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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 13

Two Variables, One Line

Complete Exercise Solutions

Exercise Sets 13.1 – 13.5 & End-of-Chapter Exercises

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Every question is followed by its answer. Question numbers above are clickable and the PDF bookmarks panel lets you jump to any exercise set or question. T1, T2… are the “Think and Reflect” in-text questions.

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IT

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 13

In-Text Questions, Examples & Activities

17 Think and Reflect · 13 Examples · 1 Activity

Solutions to the Think and Reflect questions, worked Examples and the graphing Activity, in the order in which they appear in the chapter.

EXAMPLE 1

Write each of the following equations in standard form, $ax + by + c = 0$, and indicate the values of a, b and c in each case: (i) $5x + 2y = 2.7$ (ii) $\frac{x}{2} - 4 = \frac{3}{2}y$ (iii) $4 = 1.5x - 1.3y$ (iv) $3y = 5$

ANSWER

(i) 5x + 2y = 2.7

$5x + 2y - 2.7 = 0$, so $a = 5$, $b = 2$, $c = -2.7$. (Also $-5x - 2y + 2.7 = 0$ with $a = -5$, $b = -2$, $c = 2.7$.)

(ii) x/2 – 4 = 3y/2

$\frac{x}{2} - \frac{3}{2}y - 4 = 0$, so $a = \frac{1}{2}$, $b = -\frac{3}{2}$, $c = -4$.

(iii) 4 = 1.5x – 1.3y

$1.5x - 1.3y - 4 = 0$, so $a = 1.5$, $b = -1.3$, $c = -4$. (Also $-1.5x + 1.3y + 4 = 0$.)

(iv) 3y = 5

$0 \times x + 3y - 5 = 0$, so $a = 0$, $b = 3$, $c = -5$.

Therefore, each equation is written in the form $ax + by + c = 0$ with the values above.

THINK AND REFLECT 1

Are $(1, \frac{9}{2})$, (0, 6), (4, 1) and $(\frac{1}{2}, \frac{21}{4})$ solutions of the equation $3x + 2y = 12$? Can you find any other solution? How many solutions can you find?

ANSWER

Substitute each ordered pair in $3x + 2y$ and compare with 12.

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Ordered pair	$3x + 2y$	Solution?
$(1, \frac{9}{2})$	$3 + 9 = 12$	Yes
$(0, 6)$	$0 + 12 = 12$	Yes
$(4, 1)$	$12 + 2 = 14 \neq 12$	No
$(\frac{1}{2}, \frac{21}{4})$	$\frac{3}{2} + \frac{21}{2} = 12$	Yes

Other solutions: choose any x, then find y. For $x = 2$, $6 + 2y = 12$ gives $y = 3$, so $(2, 3)$. For $x = 6$, $18 + 2y = 12$ gives $y = -3$, so $(6, -3)$. Similarly $(4, 0)$ and $(-2, 9)$ are solutions.

Therefore, $(1, \frac{9}{2})$, $(0, 6)$ and $(\frac{1}{2}, \frac{21}{4})$ are solutions, $(4, 1)$ is not. There are infinitely many solutions, as x can be given any value.

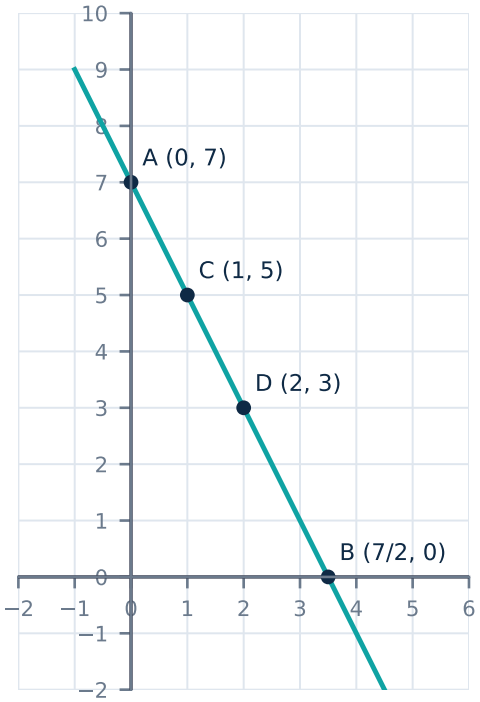
EXAMPLE 2

Find any two solutions of $2x + y = 7$. Draw the graph of this equation. Use the graph to find 4 more solutions.

ANSWER

Put $x = 0$: $y = 7$, giving $(0, 7)$. Put $y = 0$: $2x = 7$, $x = \frac{7}{2}$, giving $(\frac{7}{2}, 0)$.

x	0	$\frac{7}{2}$
y	7	0



Graph of $2x + y = 7$

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Reading points on the line: (1, 5), (2, 3), (3, 1) and $(\frac{3}{2}, 4)$. Check: $2(1) + 5 = 7$ ✓, $2(2) + 3 = 7$ ✓, $2(3) + 1 = 7$ ✓, $2(\frac{3}{2}) + 4 = 7$ ✓.

Therefore, two solutions are (0, 7) and $(\frac{7}{2}, 0)$; four more are (1, 5), (2, 3), (3, 1), $(\frac{3}{2}, 4)$.

EXAMPLE 3

Find any two solutions for each of the following equations and draw their graphs: (i) $4x + 3y = 12$ (ii) $2x + 5y = 0$

ANSWER

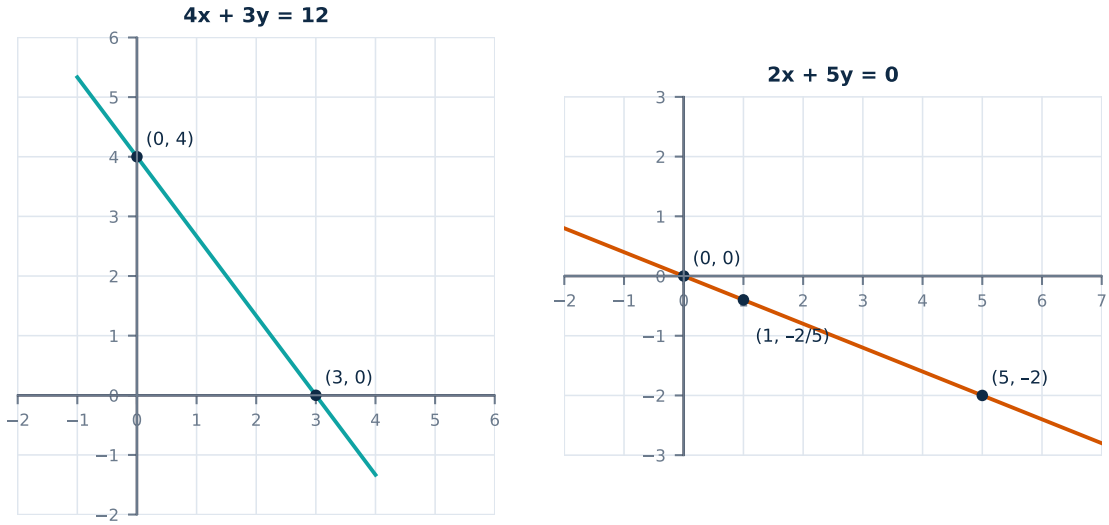
(i) $4x + 3y = 12$

$x = 0$ gives $3y = 12$, $y = 4$: (0, 4). $y = 0$ gives $x = 3$: (3, 0).

(ii) $2x + 5y = 0$

$x = 0$ gives $y = 0$: (0, 0). $x = 1$ gives $y = -\frac{2}{5}$: $(1, -\frac{2}{5})$. $x = 5$ gives $y = -2$: (5, –2).

x	0	1	5
y	0	$-\frac{2}{5}$	–2



Graphs of the two equations

Therefore, (i) (0, 4) and (3, 0); (ii) (0, 0) and $(1, -\frac{2}{5})$ (also (5, –2)). Each pair of points gives the line shown.

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EXAMPLE 4

In the equations $3ax + 4y = -2$ and $2x + by = 14$, a and b are unknown numbers. If (–3, 4) is the solution of both equations, find the values of a and b.

ANSWER

$3a(-3) + 4(4) = -2$, so $-9a + 16 = -2$, $-9a = -18$, $a = 2$
$2(-3) + b(4) = 14$, so $-6 + 4b = 14$, $4b = 20$, $b = 5$

Therefore, $a = 2$ and $b = 5$.

THINK AND REFLECT 2

Ranju, Farhan, Toshi and Meena conclude: whenever $c = 0$, the line $ax + by + c = 0$ will always pass through the origin, irrespective of the values of a or b. Do you agree with them?

ANSWER

✓ **Check the claim** YES, WE AGREE

If $c = 0$ the equation is $ax + by = 0$. Putting $x = 0$ and $y = 0$ gives $a(0) + b(0) = 0$, which is true for every a and b. So (0, 0) always lies on the line.

This also covers the special cases: for $b = c = 0$ the equation is $x = 0$ (the y-axis) and for $a = c = 0$ it is $y = 0$ (the x-axis); both pass through the origin. (a and b cannot both be zero.)

Therefore, the friends' conclusion is correct: a line $ax + by + c = 0$ passes through the origin if and only if $c = 0$.

THINK AND REFLECT 3

Consider the line AB through A (2, 0) and B (4, 3).
1. Consider the point on the line AB whose x-coordinate is 8. What is the y-coordinate of this point? Express this as an ordered pair.
2. What is the x-coordinate of the point on the line AB whose y-coordinate is 6? Express this as an ordered pair.

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ANSWER

Slope of AB = $\frac{3-0}{4-2} = \frac{3}{2}$. For any point (x, y) on the line, the slope between A and that point is also $\frac{3}{2}$: $\frac{y-0}{x-2} = \frac{3}{2}$, i.e. $y = \frac{3}{2}(x - 2)$.

1 **$x = 8$**

$y = \frac{3}{2}(8 - 2) = \frac{3}{2} \times 6 = 9$. The point is (8, 9).

2 **$y = 6$**

$6 = \frac{3}{2}(x - 2)$ gives $x - 2 = 4$, so $x = 6$. The point is (6, 6).

Therefore, the ordered pairs are (8, 9) and (6, 6) (these are the points D and C of Fig. 13.8).

EXAMPLE 5

Consider the straight line that passes through A (–1, 0), B (1, 2) and C (4, 5). Plot the points and find the slope of the line.

ANSWER

Take B (1, 2) and C (4, 5): slope = $\frac{\text{Rise}}{\text{Run}} = \frac{5-2}{4-1} = \frac{3}{3} = 1$. Using A and B: $(2 - 0) \div (1 - (-1)) = 2 \div 2 = 1$, the same.

Therefore, the slope of the line is 1 (the line rises 1 unit for every 1 unit to the right).

EXAMPLE 6

Consider the line passing through A (–4, 5) and B (–1, 2). Find the slope of the line.

ANSWER

A as (x_1, y_1), B as (x_2, y_2): slope = $(2 - 5)/(-1 - (-4)) = -3/3 = -1$
B first, then A: slope = $(5 - 2)/(-4 - (-1)) = 3/(-3) = -1$

Therefore, the slope is –1, and it does not depend on the order in which the points are taken.

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THINK AND REFLECT 4

Can you use the points A (0, 3), B (1, –2) and C (–1, 8) to verify that the slope of the line $y = -5x + 3$ is indeed –5?

ANSWER

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$ for any two points on the line.

A, B: $(-2 - 3)/(1 - 0) = -5$
B, C: $(8 - (-2))/(-1 - 1) = 10/(-2) = -5$
A, C: $(8 - 3)/(-1 - 0) = 5/(-1) = -5$

Therefore, every pair of the points gives slope –5, as expected from $m = -5$ in $y = -5x + 3$.

THINK AND REFLECT 5

The cost of one paratha and 2 bowls of *dahi* is ₹90. Also, the cost of 2 parathas and 7 bowls of *dahi* is ₹260. Can you figure out the cost of one paratha and one bowl of *dahi* and explain your reasoning?

