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SOLUTIONS BOOK

# CLASS 9

## Mathematics · Ganita Manjari

CHAPTER 8

## Predicting What Comes Next: Exploring Sequences and Progressions

Complete Chapter Solutions

Examples & In-text Questions | Exercise Sets 8.1 – 8.3 | End-of-Chapter Exercises

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§8.1

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 8

# Sequences, Explicit and Recursive Rules

Examples, exercises & in-text questions · 17 items

**Note:** Solutions of the Examples, in-text Exercises, Think and Reflect prompts and in-text questions of Sections 8.1 (Introduction to Sequences), 8.2 (Explicit Rule) and 8.3 (Recursive Rule), in the order in which they appear in the chapter.

THINK AND REFLECT

**Natural numbers 1, 2, 3, 4, 5, 6, …; Odd numbers 1, 3, 5, 7, 9, 11, …; Triangular numbers 1, 3, 6, 10, 15, 21, …; Square numbers 1, 4, 9, 16, 25, 36, …**

**Can you describe the pattern in each of the above sequences? Can you predict the next few numbers in these sequences?**

ANSWER

1 **Natural numbers: 1, 2, 3, 4, 5, 6, …**

Each term is 1 more than the previous term. Next terms: 7, 8, 9.

2 **Odd numbers: 1, 3, 5, 7, 9, 11, …**

Each term is 2 more than the previous term. Next terms: 13, 15, 17.

3 **Triangular numbers: 1, 3, 6, 10, 15, 21, …**

The differences between consecutive terms are 2, 3, 4, 5, 6, so the differences increase by 1 each time. Each term is the sum of the natural numbers up to that position (1, 1 + 2, 1 + 2 + 3, …). The next differences are 7, 8, 9, so the next terms are 21 + 7 = 28, 28 + 8 = 36 and 36 + 9 = 45.

4 **Square numbers: 1, 4, 9, 16, 25, 36, …**

Each term is a perfect square ($1^2$, $2^2$, $3^2$, …). The differences are 3, 5, 7, 9, 11, the odd numbers. Next terms: $7^2 = 49$, $8^2 = 64$ and $9^2 = 81$.

**Next three terms: natural 7, 8, 9; odd 13, 15, 17; triangular 28, 36, 45; square 49, 64, 81.**

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IN-TEXT QUESTION

**Can you think of other finite sequences that you see in your daily life?**

ANSWER

A finite sequence has a last term. Examples from daily life:

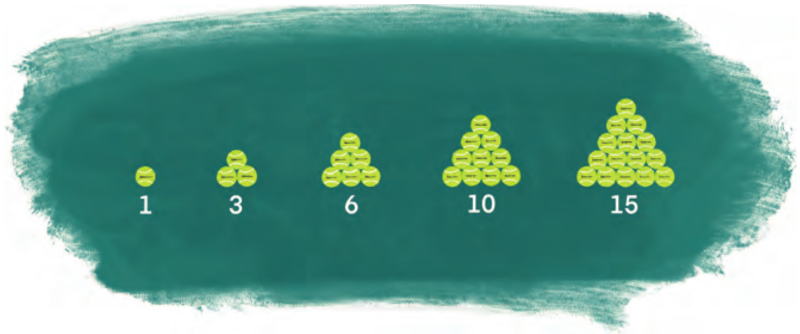
The number of days in the 12 months of a year: 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31
The days of a week numbered 1 to 7: 1, 2, 3, 4, 5, 6, 7
The roll numbers of the students of a class: 1, 2, 3, ..., 40
The house numbers on one side of a street: 2, 4, 6, 8, 10, 12
The pages of a book numbered from 1 to its last page

**Any list of numbers that stops after a fixed number of terms is a finite sequence.**

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IN-TEXT QUESTION

**Fig. 8.1 represents each triangular number by a triangular array of dots: 1, 3, 6, 10, 15. Can you draw the patterns for the next two terms of the sequence?**



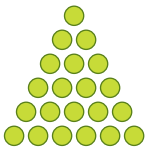

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ANSWER

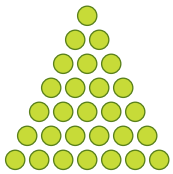
**1 Find the next two terms**

Triangular numbers are 1, 1 + 2, 1 + 2 + 3, ... so the rows of the array have 1, 2, 3, ... dots.
6th term = 1 + 2 + 3 + 4 + 5 + 6 = 21 (rows of 1, 2, 3, 4, 5, 6 dots)
7th term = 21 + 7 = 28 (rows of 1, 2, 3, 4, 5, 6, 7 dots)

