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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 9

Propositions and their Converses

Complete Exercise Solutions

Exercise Set 9.1 | In-Text Questions

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9.1

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 9

Exercise Set 9.1

Solutions · 17 questions

Directions (Questions 1–12): Frame the converse for each of the propositions in Questions 1–12. Then, determine if each of the two statements is true or not. Justify the true statements and give a counterexample for each false statement.

QUESTION 1

If two lines are parallel, then the corresponding angles formed by a transversal are equal.

ANSWER

Proposition P: If two lines are parallel, then the corresponding angles formed by a transversal are equal.

Converse Q: If the corresponding angles formed by a transversal of two lines are equal, then the two lines are parallel.

P Proposition P

TRUE

When a transversal cuts two parallel lines, each pair of corresponding angles is equal. This is the corresponding angles property of parallel lines, which is accepted as a basic result (axiom) of Euclidean geometry.

Q Converse Q

TRUE

Let a transversal t meet lines *l* and *m* at A and B, and let a pair of corresponding angles be equal, say each equal to α. The angle next to the one at A on the line *l* (forming a straight line with it) is 180° – α, so on one side of t the interior angles are 180° – α and α:

Interior angles on one side of t: (180° – α) and α

Their sum = (180° – α) + α = 180°

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Now suppose *l* and *m* were not parallel, so they meet at a point P on that side. Then ΔPAB would have angles at A and B adding up to 180°, leaving ∠P = 0°, which is impossible because the angle sum of a triangle is 180° and each angle is positive. (If they meet on the other side, the same argument applies.) Hence *l* and *m* cannot meet, i.e. they are parallel.

Therefore, both the proposition P and its converse Q are true.

QUESTION 2

If a quadrilateral is a square, then all its angles are equal.

ANSWER

Proposition P: If a quadrilateral is a square, then all its angles are equal.

Converse Q: If all the angles of a quadrilateral are equal, then it is a square.

P Proposition P TRUE

Every angle of a square is a right angle. So all four angles are equal (each is 90°).

Q Converse Q FALSE

Counterexample: a rectangle with sides 3 cm and 2 cm.

All four angles of this rectangle are 90°, so they are equal. But its sides are not all equal, so it is not a square.

Therefore, P is true but its converse Q is false.

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QUESTION 3 *

Given any ΔABC, let us bisect the angles at B and C. The bisectors meet at the incentre I of the triangle.

Now extend the bisectors beyond I till they meet the opposite sides at E and F respectively, as shown.

Proposition: If AB = AC, then IE = IF.

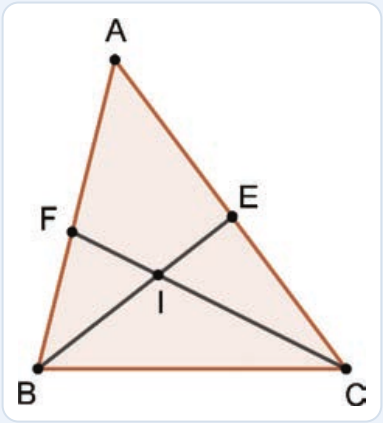


Fig. 9.1

ANSWER

Proposition P: If AB = AC, then IE = IF.
Converse Q: If IE = IF, then AB = AC.

P Proposition P — TRUE

Let AB = AC. Then the angles opposite these equal sides are equal: ∠ABC = ∠ACB. BE and CF bisect these equal angles, so their halves are equal:

∠FBI = ½∠ABC = ½∠ACB = ∠ECI ...(1)
∠IBC = ∠ICB, so in ΔIBC, IB = IC ...(2) (sides opposite equal angles)

Now compare ΔBIF and ΔCIE:

∠FBI = ∠ECI [from (1)]
BI = CI [from (2)]
∠BIF = ∠CIE [vertically opposite angles]

So ΔBIF ≅ ΔCIE by the ASA criterion. Therefore IF = IE (corresponding parts of congruent triangles).

Q Converse Q — FALSE

Counterexample: a triangle ABC with ∠A = 60°, ∠B = 80°, ∠C = 40°.