ANSWER

Let one paratha cost ₹p and one bowl of *dahi* cost ₹d.

$p + 2d = 90$ ...(1)
$2p + 7d = 260$ ...(2)

Multiply (1) by 2: $2p + 4d = 180$ ...(3). Subtract (3) from (2): $3d = 80$, so $d = \frac{80}{3}$.

From (1): $p = 90 - 2 \times \frac{80}{3} = \frac{270 - 160}{3} = \frac{110}{3}$.

Check: $\frac{110}{3} + \frac{160}{3} = 90$ ✓ and $\frac{220}{3} + \frac{560}{3} = 260$ ✓.

Therefore, one paratha costs ₹$\frac{110}{3}$ ≈ ₹36.67 and one bowl of *dahi* costs ₹$\frac{80}{3}$ ≈ ₹26.67 (together ₹$\frac{190}{3}$ ≈ ₹63.33).

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THINK AND REFLECT 6

Group A: 2 adult tickets and 3 child tickets for ₹600. Group B: 3 adult tickets and 2 child tickets for ₹700. With adult ticket ₹x and child ticket ₹y: $2x + 3y = 600$, $3x + 2y = 700$. A solution to these equations will give us the answer that we seek. Can you see why?

ANSWER

Both groups paid at the same prices, so the actual prices must make both statements true at once. An ordered pair (x, y) that satisfies both equations is exactly such a price pair. Solving:

Add: $5x + 5y = 1300$, so $x + y = 260$
Subtract: $x - y = 100$
So $x = 180$ and $y = 80$

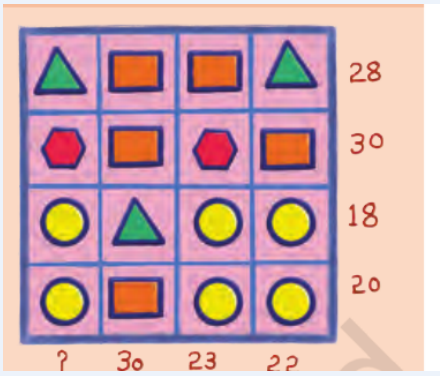
Check: $2(180) + 3(80) = 600$ ✓ and $3(180) + 2(80) = 700$ ✓.

Therefore, an adult ticket costs ₹180 and a child ticket costs ₹80.

THINK AND REFLECT 7

Solve the puzzle and discuss your strategy. Can you determine the value represented by each shape?

Rows give 28, 30, 18, 20 and columns give ?, 30, 23, 22. Row 1: △ ▭ ▭ △; Row 2: ⬣ ▭ ⬣ ▭; Row 3: ○ △ ○ ○; Row 4: ○ ▭ ○ ○.



ANSWER

Let T, R, C, H be the values of a triangle, rectangle, circle and hexagon.

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Row 1: 2T + 2R = 28, so T + R = 14
Column 4: T + R + 2C = 22, so 14 + 2C = 22 and C = 4
Row 3: 3C + T = 18, so T = 18 – 12 = 6
Row 4: 3C + R = 20, so R = 20 – 12 = 8
Row 2: 2H + 2R = 30, so H = 15 – 8 = 7
Column 1: T + H + 2C = 6 + 7 + 8 = 21

Check: T + R = 6 + 8 = 14 ✓; column 2: 3R + T = 24 + 6 = 30 ✓; column 3: R + H + 2C = 8 + 7 + 8 = 23 ✓.

Therefore, triangle = 6, rectangle = 8, circle = 4, hexagon = 7, and the missing column total (?) is 21.

EXAMPLE 7

Solve the following pair of equations by the substitution method: $7x - 15y = 2$ (1); $x + 2y = 3$ (2).

ANSWER

1 Express x in terms of y from (2)

$x = 3 - 2y$ ...(3)

2 Substitute in (1)

$7(3 - 2y) - 15y = 2$
$21 - 14y - 15y = 2$
$-29y = -19$, so $y = \frac{19}{29}$

3 Find x from (3)

$x = 3 - 2(\frac{19}{29}) = \frac{87 - 38}{29} = \frac{49}{29}$.

Check: $7(\frac{49}{29}) - 15(\frac{19}{29}) = \frac{343 - 285}{29} = 2$ ✓; $\frac{49}{29} + \frac{38}{29} = 3$ ✓.

Therefore, $x = \frac{49}{29}$ and $y = \frac{19}{29}$.

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THINK AND REFLECT 8

Verify the solution $x = \frac{49}{29}$, $y = \frac{19}{29}$ of Example 7 in both equations: $7x - 15y = 2$ and $x + 2y = 3$.

What if we had expressed y in terms of x, i.e. $y = \frac{1}{2}(3 - x)$ from Equation (2)? Would we still get the same solution?

ANSWER

$7x - 15y = (7\times49 - 15\times19)/29 = (343 - 285)/29 = 58/29 = 2$ ✓
$x + 2y = (49 + 2\times19)/29 = 87/29 = 3$ ✓

Using $y = \frac{1}{2}(3 - x)$ in Equation (1): $7x - \frac{15}{2}(3 - x) = 2$. Multiply by 2: $14x - 45 + 15x = 4$, so $29x = 49$ and $x = \frac{49}{29}$. Then $y = \frac{1}{2}(3 - \frac{49}{29}) = \frac{1}{2} \times \frac{38}{29} = \frac{19}{29}$.

Therefore, the solution is verified, and yes, we get the same solution x = 49/29, y = 19/29 whichever variable we express in terms of the other.

EXAMPLE 8

If two angles of a triangle are $x°$ and $y°$ with $y = 4x$, and the third angle is 50°, what are the angles?

ANSWER

$x + y + 50 = 180$ (angle sum)
$x + 4x + 50 = 180$, so $5x = 130$ and $x = 26$
$y = 4 \times 26 = 104$

Check: 26 + 104 + 50 = 180 ✓.

Therefore, the angles are 26°, 104° and 50°.

EXAMPLE 9

The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹2000 per month, find their monthly incomes.

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ANSWER

Incomes $9x$ and $7x$; expenditures $4y$ and $3y$.

$$9x - 4y = 2000 \quad ...(1)$$
$$7x - 3y = 2000 \quad ...(2)$$

Multiply (1) by 3 and (2) by 4: $27x - 12y = 6000$ and $28x - 12y = 8000$. Subtract: $x = 2000$. From (1): $18000 - 4y = 2000$, so $y = 4000$.

Incomes: $9 \times 2000 = ₹18{,}000$ and $7 \times 2000 = ₹14{,}000$. Check: expenditures ₹16,000 and ₹12,000 (ratio 4 : 3), savings ₹2000 each ✓.

Therefore, the monthly incomes are ₹18,000 and ₹14,000.

THINK AND REFLECT 9

In Example 9 ($9x - 4y = 2000$ and $7x - 3y = 2000$), try the same problem by eliminating x instead of y. Also try to solve the two equations using the Substitution Method. Assess the pros and cons of each method.

ANSWER

1 Eliminating x

Multiply (1) by 7 and (2) by 9: $63x - 28y = 14000$ and $63x - 27y = 18000$. Subtract the first from the second: $y = 4000$. Then $9x = 2000 + 4 \times 4000 = 18000$, so $x = 2000$.

2 Substitution

From (1): $x = \frac{2000 + 4y}{9}$. Put in (2): $7(2000 + 4y)/9 - 3y = 2000$, i.e. $14000 + 28y - 27y = 18000$, so $y = 4000$ and $x = 2000$.

Pros and cons. Elimination avoids fractions and is quick when coefficients can easily be matched. Substitution gave the fraction $\frac{2000 + 4y}{9}$ and needed more arithmetic here, but it is convenient when one equation already has a variable with coefficient 1 (as in Example 7).

Therefore, both methods give $x = 2000$, $y = 4000$ (incomes ₹18,000 and ₹14,000).

EXAMPLE 10

The sum of a two-digit number and the number obtained by reversing its digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

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ANSWER

Let the tens and units digits be x and y with x the larger digit. The number is $10x + y$; reversed, $10y + x$.

$(10x + y) + (10y + x) = 66$, so $x + y = 6$ ...(1)
$x - y = 2$ ...(2)

Adding: $2x = 8$, so $x = 4$ and $y = 2$. The number is 42; 42 + 24 = 66 ✓ and 4 – 2 = 2 ✓.

Therefore, the number is 42 (with the larger digit in the tens place). Its reverse 24 also satisfies both conditions, so the two numbers are 42 and 24.

THINK AND REFLECT 10

Does a pair of linear equations always have a unique solution? Can you analyse this using the Elimination Method? What if the second variable gets eliminated too while eliminating the first variable? Can you think of a situation where this happens?

ANSWER

No. In elimination, if after subtracting one variable remains, we get a linear equation in it and hence a unique solution. If both variables vanish, we are left with a statement like 0 = 0 (true for all values) or 0 = a non-zero number (never true).

Example: $x - y = 10$ and $10x - 10y = 100$. Multiplying the first by 10 and subtracting leaves 0 = 0, so there are infinitely many solutions. With $10x - 10y = 101$ we get 0 = 1, so there is no solution.

Therefore, a pair of linear equations need not have a unique solution; it can also have infinitely many solutions or none.

THINK AND REFLECT 11

How many solutions are there to the equations $10x - 5y = 20$ and $4x - 2y = 8$? Give 3 more examples of pairs of equations that lead to 0 = 0 after subtraction, and therefore have infinitely many solutions. Can you find a simple rule to check when this happens?

ANSWER

Divide the first equation by 5 and the second by 2: both become $2x - y = 4$. They are the same equation, so every solution of one is a solution of the other.

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Three more examples: (a) $x + y = 3$ and $2x + 2y = 6$; (b) $x - 2y = 1$ and $3x - 6y = 3$; (c) $2x + 3y = 5$ and $4x + 6y = 10$. In each, one equation is a constant multiple of the other.

Therefore, $10x - 5y = 20$ and $4x - 2y = 8$ have infinitely many solutions. Rule: this happens when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

THINK AND REFLECT 12

What if one of a_2, b_2, c_2 is zero (in the condition $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$)?

ANSWER

Then a ratio with that denominator is not defined, so we use the equivalent statement that the equations are proportional: $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$ for some $k \neq 0$. If, say, $c_2 = 0$, this forces $c_1 = 0$ as well, and the equations are the same line when $a_1b_2 = a_2b_1$. Similarly if $a_2 = 0$ then $a_1 = 0$.

Therefore, when a denominator is zero we compare the equations by proportionality (cross-multiplication) rather than by ratios.

THINK AND REFLECT 13

Does a pair of linear equations always have either a unique solution or infinitely many solutions?

ANSWER

No. When $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ the pair has no solution. Example: $x - y = 10$ and $10x - 10y = 101$ lead to $0 = 1$, which is impossible.

Therefore, a pair can have a unique solution, infinitely many solutions, or no solution.

THINK AND REFLECT 14

Give 3 more examples of pairs of equations that have no solution. Can you find a simple rule to check when this will happen?

ANSWER

Examples: (a) $x + y = 2$ and $2x + 2y = 5$; (b) $x - 2y = 3$ and $2x - 4y = 7$; (c) $3x + y = 1$ and $6x + 2y = 5$. In each pair the left sides are proportional but the right sides are not (e.g. $2x + 2y$ would have to equal 4).

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Therefore, a pair has no solution when $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

THINK AND REFLECT 15

Are the converses of the three statements true? (1) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair has a unique solution. (2) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, it has infinitely many solutions. (3) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, it has no solution.

ANSWER

The three cases cover all possibilities and cannot hold together. Suppose a pair has a unique solution; if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ held, the pair would have either none or infinitely many solutions, a contradiction. So $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. In the same way, infinitely many solutions force all three ratios to be equal, and no solution forces the ratio pattern of (3).

Therefore, the converses of (1), (2) and (3) are all true (for a_2, b_2, c_2 not zero).