**2** **The patterns for the 6th and 7th terms**



21 (6 rows)

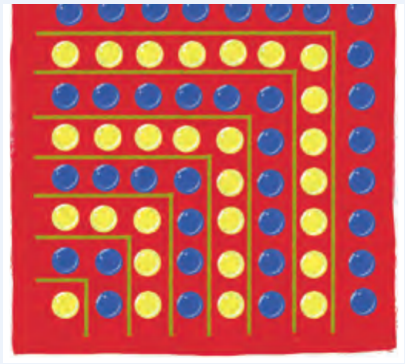


28 (7 rows)

**The next two patterns are triangular arrays of 21 dots (6 rows) and 28 dots (7 rows).**

IN-TEXT QUESTION

**The relationship between the odd numbers and square numbers can be represented by the diagram in Fig. 8.2. Can you explain the relationship?**



ANSWER

**1** **The diagram**

The 8 × 8 square of dots is built from L-shaped layers (alternating yellow and blue). The first layer is 1 dot, the next L has 3 dots, the next has 5, then 7, and so on: each new L-shaped layer has the next odd number of dots.

$1 = 1^2 = 1$
$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$
$1 + 3 + 5 + \ldots + (2n - 1) = n^2$

**2** **Why it works**

Going from an n × n square to an (n + 1) × (n + 1) square, one row of n + 1 dots and one column of n dots are added: n + 1 + n = 2n + 1 dots, which is the next odd number. So the sum of the first n odd numbers is the n-th square number.

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**The sum of the first n odd numbers is $n^2$, so each square number is the sum of the odd numbers up to that term.**

EXERCISE

**Consider the sequence 1, 4, 7, 10, 13, … Can you predict the next four terms? Can you derive the first 10 terms of the sequence obtained by adding all the terms up to a given term of this sequence? (Hint: The first term is 1. The second term is 1 + 4 = 5, the third term is 1 + 4 + 7 = 12, and so on.)**

ANSWER

**1 Next four terms**

Each term is 3 more than the previous term, so after 13 come 16, 19, 22 and 25.

**2 First 10 terms of the sequence 1, 4, 7, 10, 13, 16, 19, 22, 25, 28**

Sum up to the 1st term: 1
Sum up to the 2nd term: 1 + 4 = 5
Sum up to the 3rd term: 5 + 7 = 12
Sum up to the 4th term: 12 + 10 = 22
Sum up to the 5th term: 22 + 13 = 35
Sum up to the 6th term: 35 + 16 = 51
Sum up to the 7th term: 51 + 19 = 70
Sum up to the 8th term: 70 + 22 = 92
Sum up to the 9th term: 92 + 25 = 117
Sum up to the 10th term: 117 + 28 = 145

**Next four terms: 16, 19, 22, 25. The new sequence of sums begins 1, 5, 12, 22, 35, 51, 70, 92, 117, 145.**

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EXERCISE

**Can you write $t_5$, $t_6$, $t_7$ and $t_8$ for the sequence of triangular numbers?**

ANSWER

The triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, ...
$t_5 = 1 + 2 + 3 + 4 + 5 = 15$
$t_6 = 15 + 6 = 21$
$t_7 = 21 + 7 = 28$
$t_8 = 28 + 8 = 36$

**$t_5 = 15$, $t_6 = 21$, $t_7 = 28$, $t_8 = 36$.**

THINK AND REFLECT

**Can you think of any other kinds of sequences? List out five different types of sequences and discuss their properties with your friends.**

ANSWER

### 1 Five types of sequences

1. Increasing by a fixed amount: 2, 5, 8, 11, ... (difference 3 every time)
2. Decreasing by a fixed amount: 11, 7, 3, –1, –5, ... (difference –4)
3. Each term is a fixed multiple of the previous one: 3, 6, 12, 24, ... (multiplied by 2)
4. Decreasing fractions: 1, 1/2, 1/3, 1/4, ... (unit fractions)
5. Each term from the two previous terms: 1, 2, 3, 5, 8, 13, ... (add the previous two)

### 2 Some properties

Sequences 1 and 2 have a constant difference; sequence 3 has a constant ratio; sequence 4 gets smaller and smaller (the terms are positive but approach 0); sequence 5 grows faster and faster. Other kinds: the primes 2, 3, 5, 7, 11, ... (no simple rule), alternating sequences such as 1, –1, 1, –1, ... and constant sequences such as 5, 5, 5, ....