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$\angle IBC + \angle ICB = \frac{1}{2}(80° + 40°) = 60°$
$\angle BIC = 180° - 60° = 120°$
$\angle FIE = \angle BIC = 120°$ (vertically opposite angles)
$\angle FAE + \angle FIE = 60° + 120° = 180°$

The opposite angles of quadrilateral AFIE add up to 180°, so AFIE is a cyclic quadrilateral. Since I is the incentre, AI bisects ∠A: ∠FAI = ∠IAE = 30°. Equal angles in a circle stand on equal chords, so IF = IE.

But ∠B ≠ ∠C, so the sides opposite them are unequal: AB ≠ AC. (Check with BC = 1: AB ≈ 0.742, AC ≈ 1.137, while IE = IF ≈ 0.258.)

Therefore, P is true but its converse Q is false. (The same argument shows IE = IF whenever ∠A = 60°, even if AB ≠ AC.)

QUESTION 4

If $x = y$, then $a + x = a + y$, where x, y and a are any three numbers.

This proposition and its converse are routinely used while solving equations.

ANSWER

Proposition P: If $x = y$, then $a + x = a + y$.
Converse Q: If $a + x = a + y$, then $x = y$.

P Proposition P TRUE

If x and y are equal numbers, adding the same number a to each gives equal results. So $a + x = a + y$.

Q Converse Q TRUE

Suppose $a + x = a + y$. Add the number $(-a)$ to both sides (this is allowed by Proposition P, applied to the number $-a$):

$(-a) + (a + x) = (-a) + (a + y)$
$0 + x = 0 + y$
$x = y$

Therefore, both the proposition P and its converse Q are true.

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QUESTION 5

If *a* and *b* are perfect squares, then *ab* is a perfect square.

ANSWER

Proposition P: If a and b are perfect squares, then ab is a perfect square.
Converse Q: If ab is a perfect square, then a and b are perfect squares.

P Proposition P — TRUE

Let $a = m^2$ and $b = n^2$ for some integers m and n. Then

$$ab = m^2 \times n^2 = (mn)^2$$

Since mn is an integer, ab is a perfect square.

Q Converse Q — FALSE

Counterexample: $a = 2$ and $b = 8$.

Here $ab = 2 \times 8 = 16 = 4^2$, a perfect square. But 2 and 8 are not perfect squares.

Therefore, P is true but its converse Q is false.

QUESTION 6

In Questions 6 and 7, x and y are real numbers.

If $x = y$, then $x^2 = y^2$.

ANSWER

Proposition P: If $x = y$, then $x^2 = y^2$.
Converse Q: If $x^2 = y^2$, then $x = y$.

P Proposition P — TRUE

If $x = y$, then

$$x^2 = x \times x = y \times y = y^2$$

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Q Converse Q — FALSE

Counterexample: $x = 2$ and $y = -2$.

Here $x^2 = 4 = y^2$, but $x \neq y$.

Therefore, P is true but its converse Q is false.

QUESTION 7

In Questions 6 and 7, x and y are real numbers.

If $x = y$, then $x^3 = y^3$.

ANSWER

Proposition P: If $x = y$, then $x^3 = y^3$.
Converse Q: If $x^3 = y^3$, then $x = y$.

P Proposition P — TRUE

If $x = y$, then $x^3 = x \times x \times x = y \times y \times y = y^3$.

Q Converse Q — TRUE

Let $x^3 = y^3$ for real numbers x and y. Then $x^3 - y^3 = 0$, and factorising,

$$(x - y)(x^2 + xy + y^2) = 0$$

So either $x - y = 0$, or $x^2 + xy + y^2 = 0$. Suppose $x^2 + xy + y^2 = 0$. Since

$$x^2 + xy + y^2 = (x + y/2)^2 + (3/4)y^2$$

is a sum of two non-negative terms, both terms must be 0. Thus $y = 0$ and then $x = 0$, so again $x = y$. In both cases $x = y$.

Therefore, both the proposition P and its converse Q are true.

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QUESTION 8

In Questions 8–12, n is a positive integer.

If n is divisible by 24, then it is divisible by both 4 and 6.

ANSWER

Proposition P: If n is divisible by 24, then it is divisible by both 4 and 6.

Converse Q: If n is divisible by both 4 and 6, then it is divisible by 24.