THINK AND REFLECT 16

Can there be methods other than elimination and substitution to reduce a pair of linear equations in two variables to a linear equation in one variable? If so, describe them.

ANSWER

Graphical method: plot both lines; the common point (if any) gives the solution. **Cross-multiplication:** for $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ with $a_1b_2 - a_2b_1 \neq 0$, the solution is $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$. Other ways: adding/subtracting the equations directly when coefficients are interchanged (as in Exercise 13.5 Q7), and matrix/determinant methods.

Therefore, yes: besides substitution and elimination there are the graphical and cross-multiplication methods.

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ACTIVITY

For each pair prepare a table of values (with at least two ordered pairs), plot the points on the Cartesian plane and draw the straight lines. How do we find solutions from the graph? What can we say about the number of solutions when the two lines (i) intersect at a point, (ii) are parallel, (iii) are coincident?
Pair 1: $2x + y = 6$ and $x - y = 2$ Pair 2: $x + y = 4$ and $2x + 2y = 8$
Pair 3: $3x - 2y = 6$ and $6x - 4y = 12$ Pair 4: $x - 2y = 4$ and $2x - 4y = 6$

ANSWER

Pair	Equation	x	y	x	y
1	$2x + y = 6$	0	6	3	0
1	$x - y = 2$	0	–2	2	0
2	$x + y = 4$	0	4	4	0
2	$2x + 2y = 8$	0	4	4	0
3	$3x - 2y = 6$	0	–3	2	0
3	$6x - 4y = 12$	0	–3	2	0
4	$x - 2y = 4$	0	–2	4	0
4	$2x - 4y = 6$	0	$-\frac{3}{2}$	3	0

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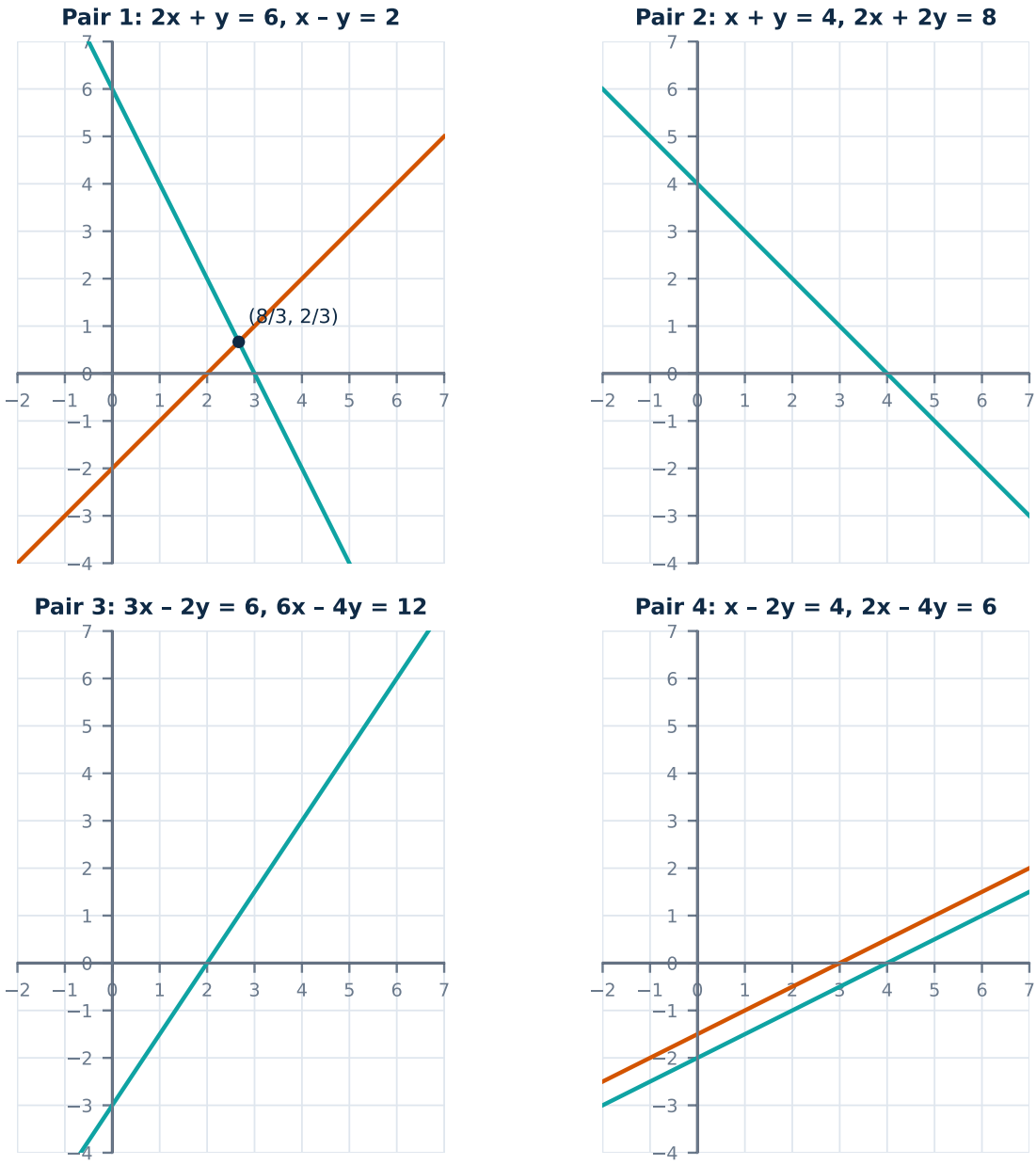

Graphs of Pairs 1 to 4

Pair 1: lines cross at one point; adding the equations gives $3x = 8$, $x = \frac{8}{3}$, $y = \frac{2}{3}$. Pair 2 and Pair 3: the two lines coincide (ratios 1/2 = 1/2 = 1/2 and 1/2 = 1/2 = 1/2). Pair 4: $\frac{1}{2} = \frac{2}{4} \neq \frac{4}{6}$: parallel lines.

A solution of the pair is a point lying on both lines. (i) Intersecting lines: exactly one common point, a unique solution. (ii) Parallel lines: no common point, no solution. (iii) Coincident lines: every point is common, infinitely many solutions.

Therefore, Pair 1 has the unique solution ($\frac{8}{3}$, $\frac{2}{3}$), Pairs 2 and 3 have infinitely many solutions, and Pair 4 has no solution.

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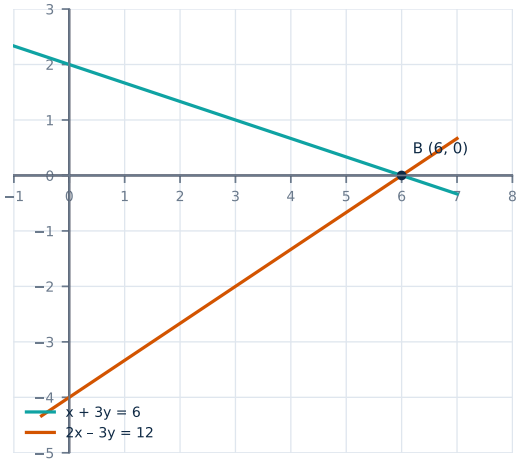
EXAMPLE 11

Solve the pair of linear equations $x + 3y = 6$ and $2x - 3y = 12$ graphically. Comment on the nature of the solution.

ANSWER

$x + 3y = 6$: x	0	6
y	2	0

$2x - 3y = 12$: x	0	3
y	–4	–2



The lines meet at B (6, 0)

The lines AB (through (0, 2), (6, 0)) and PQ (through (0, –4), (3, –2)) intersect at B (6, 0). Ratios: $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{-3} = -1$, so $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Therefore, the pair has a unique solution: $x = 6$, $y = 0$.

EXAMPLE 12

Determine if the lines $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ intersect, coincide or are parallel to each other. Then verify your answer by graphing the equations.

ANSWER

$\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-4}{-12} = \frac{1}{3}$. Since $\frac{1}{2} = \frac{1}{2} \neq \frac{1}{3}$, the lines are parallel and the pair has no solution.

$x + 2y - 4 = 0$: x	0	4
y	2	0

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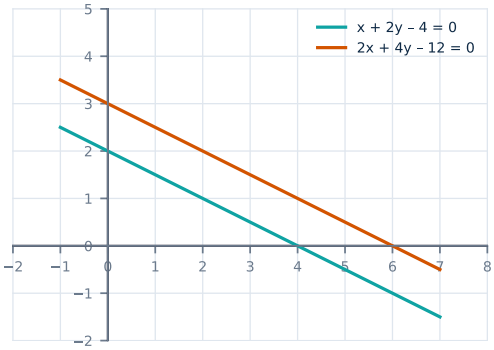
$2x + 4y - 12 = 0$: x	0	6
y	3	0



Parallel lines

Therefore, the lines are parallel; the graph shows that they never meet.

THINK AND REFLECT 17

Can you prove that two lines of equal slope are parallel? (Hint: Consider the equations of the two lines to be $y = mx + d_1$ and $y = mx + d_2$.)

ANSWER

Take distinct lines, so $d_1 \neq d_2$. Suppose they had a common point (p, q). Then $q = mp + d_1$ and $q = mp + d_2$, so $d_1 = d_2$, contradicting $d_1 \neq d_2$. Hence the lines have no common point, i.e. they never meet.

Therefore, two distinct lines with the same slope are parallel (if $d_1 = d_2$ they are the same line).

EXAMPLE 13

Romila purchased 2 erasers and 3 A4 sheets for ₹9. Her friend Sonali bought 4 erasers and 6 A4 sheets of the same kind for ₹18. Represent this situation graphically.

ANSWER

Let an eraser cost ₹x and an A4 sheet ₹y: $2x + 3y = 9$ and $4x + 6y = 18$, i.e. $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$. Points on both: (0, 3) and (4.5, 0).

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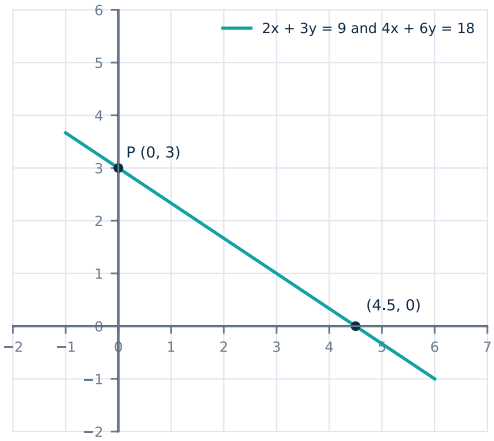

The two lines coincide

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$.

Therefore, both equations are represented by the same line (coincident lines); the pair has infinitely many solutions.

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13.1

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 13

Exercise Set 13.1

Solutions · 3 questions

Directions: Solve every question of Exercise Set 13.1 (Standard Form of Linear Equations in Two Variables, ax + by + c = 0).

QUESTION 1

Write a linear equation in two variables in which $a = 3$, $b = 0$ and $c = -\frac{1}{5}$.

ANSWER

✓ **Using ax + by + c = 0**

Substitute $a = 3$, $b = 0$ and $c = -\frac{1}{5}$ in $ax + by + c = 0$.

$$3x + 0 \times y - \frac{1}{5} = 0$$

Multiplying throughout by 5 to remove the fraction gives an equivalent equation with integer coefficients.

$$15x - 1 = 0$$

Therefore, the equation is $3x + 0 \times y - \frac{1}{5} = 0$, i.e. $3x - \frac{1}{5} = 0$ (or $15x - 1 = 0$).

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QUESTION 2

Complete the following table after expressing the given linear equations in standard form.

Linear equation in two variables	Standard Form	Coefficient of x	Coefficient of y	Constant term
$y - 15 = \sqrt{2}x$				
$3y - 2x = 0$				
$5x = 3y$				
$x = 8$				
$3y = 1$				

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ANSWER

Move every term to the left-hand side so that the right-hand side becomes 0, then read off a, b and c.