**Sequences may be increasing, decreasing, alternating or constant, with or without a simple rule.**

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**EXAMPLE 1**

**Consider the expression $u_n = 2n - 1$. This states that the nth term of the sequence is given by the rule 2n – 1. When we substitute 1, 2, 3, … for n in the expression 2n – 1, we get $u_1$, $u_2$, $u_3$, etc. Which sequence is it?**

**ANSWER**

**1** **Substitute n = 1, 2, 3, …**

$u_1 = 2 \times 1 - 1 = 1$
$u_2 = 2 \times 2 - 1 = 3$
$u_3 = 2 \times 3 - 1 = 5$
$u_4 = 2 \times 4 - 1 = 7$

**$u_n = 2n - 1$ is the explicit rule for the nth term of the sequence of odd numbers 1, 3, 5, 7, ….**

**THINK AND REFLECT**

**Why is it useful to have an explicit formula for the nth term of a sequence?**

**ANSWER**

With an explicit formula we can find any term directly by substituting its position n: the 20th term, the 53rd term or the 300th term, without calculating all the earlier terms. We can also check whether a number is a term of the sequence and find its position by solving an equation for n.

**An explicit formula gives any term directly, without knowing the earlier terms.**

**EXERCISE**

**Using the explicit rule $u_n = 2n - 1$, find the 53rd term, the 108th term, and the 1170th term of the odd number sequence.**

**ANSWER**

$u_{53} = 2 \times 53 - 1 = 106 - 1 = 105$
$u_{108} = 2 \times 108 - 1 = 216 - 1 = 215$
$u_{1170} = 2 \times 1170 - 1 = 2340 - 1 = 2339$

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**The 53rd, 108th and 1170th terms are 105, 215 and 2339.**

EXAMPLE 2

**Consider the sequence that is generated by the explicit formula $s_n = 5n - 2$. Can you write the first 6 terms of this sequence? What is the 100th term? The 1000th term? Let us check if the numbers 308 and 473 are terms of this sequence. Can you explain why we need n to be a natural number?**

ANSWER

**1 First 6 terms**

$s_1 = 3$, $s_2 = 8$, $s_3 = 13$, $s_4 = 18$, $s_5 = 23$, $s_6 = 28$

**2 100th and 1000th terms**

$s_{100} = 5 \times 100 - 2 = 498$
$s_{1000} = 5 \times 1000 - 2 = 4998$

**3 Is 308 a term?** YES

$5n - 2 = 308$, so $5n = 310$ and $n = 62$
308 is the 62nd term

**4 Is 473 a term?** YES

$5n - 2 = 473$, so $5n = 475$ and $n = 95$
473 is the 95th term

(The chapter text works out the equation $5n - 2 = 471$, which gives $n = 94.6$, not a natural number, so 471 is not a term. The number 473 mentioned in the question is a term: it is the 95th.)

**5 Why must n be a natural number?**

The position of a term is a counting number 1, 2, 3, ... (first term, second term, ...). A value such as 94.6 does not correspond to any position in the sequence, so a number is a term only if solving for n gives a natural number.

**$s_{100} = 498$ and $s_{1000} = 4998$; 308 is the 62nd term and 473 the 95th term; 471 is not a term ($n = 94.6$).**

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THINK AND REFLECT

**Can you find the rule describing the nth term of the sequence of square numbers?**

ANSWER

1st term $1 = 1^2$, 2nd term $4 = 2^2$, 3rd term $9 = 3^2$, 4th term $16 = 4^2$

**The nth term of the sequence of square numbers is $t_n = n^2$.**

IN-TEXT QUESTION

**Here is the sequence of the first ten prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Do you see any pattern in this sequence? Can you think of a rule that can predict the next few prime numbers?**

ANSWER

**1 Is there a pattern?** NO SIMPLE RULE

The differences between consecutive primes are 1, 2, 2, 4, 2, 4, 2, 4, 6: they are irregular, so there is no clear pattern or simple formula for the nth prime. (All primes except 2 are odd.)

**2 Predicting the next primes**

We can continue the list by testing numbers one by one for factors. After 29 the next primes are 31, 37, 41, 43, 47, ....