P Proposition P — TRUE

Let $n = 24k$ for some integer k. Then

$$n = 4 \times (6k) \text{ and } n = 6 \times (4k)$$

So n is divisible by both 4 and 6.

Q Converse Q — FALSE

Counterexample: $n = 12$.

$12 = 4 \times 3$ and $12 = 6 \times 2$, so 12 is divisible by both 4 and 6. But 12 is not divisible by 24 (it is smaller than 24).

Therefore, P is true but its converse Q is false.

QUESTION 9

In Questions 8–12, n is a positive integer.

If n is divisible by 60, then it is divisible by both 5 and 12.

ANSWER

Proposition P: If n is divisible by 60, then it is divisible by both 5 and 12.

Converse Q: If n is divisible by both 5 and 12, then it is divisible by 60.

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P **Proposition P** TRUE

Let $n = 60k$ for some integer k. Then

$$n = 5 \times (12k) \text{ and } n = 12 \times (5k)$$

So n is divisible by both 5 and 12.

Q **Converse Q** TRUE

Let n be divisible by both 5 and 12. Since 12 divides n, write $n = 12m$ for some integer m. Now 5 divides $12m$. The number 5 is prime and does not divide 12 ($12 = 2^2 \times 3$), so 5 must divide m. Let $m = 5k$. Then

$$n = 12 \times 5k = 60k$$

So n is divisible by 60.

Therefore, both the proposition P and its converse Q are true.

QUESTION 10

In Questions 8–12, n is a positive integer.

If n is the square of a prime number, then it has exactly 3 factors.

ANSWER

Proposition P: If n is the square of a prime number, then it has exactly 3 factors.

Converse Q: If n has exactly 3 factors, then n is the square of a prime number.

P **Proposition P** TRUE

Let $n = p^2$, where p is prime. The only prime factor of n is p, so every factor of n is a product of p's: 1, p or p^2.

Factors of p^2: 1, p, p^2 (exactly 3)

Q **Converse Q** TRUE

Let n have exactly 3 factors. These are 1, n and one more factor, say f, where $1 < f < n$. The partner factor n/f is also a factor of n. It is not 1 or n (otherwise f would be n or 1), so it must be f itself. Hence $n = f \times f = f^2$.

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Also f is prime. If f had a factor g with $1 < g < f$, then g would be a fourth factor of n, which is not possible.

Therefore, both the proposition P and its converse Q are true.

QUESTION 11

In Questions 8–12, n is a positive integer.

If n is a product of two unequal prime numbers, then it has exactly 4 divisors.

ANSWER

Proposition P: If n is a product of two unequal prime numbers, then it has exactly 4 divisors.
Converse Q: If n has exactly 4 divisors, then it is a product of two unequal prime numbers.

P Proposition P — TRUE

Let $n = pq$, where p and q are different primes. The only prime factors of n are p and q, each occurring once, so its divisors are products of some of them:

Divisors of pq: $1, p, q, pq$ (exactly 4)

Q Converse Q — FALSE

Counterexample: $n = 8$.

The divisors of 8 are 1, 2, 4, 8, so it has exactly 4 divisors. But $8 = 2^3$ has only one prime factor, so it is not a product of two unequal primes.

Therefore, P is true but its converse Q is false.

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QUESTION 12

In Questions 8–12, n is a positive integer.

If n and $n + 3$ have no factors in common, then n is not a multiple of 3.

ANSWER

Here "no factors in common" means no common factor other than 1, since 1 is a factor of every number.

Proposition P: If n and $n + 3$ have no factors in common, then n is not a multiple of 3.
Converse Q: If n is not a multiple of 3, then n and $n + 3$ have no factors in common.

P Proposition P TRUE

Suppose n is a multiple of 3, say $n = 3k$. Then $n + 3 = 3k + 3 = 3(k + 1)$ is also a multiple of 3. So n and $n + 3$ would have the common factor 3. Hence if they have no common factor, n cannot be a multiple of 3.

Q Converse Q TRUE

Let n not be a multiple of 3. Suppose a number $d > 1$ divides both n and $n + 3$. Then d divides their difference:

$$(n + 3) - n = 3$$

The only divisor of 3 greater than 1 is 3, so $d = 3$ and 3 divides n. This contradicts the fact that n is not a multiple of 3. Hence n and $n + 3$ have no common factor other than 1.