Linear equation in two variables	Standard Form	Coefficient of x	Coefficient of y	Constant term
$y - 15 = \sqrt{2}x$	$\sqrt{2}x - y + 15 = 0$	$\sqrt{2}$	-1	15
$3y - 2x = 0$	$-2x + 3y + 0 = 0$	-2	3	0
$5x = 3y$	$5x - 3y + 0 = 0$	5	-3	0
$x = 8$	$x + 0 \times y - 8 = 0$	1	0	-8
$3y = 1$	$0 \times x + 3y - 1 = 0$	0	3	-1

(i) $y - 15 = \sqrt{2}x$

$y - 15 - \sqrt{2}x = 0$, i.e. $\sqrt{2}x - y + 15 = 0$. Here $a = \sqrt{2}$, $b = -1$, $c = 15$.

(ii) $3y - 2x = 0$

Rearranged: $-2x + 3y + 0 = 0$. Here $a = -2$, $b = 3$, $c = 0$.

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(iii) **$5x = 3y$**

$5x - 3y = 0$, so $a = 5$, $b = -3$, $c = 0$.

(iv) **$x = 8$**

$x - 8 = 0$, written as $x + 0 \times y - 8 = 0$. So $a = 1$, $b = 0$, $c = -8$.

(v) **$3y = 1$**

$3y - 1 = 0$, written as $0 \times x + 3y - 1 = 0$. So $a = 0$, $b = 3$, $c = -1$.

Therefore, the completed table is as shown in the question above. (Multiplying an equation by –1 gives an equally valid standard form, e.g. $-\sqrt{2}x + y - 15 = 0$.)

QUESTION 3

(i) The cost of a notebook is twice the cost of a pen. Consider the cost of a notebook to be ₹t and that of a pen to be ₹p. Charlie wrote $t = 2p$, whereas Meera wrote $p = 2t$. Which of these two representations is correct?

(ii) In a one-day International Cricket match between India and Sri Lanka played in Nagpur, two Indian batsmen together scored 176 runs. Manisha expressed this situation as $x + y = 176$, where the number of runs scored by one batsman is x, and the number of runs scored by the other is y. Is this a correct representation?

ANSWER

(i) **Notebook and pen** — CHARLIE IS CORRECT

The notebook costs twice as much as the pen, so cost of notebook = 2 × cost of pen, i.e. $t = 2p$.

Meera's equation $p = 2t$ says the pen costs twice the notebook, which is the opposite of what is given. (Check: if a pen costs ₹10, the notebook costs ₹20; $t = 2p$ gives $20 = 20$ ✓, while $p = 2t$ gives $10 = 40$ ✗.)

Therefore, Charlie's representation $t = 2p$ is correct.

(ii) **Runs of two batsmen** — CORRECT

The runs of the two batsmen are x and y. Their total is $x + y$, which is given as 176.

Therefore, $x + y = 176$ is a correct representation (it is a linear equation in two variables, with x and y whole numbers).

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13.2

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 13

Exercise Set 13.2

Solutions · 7 questions

Exercise Set 13.2: Solutions of a linear equation in two variables.

QUESTION 1

Verify if the ordered pair (4, 3) is a solution of $5x - 6y = 2$. Explain your reasoning.

ANSWER

A pair is a solution if, on substituting x and y, both sides become equal. Put $x = 4$ and $y = 3$:

$$\text{LHS} = 5x - 6y = 5(4) - 6(3) = 20 - 18 = 2$$
$$\text{RHS} = 2$$

Therefore, LHS = RHS, so (4, 3) is a solution of $5x - 6y = 2$.

QUESTION 2

Find any two solutions for each of the following equations:

(i) $7x - 3y = 21$ (ii) $2x + 3y = 5$

ANSWER

(i) **$7x - 3y = 21$**

Put $x = 0$: $-3y = 21$, so $y = -7$. Put $y = 0$: $7x = 21$, so $x = 3$.

x	0	3
y	–7	0

Check: $7(0) - 3(-7) = 21$ ✓ and $7(3) - 3(0) = 21$ ✓.

Two solutions: (0, –7) and (3, 0).

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(ii) **2x + 3y = 5**

Put $x = 1$: $2 + 3y = 5$, so $y = 1$. Put $x = -2$: $-4 + 3y = 5$, so $y = 3$.

x	1	–2
y	1	3

Check: $2(1) + 3(1) = 5$ ✓ and $2(-2) + 3(3) = 5$ ✓.

Two solutions: (1, 1) and (–2, 3). (Other answers are also correct, e.g. (0, $\frac{5}{3}$) and ($\frac{5}{2}$, 0).)

QUESTION 3

In the equations given below, *m* and *n* are unknown constants: 2*mx* + 3*y* = 7; 4*x* + *ny* = –10. If (2, –1) is the solution of both equations, find the values of *m* and *n*.

ANSWER

1 **Substitute (2, –1) in the first equation**

$2m(2) + 3(-1) = 7$
$4m - 3 = 7$
$4m = 10$, so $m = \frac{5}{2}$

2 **Substitute (2, –1) in the second equation**

$4(2) + n(-1) = -10$
$8 - n = -10$
$n = 18$

Verification: $2 \cdot \frac{5}{2} \cdot 2 + 3(-1) = 10 - 3 = 7$ ✓ and $4(2) + 18(-1) = -10$ ✓.

Therefore, *m* = $\frac{5}{2}$ and *n* = 18.

QUESTION 4

Find two solutions which lie in different quadrants for each of the following linear equations. Identify the quadrants in which the points lie.

(i) 5*x* + 3*y* = 7 (ii) 5*x* – 3*y* = 7 (iii) –5*x* + 3*y* = 7 (iv) –5*x* – 3*y* = 7

Verify your solutions by representing the linear equations on a graph paper.

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ANSWER

Give x a value, solve for y, and look at the signs of (x, y): (+, +) Quadrant I, (–, +) Quadrant II, (–, –) Quadrant III, (+, –) Quadrant IV.

(i) 5x + 3y = 7

$x = 2$: $10 + 3y = 7$ gives $y = -1$, so (2, –1) lies in Quadrant IV. $x = -1$: $-5 + 3y = 7$ gives $y = 4$, so (–1, 4) lies in Quadrant II.

(ii) 5x – 3y = 7

$x = 2$: $10 - 3y = 7$ gives $y = 1$, so (2, 1) lies in Quadrant I. $x = -1$: $-5 - 3y = 7$ gives $y = -4$, so (–1, –4) lies in Quadrant III.

(iii) –5x + 3y = 7

$x = 1$: $-5 + 3y = 7$ gives $y = 4$, so (1, 4) lies in Quadrant I. $x = -3$: $15 + 3y = 7$ gives $y = -\frac{8}{3}$, so $(-3, -\frac{8}{3})$ lies in Quadrant III.

(iv) –5x – 3y = 7

$x = -2$: $10 - 3y = 7$ gives $y = 1$, so (–2, 1) lies in Quadrant II. $x = 1$: $-5 - 3y = 7$ gives $y = -4$, so (1, –4) lies in Quadrant IV.

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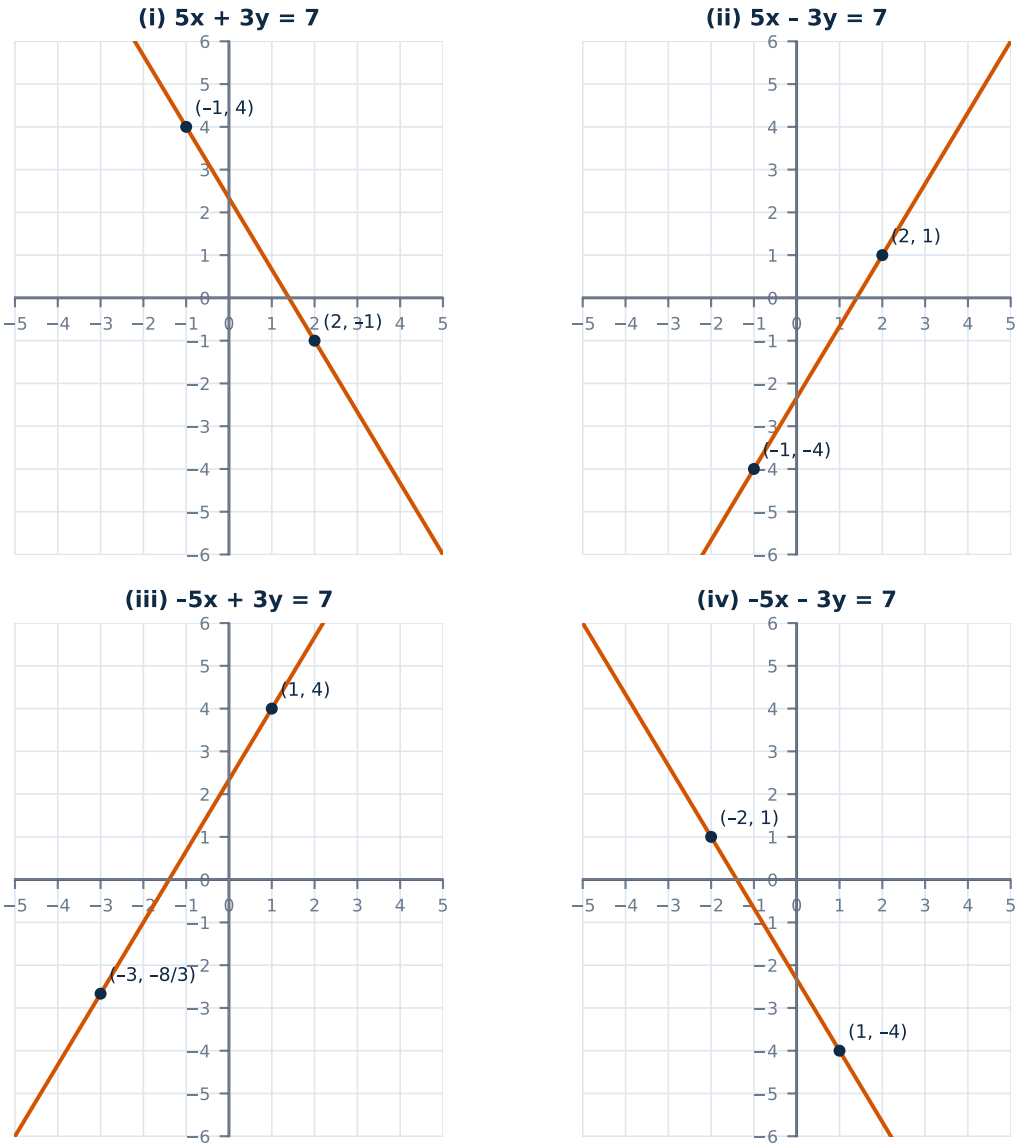

Graphs of the four equations with the chosen points marked (each point lies on its line).

Therefore, the points found lie in the quadrants stated above, and each plotted point lies on its line.

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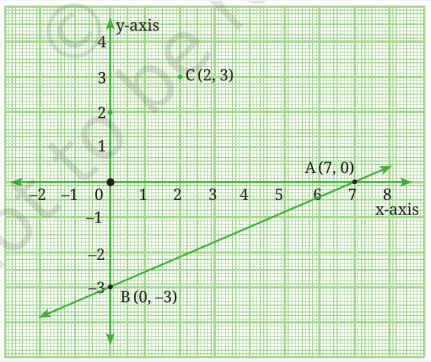
QUESTION 5

Consider the graph of the equation $3x - 7y = 21$ shown alongside. Does the point C (2, 3) lie on the line? Does it satisfy the equation? Can points that do not lie on the line satisfy the equation?



ANSWER

The line passes through A (7, 0) and B (0, –3). Check C (2, 3) in the equation:

$$3x - 7y = 3(2) - 7(3) = 6 - 21 = -15 \neq 21$$

So C does not satisfy the equation, and in the graph C is not on the line (the line at $x = 2$ has $y = \frac{3 \cdot 2 - 21}{7} = -\frac{15}{7}$, well below C).

Every solution of the equation is a point on the line and every point on the line is a solution. Hence a point that is not on the line cannot satisfy the equation.