**There is no simple rule that predicts the primes; they are found by checking each number.**

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EXERCISE

**Consider the expression $t_n = 3n - 7$.**

**(i) Find its first, second, third, 12th, 18th and 50th terms.**

**(ii) Which term of the sequence is 332?**

**(iii) Is 557 a term of this sequence? Why or why not?**

ANSWER

**(i) Terms of $t_n = 3n - 7$**

$t_1 = 3 - 7 = -4$
$t_2 = 6 - 7 = -1$
$t_3 = 9 - 7 = 2$
$t_{12} = 36 - 7 = 29$
$t_{18} = 54 - 7 = 47$
$t_{50} = 150 - 7 = 143$

**(ii) Which term is 332?** 113TH TERM

$3n - 7 = 332$
$3n = 339$, so $n = 113$

**(iii) Is 557 a term?** YES, 188TH

$3n - 7 = 557$
$3n = 564$, so $n = 188$

$n = 188$ is a natural number, so 557 is a term of the sequence (the 188th term).

**(i) -4, -1, 2, 29, 47, 143; (ii) 332 is the 113th term; (iii) yes, 557 is the 188th term.**

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**EXAMPLE 3**

**Find the first four terms of the sequence given by the recursive rule $u_1 = 1$, $u_n = 2u_{n-1} + 3$ for $n \geq 2$. Is 133 a term of this sequence?**

**ANSWER**

**1 First four terms**

$u_2 = 2u_1 + 3 = 2 \times 1 + 3 = 5$
$u_3 = 2u_2 + 3 = 2 \times 5 + 3 = 13$
$u_4 = 2u_3 + 3 = 2 \times 13 + 3 = 29$

**2 Is 133 a term?** NO

$u_5 = 2 \times 29 + 3 = 61$
$u_6 = 2 \times 61 + 3 = 125$
$u_7 = 2 \times 125 + 3 = 253$

The terms increase: 125 is the 6th term and the 7th term is already 253, so 133 does not occur. (Another way: $u_n = 2^{n+1} - 3$, and $133 + 3 = 136$ is not a power of 2.)

**The first four terms are 1, 5, 13, 29. The number 133 is not a term.**

**EXAMPLE 4**

**Find the first four terms of the sequence given by the recursive rule $s_1 = 3$, $s_n = s_{n-1}(s_{n-1} - 1)$ for $n \geq 2$.**

**ANSWER**

$s_2 = s_1(s_1 - 1) = 3 \times (3 - 1) = 3 \times 2 = 6$
$s_3 = s_2(s_2 - 1) = 6 \times (6 - 1) = 6 \times 5 = 30$
$s_4 = s_3(s_3 - 1) = 30 \times (30 - 1) = 30 \times 29 = 870$

**The first four terms are 3, 6, 30, 870.**

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IN-TEXT QUESTION

**The Virahānka–Fibonacci sequence is $V_1 = 1$, $V_2 = 2$, and $V_n = V_{n-1} + V_{n-2}$ for $n \geq 3$, giving 1, 2, 3, 5, 8, 13, 21, 34, … Can you write the next two terms of this sequence?**

ANSWER

$V_9 = V_8 + V_7 = 34 + 21 = 55$
$V_{10} = V_9 + V_8 = 55 + 34 = 89$

**The next two terms are 55 and 89.**

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8.1

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# Exercise Set 8.1

Solutions · 6 questions

**QUESTION 1**

**Find the first five terms of the sequence in which the nth term is given by (i) $t_n = 3n - 4$, (ii) $t_n = 2 - 5n$, and (iii) $t_n = n^2 - 2n + 3$ for $n \geq 1$.**

**ANSWER**

**(i)** $t_n = 3n - 4$

$t_1 = -1, t_2 = 2, t_3 = 5, t_4 = 8, t_5 = 11$

**(ii)** $t_n = 2 - 5n$

$t_1 = 2 - 5 = -3$
$t_2 = 2 - 10 = -8$
$t_3 = 2 - 15 = -13$
$t_4 = 2 - 20 = -18$
$t_5 = 2 - 25 = -23$

**(iii)** $t_n = n^2 - 2n + 3$

$t_1 = 1 - 2 + 3 = 2$
$t_2 = 4 - 4 + 3 = 3$
$t_3 = 9 - 6 + 3 = 6$
$t_4 = 16 - 8 + 3 = 11$
$t_5 = 25 - 10 + 3 = 18$