Therefore, both the proposition P and its converse Q are true.

QUESTION 13

There are no known 'neat' expressions that generate only primes! Find counterexamples to the following claims.

(i) All numbers of the form $4n^2 + 1$ are prime.

(ii) All numbers of the form $n^2 + n + 11$ are prime.

(iii) All numbers of the form $4^n + 3$ are prime.

ANSWER

A single value of n for which the number is not prime is enough to disprove each claim.

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(i) $4n^2 + 1$ — **FALSE**

For n = 1, 2, 3 we get 5, 17, 37, which are all prime. But for n = 4:

$$4 \times 4^2 + 1 = 4 \times 16 + 1 = 65 = 5 \times 13$$

65 is composite. So n = 4 is a counterexample.

(ii) $n^2 + n + 11$ — **FALSE**

For n = 1, 2, ..., 9 the values 13, 17, 23, 31, 41, 53, 67, 83, 101 are all prime. But for n = 10:

$$10^2 + 10 + 11 = 100 + 10 + 11 = 121 = 11 \times 11$$

121 is composite. So n = 10 is a counterexample.

(iii) $4^n + 3$ — **FALSE**

For n = 1, 2, 3 we get 7, 19, 67, which are all prime. But for n = 4:

$$4^4 + 3 = 256 + 3 = 259 = 7 \times 37$$

259 is composite. So n = 4 is a counterexample.

QUESTION 14

Find counterexamples to the following statements.

(i) If n is a prime number, then $2^n - 1$ is a prime number.

(ii) If n is an even number, then $2^n + 1$ is a prime number.

ANSWER

(i) **If n is prime, then $2^n - 1$ is prime** — **FALSE**

For n = 2, 3, 5, 7 the numbers 3, 7, 31, 127 are all prime. But take the prime n = 11:

$$2^{11} - 1 = 2048 - 1 = 2047 = 23 \times 89$$

11 is prime but 2047 is composite. So n = 11 is a counterexample.

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(ii) If n is even, then $2^n + 1$ is prime — FALSE

For $n = 2$ and 4 we get 5 and 17, which are prime. But take the even number $n = 6$:

$$2^6 + 1 = 64 + 1 = 65 = 5 \times 13$$

6 is even but 65 is composite. So $n = 6$ is a counterexample.

QUESTION 15

Consider the statement: 'If a number is divisible by 8, then it is divisible by both 2 and 4'.

(i) Justify the statement.

(ii) Recall the divisibility shortcuts that we have studied for different numbers. To check whether a given number is divisible by 8, is it enough to check whether it is divisible by 2 and 4? Why or why not?

ANSWER

(i) Justify the statement

Let a number n be divisible by 8, so $n = 8k$ for some integer k. Then

$$n = 2 \times (4k) \text{ and } n = 4 \times (2k)$$

So n is divisible by both 2 and 4. The statement is true.

(ii) Is checking divisibility by 2 and 4 enough? — NO

The statement in part (i) is "divisible by 8 ⇒ divisible by 2 and 4". For the test we need its converse: "divisible by 2 and 4 ⇒ divisible by 8". This converse is false.

Counterexample: 12 (or 4).

12 is divisible by both 2 and 4, but 12 ÷ 8 leaves a remainder of 4, so it is not divisible by 8. (Also, any number divisible by 4 is already divisible by 2, so checking 2 adds nothing.)

Therefore, it is not enough to check divisibility by 2 and 4. The correct test for 8 is to check whether the number formed by its last three digits is divisible by 8.

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QUESTION 16

Recall that a shortcut to check whether a given number is divisible by 3 is to add the digits of the number and check if the sum is a multiple of 3. Express the relationship between 'a number is divisible by 3' and 'sum of the digits is a multiple of 3' using 'If-then' sentences.

ANSWER

⇒ If-then sentences

(1) If a number is divisible by 3, then the sum of its digits is a multiple of 3.

(2) If the sum of the digits of a number is a multiple of 3, then the number is divisible by 3.