Therefore, C (2, 3) does not lie on the line and does not satisfy the equation. No point off the line can satisfy the equation.

QUESTION 6

State whether the following sentences are True or False. Justify your answer.

(i) A linear equation in two variables has only one solution.
(ii) The graph of a linear equation in two variables always passes through the origin.
(iii) A linear equation in two variables can never have rational solutions.
(iv) $x = 3$ is a valid linear equation in two variables.
(v) The equation $2x + 3y = 7$ has infinitely many solutions.
(vi) The point (1, 2) is a solution of the equation $2x + 3y = 7$.

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ANSWER

(i) Only one solution — FALSE

For any value u given to x we can find y so that (u, v) is a solution. E.g. $x + y = 3$ has (0, 3), (1, 2), (2, 1), ... So there are infinitely many solutions.

(ii) Always passes through the origin — FALSE

The graph passes through the origin only when $c = 0$. Counterexample: $x + y = 1$; at (0, 0), $0 \neq 1$, so the origin is not on its graph.

(iii) Never has rational solutions — FALSE

$x + y = 1$ has the rational solutions $(\frac{1}{2}, \frac{1}{2})$, (0, 1), $(\frac{1}{3}, \frac{2}{3})$, ...

(iv) $x = 3$ is a valid linear equation — TRUE

$x = 3$ can be written as $1 \times x + 0 \times y - 3 = 0$, which is of the form $ax + by + c = 0$ with $a = 1$, $b = 0$ (a and b not both zero).

(v) 2x + 3y = 7 has infinitely many solutions — TRUE

Every value of x gives $y = \frac{7 - 2x}{3}$, so there are infinitely many solutions, e.g. (2, 1), (–1, 3), $(\frac{7}{2}, 0)$.

(vi) (1, 2) is a solution of 2x + 3y = 7 — FALSE

$2(1) + 3(2) = 2 + 6 = 8 \neq 7$, so (1, 2) is not a solution.

Therefore: (i) False, (ii) False, (iii) False, (iv) True, (v) True, (vi) False.

QUESTION 7

(i) Compare the solutions of the equations $3x + 4y = 7$ and $6x + 8y = 14$. Argue that they have the same set of solutions, that is, every solution of one is also a solution of the other.

(ii) Show that the equations $ax + by = c$ and $kax + kby = kc$, with $k \neq 0$, have the same set of solutions.

ANSWER

(i) 3x + 4y = 7 and 6x + 8y = 14

Note that $6x + 8y = 14$ is $2 \times (3x + 4y = 7)$.

If (p, q) satisfies $3x + 4y = 7$, then $3p + 4q = 7$, so $6p + 8q = 2(3p + 4q) = 14$. Hence (p, q) satisfies $6x + 8y = 14$.

Conversely, if $6p + 8q = 14$, dividing both sides by 2 gives $3p + 4q = 7$. So (p, q) satisfies the first equation.

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(ii) ax + by = c and kax + kby = kc, k ≠ 0

Let (p, q) satisfy $ax + by = c$, i.e. $ap + bq = c$. Multiplying by k: $kap + kbq = kc$, so (p, q) satisfies the second equation.

Conversely, if $kap + kbq = kc$, dividing by k (possible because $k \neq 0$) gives $ap + bq = c$.

Therefore, multiplying an equation by a non-zero number does not change its set of solutions; both equations have the same solutions (and the same graph).

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13.3

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 13

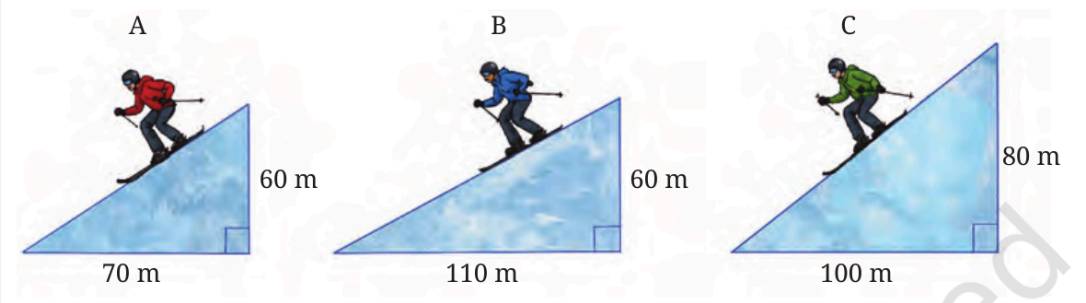
Exercise Set 13.3

Solutions · 4 questions

Exercise Set 13.3: Slope of a line.

QUESTION 1

The following diagrams represent ski hills. Rank the hills in order of their steepness, from least to greatest.



ANSWER

Steepness of a hill = slope = $\frac{\text{rise}}{\text{run}} = \frac{\text{vertical height}}{\text{horizontal length}}$.

A: $\frac{60}{70} = \frac{6}{7} \approx 0.857$
B: $\frac{60}{110} = \frac{6}{11} \approx 0.545$
C: $\frac{80}{100} = \frac{4}{5} = 0.8$

Arranging: $0.545 < 0.8 < 0.857$.

Therefore, from least to greatest steepness the order is B, C, A.

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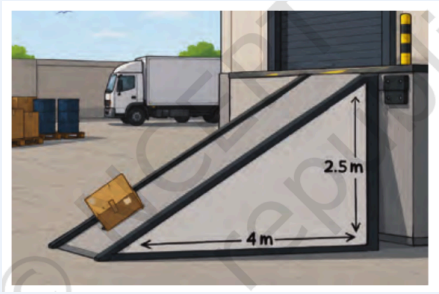
QUESTION 2

The ramp at a loading dock rises 2.5 metres over a run of 4 metres. Find the slope of the ramp.



ANSWER

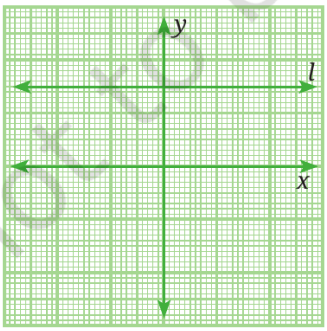
$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{2.5}{4} = \frac{25}{40} = \frac{5}{8} = 0.625$$

Therefore, the slope of the ramp is $\frac{5}{8}$ (= 0.625).

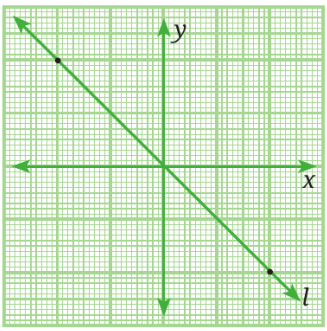
QUESTION 3

Find the slope of the line *l* in each of the following diagrams.

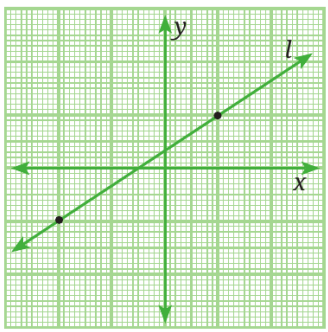
(i)



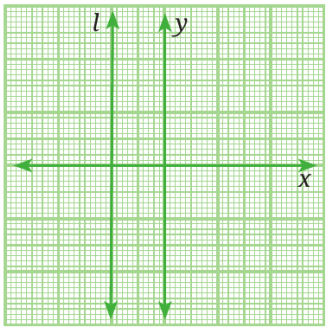
(ii)



(iii)



(iv)



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ANSWER

(i) Horizontal line — SLOPE = 0

l is parallel to the x-axis, so y stays constant. Rise = 0, hence slope = $\frac{0}{\text{Run}} = 0$.

(ii) Line through (–2, 2) and (2, –2) — SLOPE = -1

Reading the two marked points from the grid: slope = (–2 – 2) ÷ (2 – (–2)) = –4 ÷ 4 = –1.

(iii) Line through (–2, –1) and (1, 1) — SLOPE = 2/3

Reading the two marked points from the grid: slope = (1 – (–1)) ÷ (1 – (–2)) = 2 ÷ 3 = $\frac{2}{3}$.

(iv) Vertical line (parallel to the y-axis) — UNDEFINED

l is the vertical line $x = -1$. Run = 0, so the slope $\frac{\text{Rise}}{0}$ involves division by zero and is undefined.

Therefore, the slopes are: (i) 0, (ii) –1, (iii) $\frac{2}{3}$, (iv) undefined.

QUESTION 4

An accessibility ramp for wheelchairs is to be made alongside the staircase. The guidelines given for the slope of the ramp is 1 cm vertical rise to 12 cm horizontal length. What should be the horizontal length of the ramp if the total height of the stairs is 18 cm?



ANSWER

Required slope = $\frac{\text{Rise}}{\text{Run}} = \frac{1}{12}$. Let the horizontal length (run) be x cm for a rise of 18 cm.

$$\frac{18}{x} = \frac{1}{12}$$
$$x = 18 \times 12 = 216$$

Therefore, the horizontal length of the ramp should be 216 cm (2.16 m).

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13.4

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 13

Exercise Set 13.4

Solutions · 3 questions

Exercise Set 13.4: Construct a pair of linear equations in two variables for each situation.

QUESTION 1

On two different days a family buys movie tickets and snack boxes. Let the cost of one movie ticket be ₹x and the cost of one snack box be ₹y. Frame a pair of linear equations in x and y to represent the following situations:
(i) On the first day, the family buys 2 movie tickets and 3 snack boxes for ₹850.
(ii) On the second day, the family buys 4 movie tickets and 1 snack box for ₹1100.

ANSWER

(i) **First day**

Cost of 2 tickets = $2x$, cost of 3 snack boxes = $3y$; total ₹850.

$$2x + 3y = 850$$

(ii) **Second day**

Cost of 4 tickets = $4x$, cost of 1 snack box = y; total ₹1100.

$$4x + y = 1100$$

Therefore, the pair is $2x + 3y = 850$ and $4x + y = 1100$.

QUESTION 2

Two friends, Sahil and Meena, travelled by taxi. Let the fixed charge for one taxi trip be ₹x and the additional charge per kilometre be ₹y. Frame a pair of linear equations in x and y to represent the following situation:
(i) Sahil travelled 6 km and paid a total fare of ₹122.
(ii) Meena travelled 8 km and paid a total fare of ₹160.

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ANSWER

Total fare = fixed charge + (charge per km × distance).

(i) **Sahil**

$x + 6y = 122$

(ii) **Meena**

$x + 8y = 160$

Therefore, the pair is $x + 6y = 122$ and $x + 8y = 160$.

QUESTION 3

In a sports meet, tickets for adults cost ₹150 each and tickets for children cost ₹100 each. A total of 200 people attended the meet, and the total amount collected from ticket sales was ₹25,000. (Let the number of adult tickets sold be x and the number of children's tickets sold be y.) Frame a pair of linear equations in x and y to represent this situation.

ANSWER

Number of people: adults + children = 200. Money collected: 150 per adult ticket and 100 per child ticket.

$x + y = 200$
$150x + 100y = 25000$ (or $3x + 2y = 500$)

Therefore, the pair is $x + y = 200$ and $150x + 100y = 25000$.

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13.5

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 13

Exercise Set 13.5

Solutions · 9 questions

Exercise Set 13.5: Form a pair of linear equations for each problem and solve. (Questions 1(i) to 1(x) are solved one by one.)

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QUESTION 1

Form a pair of linear equations for each of the following problems and find their solutions.

(i) The sum of two integers is +5 and their difference is –21. Find the two numbers.

(ii) The difference between two numbers is 26 and one number is three times the other. Find the numbers.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹8880. Later, she buys 3 bats and 5 balls for ₹4000. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered for every km. For a distance of 10 km, the total amount paid is ₹155 and for a journey of 15 km, the total amount paid is ₹220. What is the fixed charge and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes equal to $\frac{9}{11}$ if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes equal to $\frac{5}{6}$. Find the fraction.

