 1 \times 48) = 2(16 + 768 + 48) = 1664$ cm² = 2×832

Another pair: 8 × 14 × 14 (volume 1568, surface area 840) and 1 × 28 × 28 (volume 784, surface area 1680).

Therefore, a cuboid of dimensions 8 cm × 12 cm × 16 cm becomes a cuboid of 1 cm × 16 cm × 48 cm, whose volume is halved while its surface area is doubled.

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QUESTION 22

Project: Find the volume of your house making necessary approximations. Present how you solved it.

ANSWER

Method: treat each room as a cuboid, measure its length, width and height, find its volume with V = l × w × h, and add the volumes. Sample measurements are given below (use those of your own house).

Hall: 4 m × 4 m × 3 m = 48 m^3

Bedroom: 4 m × 3.5 m × 3 m = 42 m^3

Kitchen: 3 m × 3 m × 3 m = 27 m^3

Bathroom: 2 m × 1.5 m × 3 m = 9 m^3

Total = 48 + 42 + 27 + 9 = 126 m^3

Approximations: walls, doors and furniture are ignored, and every room is taken as a perfect cuboid. If the thickness of the walls is included, the volume of the house is slightly larger.

Therefore, for these measurements, the volume of the house is about 126 m^3 (1,26,000 litres).

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