∴ Why both are true

Take a number with digits a, b, c (the idea works for any number of digits):

$$N = 100a + 10b + c$$
$$= (99a + 9b) + (a + b + c)$$
$$= 9(11a + b) + (a + b + c)$$

Numbers such as 10, 100, 1000, … are always 1 more than 9, 99, 999, …, which are multiples of 3. So in general, N = (a multiple of 9) + (sum of the digits). Hence N is a multiple of 3 exactly when the sum of its digits is a multiple of 3. This proves both sentences.

Therefore, "a number is divisible by 3" and "the sum of its digits is a multiple of 3" imply each other.

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QUESTION 17

We have identified different types of quadrilaterals — squares, rectangles, parallelograms, rhombi, kites and trapezia. One can identify more types (e.g., we could create a category of quadrilaterals that have equal-length opposite sides).

Suppose we have identified a category of quadrilaterals called Q, and we have to construct a quadrilateral of this type. For this, we are to use two thin sticks, put them together as diagonals so that the quadrilateral obtained by joining their endpoints is of type Q (see the Fig. 9.2).

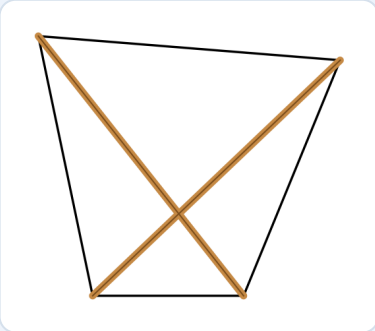


Fig. 9.2

(i) Suppose Q satisfies the following property.

If a quadrilateral is of type Q, then it has equal-length diagonals.

(a) Should the two sticks be of equal length? Why or why not?

(b) Will it matter how the two sticks are put together?

(ii) Instead of the property mentioned above, suppose Q satisfies the following property.

If a quadrilateral has equal diagonals, then it is of type Q.

What will be your answers to (a) and (b) now?

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ANSWER

(i) **Property: "If a quadrilateral is of type Q, then it has equal-length diagonals."**

(a) **Should the two sticks be of equal length?** YES

The sticks form the diagonals of the quadrilateral. Every quadrilateral of type Q has equal diagonals, so the two sticks must be of equal length. If they were unequal, the quadrilateral formed would have unequal diagonals and could not be of type Q.

(b) **Will it matter how the two sticks are put together?** YES, IT CAN

The property only says that equal diagonals are *necessary* for type Q. It does not say that every quadrilateral with equal diagonals is of type Q. So equal sticks alone may not be enough: the point where the sticks cross and the angle between them can matter, and they must be placed so that the quadrilateral formed has the other properties that type Q requires. (For example, if Q is the category of squares, the sticks must also bisect each other at right angles.)

(ii) Property: "If a quadrilateral has equal diagonals, then it is of type Q."

(a) Should the two sticks be of equal length? EQUAL STICKS WORK

With equal sticks, the quadrilateral obtained has equal diagonals, and by the property it is then of type Q. So using equal sticks is sufficient to get type Q. (The property says nothing about quadrilaterals with unequal diagonals, so with unequal sticks we cannot be sure of getting type Q.)

(b) Will it matter how the two sticks are put together? NO

No. However the two equal sticks are crossed (any crossing point, any angle), the quadrilateral obtained by joining their endpoints has equal diagonals, so it is of type Q.

Therefore, in (i) equal sticks are required but are not enough by themselves, whereas in (ii) equal sticks are sufficient and the way they are placed does not matter.

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CLASS 9 MATHS · GANITA MANJARI · CHAPTER 9

In-Text Questions

Solutions · 8 questions

IN-TEXT · CONVERSE ALWAYS TRUE?

If a proposition is true, then is its converse always true?

ANSWER

No. A proposition can be true while its converse is false. In Example 1, the proposition P "If it rains, then the road is wet" is true, but its converse Q "If the road is wet, then it has rained" may not be: the road may be wet because a tanker spilled water on it. That situation is a counterexample to Q.

Other examples: "If a quadrilateral is a square, then all its angles are equal" is true, but its converse is false (a 3 cm × 2 cm rectangle). On the other hand, for some propositions the converse is also true (for example, the Baudhāyana–Pythagoras theorem and its converse).

A true proposition need not have a true converse; each converse must be checked separately.