(vi) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes equal to $\frac{1}{2}$ if we add 1 only to the denominator. What is the fraction?

(vii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(viii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(ix) Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 did she receive?

(x) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

ANSWER

(i) **Question 1(i)**

Let the numbers be x and y.

$$x + y = 5 \quad ...(1)$$
$$x - y = -21 \quad ...(2)$$

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Adding: $2x = -16$, so $x = -8$. From (1): $y = 5 + 8 = 13$.

Check: $-8 + 13 = 5$ ✓; $-8 - 13 = -21$ ✓.

Therefore, the numbers are –8 and 13.

(ii) Question 1(ii)

Let the larger number be x and the smaller y.

$$x - y = 26 \quad ...(1)$$
$$x = 3y \quad ...(2)$$

Substitute (2) in (1): $3y - y = 26$, so $y = 13$ and $x = 39$.

Check: $39 - 13 = 26$ ✓; $39 = 3 \times 13$ ✓.

Therefore, the numbers are 39 and 13.

(iii) Question 1(iii)

Let a bat cost ₹x and a ball cost ₹y.

$$7x + 6y = 8880 \quad ...(1)$$
$$3x + 5y = 4000 \quad ...(2)$$

Multiply (1) by 3 and (2) by 7: $21x + 18y = 26640$ and $21x + 35y = 28000$. Subtract: $17y = 1360$, so $y = 80$. From (1): $7x = 8880 - 480 = 8400$, so $x = 1200$.

Check: $3(1200) + 5(80) = 3600 + 400 = 4000$ ✓.

Therefore, each bat costs ₹1200 and each ball costs ₹80.

(iv) Question 1(iv)

Let the fixed charge be ₹x and the charge per km be ₹y.

$$x + 10y = 155 \quad ...(1)$$
$$x + 15y = 220 \quad ...(2)$$

Subtract (1) from (2): $5y = 65$, so $y = 13$. From (1): $x = 155 - 130 = 25$.

For 25 km: fare $= 25 + 25 \times 13 = 25 + 325 = 350$.

Therefore, the fixed charge is ₹25, the charge per km is ₹13, and the fare for 25 km is ₹350.

(v) Question 1(v)

Let the fraction be $\frac{x}{y}$.

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$\frac{x+2}{y+2} = \frac{9}{11}$, so $11x - 9y = -4$ ...(1)
$\frac{x+3}{y+3} = \frac{5}{6}$, so $6x - 5y = -3$ ...(2)

Multiply (1) by 5 and (2) by 9: $55x - 45y = -20$ and $54x - 45y = -27$. Subtract: $x = 7$. From (2): $42 - 5y = -3$, so $y = 9$.

Check: $\frac{9}{11}$ ✓ and $\frac{10}{12} = \frac{5}{6}$ ✓.

Therefore, the fraction is $\frac{7}{9}$.

(vi) Question 1(vi)

Let the fraction be $\frac{x}{y}$.

$\frac{x+1}{y-1} = 1$, so $y = x + 2$ ...(1)
$\frac{x}{y+1} = \frac{1}{2}$, so $2x = y + 1$ ...(2)

Substitute (1) in (2): $2x = x + 3$, so $x = 3$ and $y = 5$.

Check: $\frac{4}{4} = 1$ ✓ and $\frac{3}{6} = \frac{1}{2}$ ✓.

Therefore, the fraction is $\frac{3}{5}$.

(vii) Question 1(vii)

Let Nuri's present age be x years and Sonu's be y years.

$x - 5 = 3(y - 5)$, so $x - 3y = -10$ ...(1)
$x + 10 = 2(y + 10)$, so $x - 2y = 10$ ...(2)

Subtract (1) from (2): $y = 20$. Then $x = 10 + 40 = 50$.

Check: 5 years ago 45 and 15 ($45 = 3 \times 15$ ✓); after 10 years 60 and 30 ($60 = 2 \times 30$ ✓).

Therefore, Nuri is 50 years old and Sonu is 20 years old.

(viii) Question 1(viii)

Let the tens digit be x and the units digit be y; the number is $10x + y$.

$x + y = 9$ ...(1)
$9(10x + y) = 2(10y + x)$, so $88x = 11y$, i.e. $y = 8x$ ...(2)

Substitute (2) in (1): $9x = 9$, so $x = 1$ and $y = 8$.

Check: $9 \times 18 = 162$ and $2 \times 81 = 162$ ✓.

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Therefore, the number is 18.

(ix) Question 1(ix)

Let there be x notes of ₹50 and y notes of ₹100.

$x + y = 25$ ...(1)
$50x + 100y = 2000$, i.e. $x + 2y = 40$ ...(2)

Subtract (1) from (2): $y = 15$, so $x = 10$.

Check: 10×50 + 15×100 = 500 + 1500 = 2000 ✓.

Therefore, Meena received 10 notes of ₹50 and 15 notes of ₹100.

(x) Question 1(x)

Let the fixed charge (first three days) be ₹x and the charge for each extra day be ₹y. Seven days means 4 extra days; five days means 2 extra days.

$x + 4y = 27$ ...(1)
$x + 2y = 21$ ...(2)

Subtract (2) from (1): $2y = 6$, so $y = 3$. From (2): $x = 21 - 6 = 15$.

Check: 15 + 12 = 27 ✓ and 15 + 6 = 21 ✓.

Therefore, the fixed charge is ₹15 and the charge for each extra day is ₹3.

QUESTION 2

Form a pair of linear equations and find their common solutions graphically.

10 students of Grade 9 took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

ANSWER

Let the number of boys be x and girls be y.

$x + y = 10$ ...(1)
$y = x + 4$, i.e. $y - x = 4$ ...(2)

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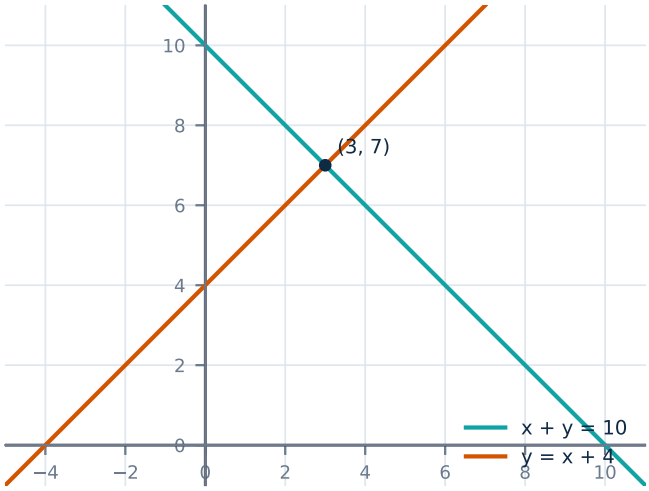
Line	x	y	x	y
$x + y = 10$	0	10	10	0
$y = x + 4$	0	4	–4	0



The lines meet at (3, 7)

The lines intersect at (3, 7). Check: 3 + 7 = 10 ✓ and 7 = 3 + 4 ✓.

Therefore, 3 boys and 7 girls took part in the quiz.

QUESTION 3

Use the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$, $\frac{c_1}{c_2}$, to determine whether the lines representing the following pairs of linear equations intersect at a point, are parallel or are coincident.

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

ANSWER

(i) **INTERSECTING**

$\frac{a_1}{a_2} = \frac{5}{7}$, $\frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$. Since $\frac{5}{7} \neq -\frac{2}{3}$, the lines intersect at a point.

(ii) **COINCIDENT**

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$. All three ratios are equal, so the lines coincide.

(iii) **PARALLEL**

$\frac{a_1}{a_2} = \frac{6}{2} = 3$, $\frac{b_1}{b_2} = \frac{-3}{-1} = 3$, $\frac{c_1}{c_2} = \frac{10}{9}$. Here $3 = 3 \neq \frac{10}{9}$, so the lines are parallel.

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Therefore, (i) intersect at a point, (ii) coincident, (iii) parallel.

QUESTION 4

Which of the following pairs of linear equations have solutions? If they have solutions, find them graphically.
(i) $x + y = 5$, $2x + 2y = 10$
(ii) $x - y = 8$, $3x - 3y = 16$
(iii) $2x + y - 6 = 0$, $4x - 2y - 4 = 0$
(iv) $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

ANSWER

(i) $x + y = 5$ and $2x + 2y = 10$ — INFINITELY MANY

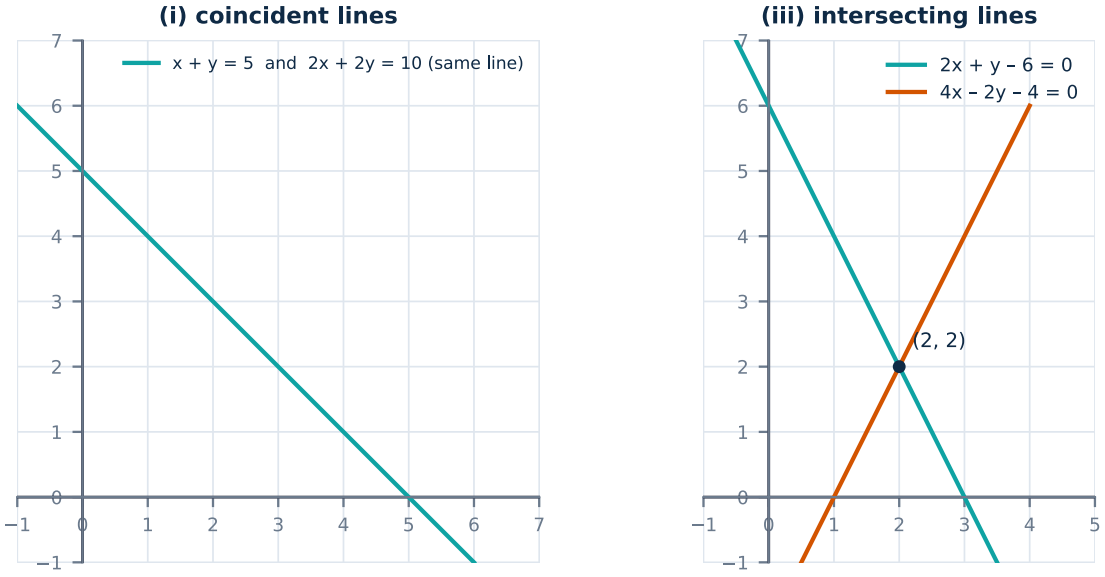
$\frac{1}{2} = \frac{1}{2} = \frac{-5}{-10} = \frac{1}{2}$: coincident lines. Both pass through (0, 5) and (5, 0); every point of the line $x + y = 5$, e.g. (1, 4), (2, 3), is a solution.

(ii) $x - y = 8$ and $3x - 3y = 16$ — NO SOLUTION

$\frac{1}{3} = \frac{-1}{-3} = \frac{1}{3}$, while $\frac{-8}{-16} = \frac{1}{2}$, and $\frac{1}{3} \neq \frac{1}{2}$ (writing in the form $ax + by + c = 0$): the lines are parallel, so there is no solution. ($x - y = 8$ passes through (8, 0), (0, –8); $3x - 3y = 16$ through ($\frac{16}{3}$, 0), (0, $-\frac{16}{3}$).)

(iii) $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$ — UNIQUE

$\frac{2}{4} = \frac{1}{2} \neq \frac{1}{-2}$: the lines intersect. Points on the first: (0, 6), (3, 0). On the second: (0, –2), (1, 0). The graphs meet at (2, 2). Check: $2(2) + 2 - 6 = 0$ ✓ and $4(2) - 2(2) - 4 = 0$ ✓.



(i) coincident lines and (iii) intersecting lines

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(iv) **2x – 2y – 2 = 0 and 4x – 4y – 5 = 0** NO SOLUTION

$\frac{2}{4} = \frac{-2}{-4} = \frac{1}{2} \neq \frac{-2}{-5} = \frac{2}{5}$: the lines are parallel, so there is no solution.