THINK AND REFLECT · STATEMENT 2

We have proved the first statement in an earlier grade. Is the second statement true? Can you prove it?

(Hint: Draw the altitude from the vertex containing the third angle.**)**

Statement 2: If two angles of a triangle are equal, then the sides opposite the equal angles have equal lengths.

ANSWER

Let ABC be a triangle with ∠B = ∠C. Draw the altitude AD from the vertex A (the vertex of the third angle) to BC, so that ∠ADB = ∠ADC = 90°. Compare ΔABD and ΔACD:

∠ABD = ∠ACD (given, ∠B = ∠C)

∠ADB = ∠ADC = 90° (AD is an altitude)

AD = AD (common side)

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So $\Delta ABD \cong \Delta ACD$ by the AAS criterion. Therefore AB = AC (corresponding sides of congruent triangles), i.e. the sides opposite the equal angles $\angle C$ and $\angle B$ are equal.

Yes, Statement 2 is true: if two angles of a triangle are equal, then the sides opposite them are equal. So Statement 1 and its converse Statement 2 are both true.

EXAMPLE 2

Proposition P: If a number is a multiple of 6, then it is a multiple of 3.

Converse Q: If a number is a multiple of 3, then it is a multiple of 6.

Proposition P is true. Justify why this is so.

Does this now mean that all multiples of 3 are also multiples of 6? This corresponds to the converse statement Q. Determine if it is true and if not, give a counterexample.

ANSWER

P **Justifying Proposition P** — **TRUE**

Let a number be a multiple of 6, so it equals $6k$ for some integer k. Then

$$6k = 3 \times (2k)$$

Since $2k$ is an integer, the number is a multiple of 3.

Q **Is the converse true?** — **FALSE**

Counterexample: the number 9 (or 3, 15, 21, …).

$9 = 3 \times 3$ is a multiple of 3, but 9 is not a multiple of 6 (9 ÷ 6 leaves remainder 3).

P is true but its converse Q is false: not all multiples of 3 are multiples of 6 (only the even ones are).

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EXAMPLE 3 AND DISCUSSION

n is any positive integer.

Proposition P: If n is a perfect square, then it has an odd number of factors.

Converse Q: If n has an odd number of factors, then it is a perfect square.

You may recall that we came across these statements in the previous grade. Which of them are true?

Discussion: What does the argument given in the chapter (about factor-partner pairs) prove? Statement P or Q?

ANSWER

Q What the argument proves — TRUE

The argument starts with "n has an odd number of factors", concludes that some factor-partner pair has the same number repeated, say (f, f), and hence that $n = f \times f$ is a perfect square. This is exactly "odd number of factors ⇒ perfect square".

(i) n has an odd number of factors
⇒ (ii) some factor-partner pair has a repeated number (f, f)
⇒ (iii) $n = f \times f$ is a perfect square

So the argument proves Q. It does not prove P (that every perfect square has an odd number of factors).

P Is Proposition P true? — TRUE

Let $n = m^2$ be a perfect square. Factors of n come in partner pairs (d, n/d). A pair has a repeated number only when $d \times d = n$, i.e. $d = m$; so exactly one pair, (m, m), has a repeated number. All the other factors fall into pairs of two different numbers (an even count), and m contributes one more factor. So the number of factors is even + 1 = odd.

Example: 36 has the pairs (1, 36), (2, 18), (3, 12), (4, 9) and (6, 6): $4 \times 2 + 1 = 9$ factors

Both P and Q are true: n is a perfect square if and only if it has an odd number of factors.

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EXAMPLE 4

Proposition P: If two triangles have the same area, then they are congruent.

Converse Q: If two triangles are congruent, then they have the same area.

Determine if these statements are true or not. Justify the true statements and give a counterexample for each false statement.

ANSWER

P Proposition P FALSE

Counterexample: a right triangle with legs 3 cm and 4 cm, and a right triangle with legs 2 cm and 6 cm.

Area of the first triangle = ½ × 3 × 4 = 6 cm²
Area of the second triangle = ½ × 2 × 6 = 6 cm²

Both triangles have the same area, but they are not congruent (their sides are different; for example, the hypotenuses are 5 cm and √40 cm).