Therefore, pairs (i) and (iii) have solutions: (i) infinitely many (all points of $x + y = 5$), (iii) the unique solution $x = 2$, $y = 2$. Pairs (ii) and (iv) have no solution.

QUESTION 5

Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

ANSWER

Let the length be l m and the width be w m.

$l = w + 4$ ...(1)
$l + w = 36$ ...(2) (half the perimeter)

Substitute (1) in (2): $2w + 4 = 36$, so $w = 16$ and $l = 20$.

Check: 20 + 16 = 36 ✓ and 20 – 16 = 4 ✓.

Therefore, the garden is 20 m long and 16 m wide.

QUESTION 6

Given the linear equation $2x + 3y – 8 = 0$, write another linear equation in two variables so that the graphs of the pair formed represent (i) intersecting lines (ii) parallel lines (iii) coincident lines.

ANSWER

Here $a_1 = 2$, $b_1 = 3$, $c_1 = –8$.

(i) **Intersecting lines**

Need $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Take $x + 2y – 5 = 0$: $\frac{2}{1} = 2 \neq \frac{3}{2}$.

(ii) **Parallel lines**

Need $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Take $4x + 6y + 7 = 0$: $\frac{2}{4} = \frac{3}{6} = \frac{1}{2} \neq \frac{-8}{7}$.

(iii) **Coincident lines**

Need all three ratios equal. Take $4x + 6y – 16 = 0$: $\frac{2}{4} = \frac{3}{6} = \frac{-8}{-16} = \frac{1}{2}$.

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Therefore, one example is: (i) $x + 2y - 5 = 0$, (ii) $4x + 6y + 7 = 0$, (iii) $4x + 6y - 16 = 0$. (Other correct equations are possible.)

QUESTION 7

Here is a problem that was posed by Mahāvīrāchārya in *Gaṇita sāra saṅgraha* (c. 850 CE).

The price of 9 citrons and 7 fragrant wood-apples taken together is 107; and the price of 7 citrons and 9 fragrant wood-apples taken together is 101. O mathematician, tell me quickly the price of each citron and of each fragrant wood-apple.

(Hint: The given problem can be modelled as $9x + 7y = 107$, $7x + 9y = 101$. Can you figure out a way of solving these equations without directly using elimination or the substitution method? What will happen if we add the pair of equations? What will happen if we subtract the pair of equations?)

ANSWER

Let a citron cost x and a wood-apple cost y. The coefficients are interchanged in the two equations.

$9x + 7y = 107$ ...(1)
$7x + 9y = 101$ ...(2)

1 **Add (1) and (2)**

$16x + 16y = 208$, so $x + y = 13$ ...(3)

2 **Subtract (2) from (1)**

$2x - 2y = 6$, so $x - y = 3$ ...(4)

Adding (3) and (4): $2x = 16$, so $x = 8$. Then $y = 13 - 8 = 5$.

Check: $9(8) + 7(5) = 72 + 35 = 107$ ✓ and $7(8) + 9(5) = 56 + 45 = 101$ ✓.

Therefore, each citron costs 8 and each fragrant wood-apple costs 5.

QUESTION 8

5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of each pencil and pen.

ANSWER

Let a pencil cost ₹x and a pen cost ₹y.

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$5x + 7y = 50$ ...(1)
$7x + 5y = 46$ ...(2)

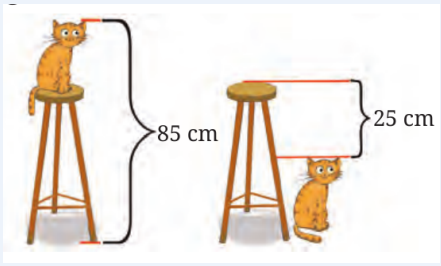
Adding: $12x + 12y = 96$, so $x + y = 8$. Subtracting (2) from (1): $-2x + 2y = 4$, so $y - x = 2$. Hence $y = 5$ and $x = 3$.

Check: $5(3) + 7(5) = 50$ ✓ and $7(3) + 5(5) = 46$ ✓.

Therefore, a pencil costs ₹3 and a pen costs ₹5.

QUESTION 9

Find the height of the stool.



ANSWER

Let the height of the stool be s cm and the height of the cat be c cm. In the first picture the cat sits on the stool, so the total height is $s + c = 85$. In the second picture the top of the stool is 25 cm above the cat's head, so $s - c = 25$.

$s + c = 85$
$s - c = 25$

Adding: $2s = 110$, so $s = 55$ and $c = 30$.

Check: $55 + 30 = 85$ ✓ and $55 - 30 = 25$ ✓.

Therefore, the height of the stool is 55 cm (and the cat is 30 cm tall).

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EC

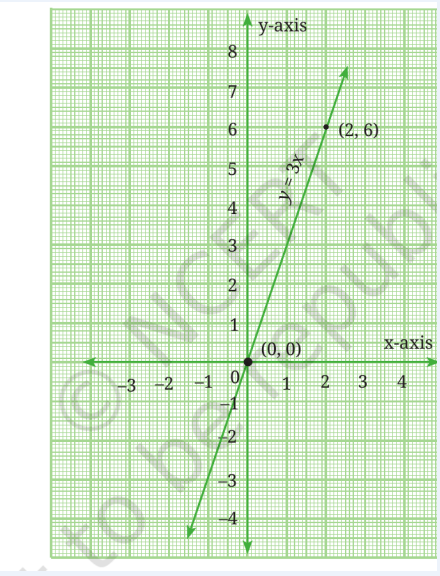
CLASS 9 MATHS · GANITA MANJARI · CHAPTER 13

End-of-Chapter Exercises

Solutions · 16 questions

QUESTION 1

The graph of the line $y = 3x$, passing through (0, 0) and (2, 6) is given alongside.
(i) Identify the slope of the line from the graph and explain how you calculated it using the two points on the line.
(ii) What is the y-intercept of the line? Explain its significance.
(iii) A water tank is being filled so that the water level y (in cm) after x minutes follows this graph.
(a) How high is the water after 5 minutes?
(b) How many minutes will it take for the water to reach a height of 21 cm?
(iv) Without drawing a new graph, determine whether the point (4, 10) lies on this line. Justify your answer.
(v) Plot the point where this line intersects the line $x = 3$. Explain how you found the coordinates.



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ANSWER

(i) **Slope**

SLOPE = 3

Using (0, 0) and (2, 6): slope = $\frac{\text{Rise}}{\text{Run}} = \frac{6 - 0}{2 - 0} = \frac{6}{2} = 3$. (It equals the coefficient of x in $y = 3x$.)

(ii) y-intercept 0

The line cuts the y-axis at (0, 0), so the y-intercept is 0. In the tank, this means that at time $x = 0$ the water level is 0 cm, i.e. the tank starts empty.

(iii) Water tank

(a) $x = 5$: $y = 3 \times 5 = 15$, so the water is 15 cm high. (b) $y = 21$: $3x = 21$, so $x = 7$ minutes.

(iv) Is (4, 10) on the line? NO

For $x = 4$, $3x = 12 \neq 10$. So (4, 10) does not satisfy $y = 3x$ and does not lie on the line (the line passes through (4, 12)).

(v) Intersection with x = 3

The point lies on both lines, so $x = 3$ and $y = 3x = 9$. It is the point (3, 9): on the graph, go to $x = 3$ and move up to the line.

Therefore: slope 3; y-intercept 0; 15 cm after 5 minutes and 7 minutes for 21 cm; (4, 10) is not on the line; the lines meet at (3, 9).

QUESTION 2

In countries like the USA, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Celsius to Fahrenheit: $F = \frac{9}{5}C + 32$

(i) Draw the graph of the linear equation above using Celsius on the x-axis and Fahrenheit on the y-axis.

(ii) If the temperature is 30 °C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95 °F, what is the temperature in Celsius?

(iv) If the temperature is 0 °C, what is the temperature in Fahrenheit and if the temperature is 0 °F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

ANSWER

(i) Graph

C	–40	0	30	35	100
F	–40	32	86	95	212

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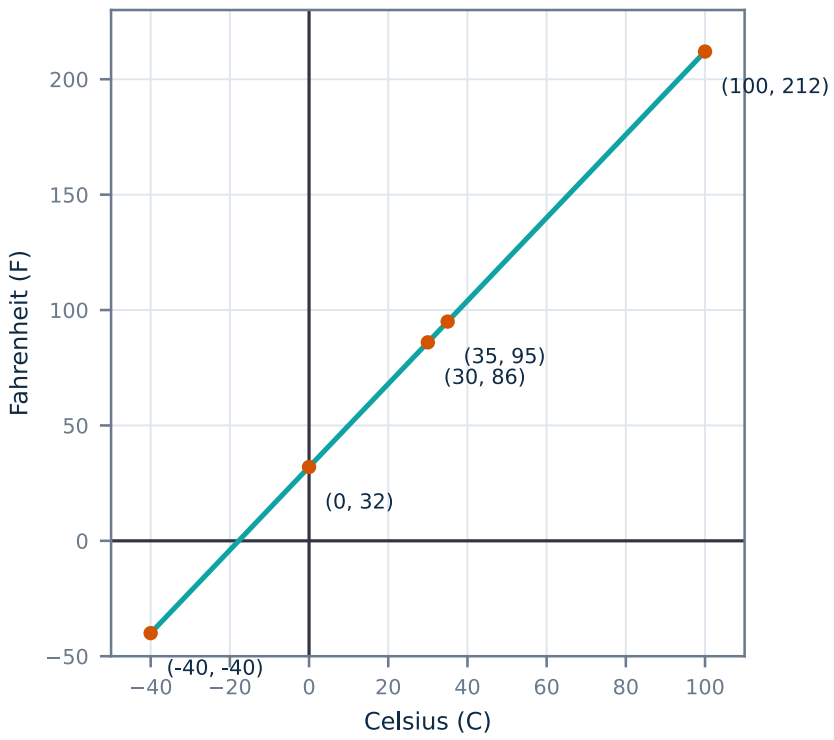

Graph of F = (9/5)C + 32

(ii) **30 °C**

F = $\frac{9}{5}$(30) + 32 = 54 + 32 = 86.

(iii) **95 °F**

95 = $\frac{9}{5}$C + 32, so $\frac{9}{5}$C = 63 and C = 63 × $\frac{5}{9}$ = 35.

(iv) **0 °C and 0 °F**

C = 0: F = 32. F = 0: 0 = $\frac{9}{5}$C + 32, so C = –$\frac{160}{9}$ ≈ –17.78.

(v) **Same number in both scales** YES

Put F = C: C = $\frac{9}{5}$C + 32, so –$\frac{4}{5}$C = 32 and C = –40. Check: $\frac{9}{5}$(–40) + 32 = –72 + 32 = –40 ✓. (On the graph, the line meets the line F = C at (–40, –40).)

Therefore: 30 °C = 86 °F; 95 °F = 35 °C; 0 °C = 32 °F and 0 °F = –$\frac{160}{9}$ °C ≈ –17.8 °C; the temperature –40 is the same in both scales.

QUESTION 3

Solve the following system of equations graphically: 2*x* + *y* = 6, 2*x* – *y* – 2 = 0.

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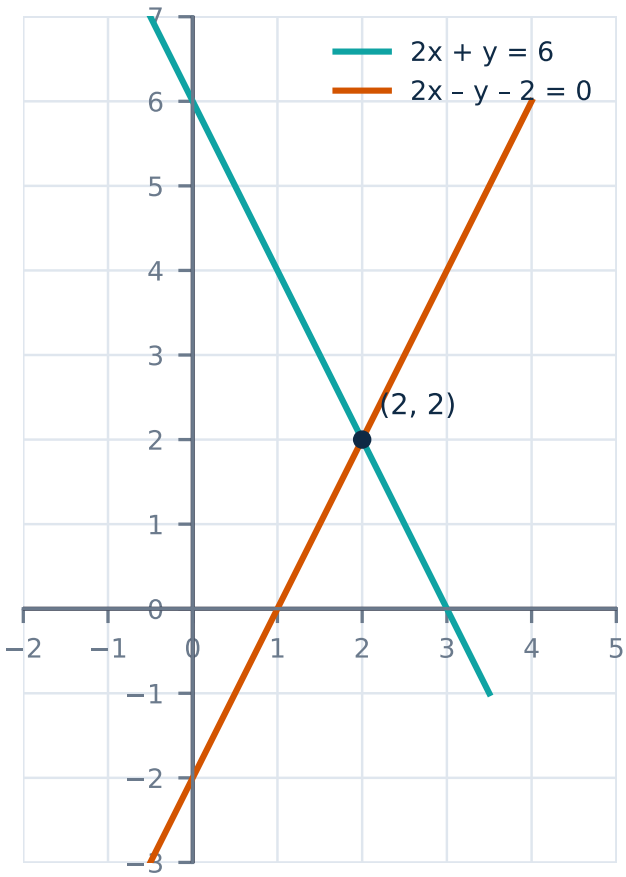
ANSWER

Line	x	y	x	y
$2x + y = 6$	0	6	3	0
$2x - y = 2$	0	–2	1	0



The lines meet at (2, 2)

The lines intersect at (2, 2). Check: 2(2) + 2 = 6 ✓ and 2(2) – 2 – 2 = 0 ✓.

Therefore, the solution is $x = 2$, $y = 2$.

QUESTION 4

Find the point of intersection of the lines shown on the cover page.

ANSWER

The lines referred to appear in the figure on the chapter’s cover page. That figure is not part of the pages supplied (the chapter PDF starts at the opening page, which shows the fruit cart), so its lines cannot be read and no numerical answer is given here, to avoid inventing data.

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Method: read two points on each line from the figure and find the slope ($m = \frac{y_2 - y_1}{x_2 - x_1}$) and the equation $y = mx + d$ of each line; equate the two expressions for y (or use substitution/elimination) to get x, then y. That ordered pair is the point of intersection.

Therefore, the point of intersection is found by solving the pair of equations of the two lines shown on the cover figure.

QUESTION 5

Give a formula to find the x-intercept of the line $y = mx + c$.

ANSWER

The x-intercept is the x-coordinate of the point where the line meets the x-axis, i.e. where $y = 0$.

$$0 = mx + c$$
$$x = -\frac{c}{m} \quad (m \neq 0)$$

Therefore, the x-intercept is $-\frac{c}{m}$, the line crossing the x-axis at $(-\frac{c}{m}, 0)$. (For $m = 0$ the line is horizontal and has no x-intercept unless $c = 0$.)

QUESTION 6

**A person is choosing between two mobile plans.
Plan A: ₹50 monthly fee + ₹0.20 per minute of call time.
Plan B: ₹30 monthly fee + ₹0.30 per minute of call time.
For how many minutes of calling per month is Plan A cheaper than Plan B? For how many minutes is Plan B cheaper? Also find the number of minutes at which both plans cost the same.**

ANSWER

For t minutes: Plan A costs $50 + 0.20t$; Plan B costs $30 + 0.30t$.

1 Equal cost

$$50 + 0.20t = 30 + 0.30t$$
$$20 = 0.10t, \text{ so } t = 200$$

2 Compare

Plan A is cheaper when $50 + 0.20t < 30 + 0.30t$, i.e. $20 < 0.10t$, i.e. $t > 200$. Plan B is cheaper when $t < 200$.

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Check: $t = 100$: A = ₹70, B = ₹60 (B cheaper). $t = 300$: A = ₹110, B = ₹120 (A cheaper). $t = 200$: both ₹90.

Therefore, Plan A is cheaper for more than 200 minutes, Plan B is cheaper for fewer than 200 minutes, and both cost the same (₹90) at 200 minutes.

QUESTION 7

How many lines exist that
(i) have a given slope?
(ii) have a given slope and pass through a given point?

ANSWER

(i) Given slope — INFINITELY MANY

Lines $y = mx + d$ with the same m and different d are all different (parallel) lines, and d can be any real number.

(ii) Given slope and a given point (p, q) — EXACTLY ONE

$q = mp + d$ fixes $d = q - mp$, so the line is $y = mx + (q - mp)$; only one such line exists.

Therefore, (i) infinitely many lines, (ii) exactly one line.

QUESTION 8

For what values of p does the pair of equations given below have a unique solution?
$4x + py + 8 = 0$; $2x + 2y + 2 = 0$.

ANSWER

A unique solution exists when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

$\frac{4}{2} \neq \frac{p}{2}$
$2 \neq \frac{p}{2}$, so $p \neq 4$

Therefore, the pair has a unique solution for every real value of p except $p = 4$.

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QUESTION 9

Find the values of a and b for which the following system of equations has infinitely many solutions: $(a + b)x - 2by = 5a + 2b + 1$; $3x - y = 14$.

ANSWER

Write in the form $a_1x + b_1y + c_1 = 0$: $(a + b)x - 2by - (5a + 2b + 1) = 0$ and $3x - y - 14 = 0$. For infinitely many solutions:

$$\frac{a + b}{3} = \frac{-2b}{-1} = \frac{5a + 2b + 1}{14}$$

From the first two: $a + b = 6b$, so $a = 5b$. From the last two: $14 \times 2b = 5a + 2b + 1$, i.e. $26b = 5a + 1 = 25b + 1$, so $b = 1$ and $a = 5$.

Check: the first equation becomes $6x - 2y = 28$, which is $2 \times (3x - y = 14)$ ✓.

Therefore, $a = 5$ and $b = 1$.

QUESTION 10

Find the value of 'k' for which the following system of equations represents a pair of coincident lines: $x + 2y = 3$; $(k - 1)x + (k + 1)y = k + 3$.

ANSWER

For coincident lines: $\frac{1}{k - 1} = \frac{2}{k + 1} = \frac{3}{k + 3}$.

$k + 1 = 2(k - 1)$, so $k = 3$
Check: $k - 1 = 2$, $k + 1 = 4$, $k + 3 = 6$ — the second equation is $2x + 4y = 6 = 2 \times (x + 2y = 3)$ ✓

Therefore, $k = 3$.

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QUESTION 11

(i) Robot 1 starts from the origin and traces a path by repeatedly moving 3 units to the right and then 4 units upward. Robot 2 starts from the point (10, 0) and repeatedly moves 1 unit to the right and then 2 units upward. Will the paths traced by these two robots intersect and if so, where?

(ii) Robot 1 starts from (3, 0) and traces a path by repeatedly moving 5 units to the right and then 5 units upward. Robot 2 starts from the point (7, 0) and repeatedly moves 5 units to the right and then 3 units downwards. Will the paths traced by these two robots intersect and if so, where?

ANSWER

Each path is modelled by the straight line through the points the robot reaches, with slope = $\frac{\text{rise}}{\text{run}}$.

(i) **Robots 1 and 2**

Robot 1: slope $\frac{4}{3}$ through (0, 0): $y = \frac{4}{3}x$. Robot 2: slope 2 through (10, 0): $y = 2(x - 10)$.

$\frac{4}{3}x = 2x - 20$
$20 = \frac{2}{3}x$, so $x = 30$ and $y = 40$

Both robots move only rightwards and upwards, and $x = 30$ is beyond both starting points (Robot 1 reaches it after 10 steps, Robot 2 after 20 steps), so the point is on both paths.

(ii) **Robots 1 and 2**

Robot 1: slope 1 through (3, 0): $y = x - 3$. Robot 2: slope $-\frac{3}{5}$ through (7, 0): $y = -\frac{3}{5}(x - 7)$.

$5(x - 3) = -3(x - 7)$
$5x - 15 = -3x + 21$, so $x = 4.5$ and $y = 1.5$

The lines meet at (4.5, 1.5). But Robot 2 starts at $x = 7$ and moves only to the right, so it never reaches $x = 4.5$. The point lies on Robot 1's path but not on Robot 2's.

Therefore, (i) yes, the paths intersect at (30, 40). (ii) The lines meet at (4.5, 1.5) but that point lies behind Robot 2's starting point, so the robots' paths do not intersect.

QUESTION 12 *

At a certain time, Jacob notices that his digital watch reads '*a*' minutes after two o'clock. Fifteen minutes later, it reads '*b*' minutes after three o'clock. He noticed that *a* is six times greater than *b*. What time was it when he looked at his watch for the second time?

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ANSWER

First time: 2 h a min. Fifteen minutes later: 3 h b min. So 2 h + (a + 15) min = 3 h + b min, giving a + 15 = 60 + b.

$a - b = 45$ ...(1)
$a = 6b$ ...(2)

Substitute (2) in (1): $5b = 45$, so $b = 9$ and $a = 54$.

Check: 2:54 + 15 min = 3:09 ✓, and 54 = 6 × 9 ✓.

Therefore, the second time was 9 minutes after three o'clock, i.e. 3:09.

QUESTION 13 *

The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.

ANSWER

Let the tens digit be x and the units digit y; the number is $10x + y$.

$x + y = 15$ ...(1)
$(10y + x) - (10x + y) = 9$, so $y - x = 1$ ...(2)

Adding: $2y = 16$, so $y = 8$ and $x = 7$.

Check: 87 – 78 = 9 ✓.

Therefore, the number is 78.

QUESTION 14 *

In a cyclic quadrilateral ABCD, ∠A = (x + 7)°, ∠B = (y + 8)°, ∠C = (3y + 23)° and ∠D = (4x + 12)°. Find all four angles of the cyclic quadrilateral.

ANSWER

Opposite angles of a cyclic quadrilateral are supplementary: ∠A + ∠C = 180° and ∠B + ∠D = 180°.

$(x + 7) + (3y + 23) = 180$, so $x + 3y = 150$ ...(1)
$(y + 8) + (4x + 12) = 180$, so $4x + y = 160$ ...(2)

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From (1): $x = 150 - 3y$. Put in (2): $600 - 12y + y = 160$, so $11y = 440$, $y = 40$ and $x = 30$.

$\angle A = 30 + 7 = 37°$, $\angle B = 40 + 8 = 48°$
$\angle C = 3(40) + 23 = 143°$, $\angle D = 4(30) + 12 = 132°$

Check: 37 + 143 = 180 ✓, 48 + 132 = 180 ✓, total 360° ✓.

Therefore, ∠A = 37°, ∠B = 48°, ∠C = 143° and ∠D = 132°.

QUESTION 15 *

A train moving with uniform speed for a certain distance takes 6 hours less if its speed is increased by 6 km/hour. It would have taken 6 hours more had its speed been decreased by 4 km/hour. Find the distance travelled and the speed of the train.

ANSWER

Let the speed be v km/h and the usual time t hours, so the distance is vt km.

1 **Speed + 6, time – 6**

$(v + 6)(t - 6) = vt$
$-6v + 6t - 36 = 0$, so $t - v = 6$ ...(1)

2 **Speed – 4, time + 6**

$(v - 4)(t + 6) = vt$
$6v - 4t - 24 = 0$, so $3v - 2t = 12$ ...(2)

Substitute $t = v + 6$ in (2): $3v - 2v - 12 = 12$, so $v = 24$ and $t = 30$. Distance = 24 × 30 = 720 km.

Check: 720 ÷ 30 = 24 h = 30 – 6 ✓ and 720 ÷ 20 = 36 h = 30 + 6 ✓.

Therefore, the speed of the train is 24 km/h and the distance travelled is 720 km.

QUESTION 16 *

The age of a father is equal to the sum of the ages of his four children. After 20 years, the sum of the ages of the children will be twice the age of the father. Find the age of the father.

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ANSWER

Let the father's age be x years and the sum of the four children's ages be y years.

$x = y$ ...(1)
After 20 years: $y + 4 \times 20 = 2(x + 20)$, so $y + 80 = 2x + 40$ ...(2)

Substitute (1) in (2): $x + 80 = 2x + 40$, so $x = 40$.

Check: the children's ages sum to 40 now and 40 + 80 = 120 after 20 years; the father is then 60 and 2 × 60 = 120 ✓.

Therefore, the father's age is 40 years.

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