Q Converse Q TRUE

If two triangles are congruent, one can be placed exactly on top of the other, so they cover the same region and hence have the same area. (Equivalently, corresponding sides, and so corresponding bases and heights, are equal, and area = ½ × base × height.)

P is false but its converse Q is true.

THINK AND REFLECT · BOTH FALSE

It can also happen that both the proposition and converse are false! Can you give an example?

ANSWER

Take the proposition P: “If a number is a multiple of 4, then it is a multiple of 6.” Its converse Q is: “If a number is a multiple of 6, then it is a multiple of 4.”

P Proposition P FALSE

Counterexample: 4 is a multiple of 4 but not of 6.

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Q **Converse Q** **FALSE**

Counterexample: 6 is a multiple of 6 but not of 4.

A geometric example: P: “If a quadrilateral has two equal sides, then it is a square”; Q: “If a quadrilateral is a square, then it has exactly two equal sides”. P is false (a 3 cm × 2 cm rectangle has equal opposite sides but is not a square) and Q is false (a square has four equal sides, not exactly two).

Yes, it can happen that both a proposition and its converse are false.

EXAMPLE 5

Let a, b, c be the sidelengths of a triangle. If the triangle is right-angled, then $a^2 + b^2 = c^2$. What is its converse? Is this converse true?

Let ΔABC be the triangle with sidelengths a, b, c such that $a^2 + b^2 = c^2$. Construct another right triangle ΔXYZ whose perpendicular sides YZ and XZ have sidelengths a and b.

What can we say about the length of XY? (Hint: Use the Baudhāyana–Pythagoras Theorem.)

What can we say about ΔABC and ΔXYZ? (Hint: Use the SSS criterion for congruence.)

ANSWER

1 **The converse**

Converse: Let a, b, c be the sidelengths of a triangle. If $a^2 + b^2 = c^2$, then the triangle is right-angled.

2 **Length of XY**

ΔXYZ is right-angled at Z with YZ = a and XZ = b. By the Baudhāyana–Pythagoras theorem,

$$XY^2 = YZ^2 + XZ^2 = a^2 + b^2 = c^2$$

so $XY = c$

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3 ΔABC and ΔXYZ

CB = a = ZY
CA = b = ZX
AB = c = XY

All three pairs of corresponding sides are equal, so ΔABC ≅ ΔXYZ by the SSS criterion. Corresponding angles are equal, so ∠C = ∠Z = 90°. Hence ΔABC had to be a right triangle after all.

The converse of the Baudhāyana–Pythagoras theorem is true: if $a^2 + b^2 = c^2$, the triangle is right-angled (the angle opposite the side c is 90°).

THINK AND REFLECT · EXAMPLES

1. Identify real-life examples in which a proposition is true but not the converse. Give nice counterexamples!

2. Give more examples from geometry as well as number theory in which a proposition is true but not its converse. Give appropriate counterexamples.

ANSWER

1. Real-life examples

P: If a person lives in Delhi, then the person lives in India. Q: If a person lives in India, then the person lives in Delhi. Counterexample: a person living in Mumbai.

P: If a student is in Class 9, then the student is in school. Q: If a student is in school, then the student is in Class 9. Counterexample: a student in Class 6.

P: If a person has a bicycle, then the person owns a vehicle. Q: If a person owns a vehicle, then the person has a bicycle. Counterexample: a person who owns only a scooter.

2. Geometry

P: If a quadrilateral is a rhombus, then its diagonals are perpendicular. Q: If the diagonals of a quadrilateral are perpendicular, then it is a rhombus. P is true; Q is false: a kite with unequal adjacent sides has perpendicular diagonals but is not a rhombus.

P: If a triangle is equilateral, then it is isosceles. Q: If a triangle is isosceles, then it is equilateral. P is true; Q is false: a triangle with sides 5, 5, 6.

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2. **Number theory**

P: If a number is divisible by 10, then it is divisible by 5. Q: If a number is divisible by 5, then it is divisible by 10. P is true; Q is false: 15.

P: If p is a prime number greater than 2, then p is odd. Q: If a number is odd, then it is a prime number greater than 2. P is true; Q is false: 9.

In each case the proposition is true, but the converse fails for at least one counterexample, so a true proposition does not guarantee a true converse.

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