**(i) -1, 2, 5, 8, 11 (ii) -3, -8, -13, -18, -23 (iii) 2, 3, 6, 11, 18**

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**QUESTION 2**

**Find the 10th and 15th terms of the sequence $t_n = 5n - 3$ for $n \geq 1$.**

**ANSWER**

$t_{10} = 5 \times 10 - 3 = 47$
$t_{15} = 5 \times 15 - 3 = 72$

**The 10th term is 47 and the 15th term is 72.**

**QUESTION 3**

**Determine whether 97 and 172 are terms of the sequence $t_n = 5n - 3$ for $n \geq 1$.**

**ANSWER**

**1 Is 97 a term?** YES

$5n - 3 = 97$, so $5n = 100$ and $n = 20$
97 is the 20th term

**2 Is 172 a term?** YES

$5n - 3 = 172$, so $5n = 175$ and $n = 35$
172 is the 35th term

**Both are terms: 97 is the 20th term and 172 is the 35th term.**

**QUESTION 4**

**Which term of the sequence $t_n = 5n - 3$ for $n \geq 1$ is 607?**

**ANSWER**

$5n - 3 = 607$
$5n = 610$, so $n = 122$

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**607 is the 122nd term.**

**QUESTION 5**

**A sequence is given by the recursive rule $t_1 = -5$, $t_{n+1} = t_n + 3$ for $n \geq 1$. Find the first five terms of the sequence. Is 52 a term of this sequence? If so, which term is it?**

**ANSWER**

**1 First five terms**

$t_1 = -5$
$t_2 = -5 + 3 = -2$
$t_3 = -2 + 3 = 1$
$t_4 = 1 + 3 = 4$
$t_5 = 4 + 3 = 7$

**2 Is 52 a term?** YES, 20TH

The terms increase by 3 each time, so $t_n = -5 + (n - 1) \times 3 = 3n - 8$.

$3n - 8 = 52$
$3n = 60$, so $n = 20$

**The first five terms are –5, –2, 1, 4, 7; 52 is a term, the 20th term.**

**QUESTION 6**

**Let $T_1 = 1$, $T_2 = 2$, $T_3 = 4$, and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Find $T_4$, $T_5$, $T_6$, $T_7$, and $T_8$.**

**ANSWER**

$T_4 = T_3 + T_2 + T_1 = 4 + 2 + 1 = 7$
$T_5 = T_4 + T_3 + T_2 = 7 + 4 + 2 = 13$
$T_6 = T_5 + T_4 + T_3 = 13 + 7 + 4 = 24$
$T_7 = T_6 + T_5 + T_4 = 24 + 13 + 7 = 44$
$T_8 = T_7 + T_6 + T_5 = 44 + 24 + 13 = 81$

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**$T_4 = 7$, $T_5 = 13$, $T_6 = 24$, $T_7 = 44$, $T_8 = 81$.**

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§8.4

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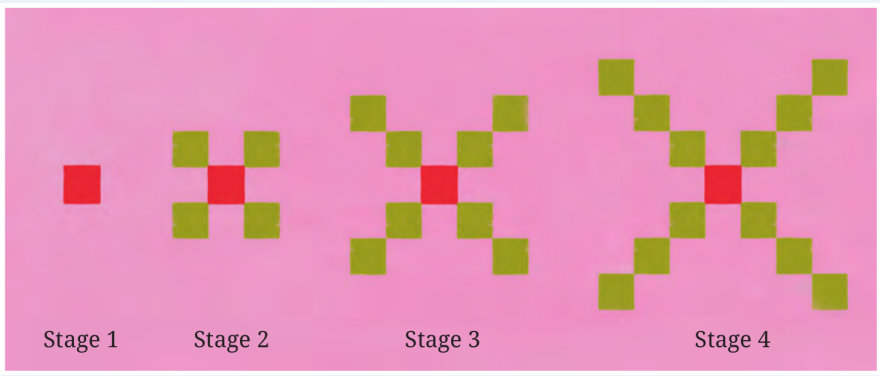
# Arithmetic Progressions and Sums

Examples, exercises & in-text questions · 11 items

**Note:** Solutions of the Examples, in-text Exercises, Think and Reflect prompts and in-text questions of Sections 8.4 (Arithmetic Progressions) and 8.5 (Sum of the First n Natural Numbers), in the order in which they appear in the chapter.

THINK AND REFLECT

**Fig. 8.3 shows a growing pattern of squares; the number of tiny squares in Stages 1 to 4 is 1, 5, 9, 13.**




**Can you predict the number of squares in Stages 5 and 6 of the sequence? In Stages 10, 11 and 12? In Stage 20? At any stage?**

ANSWER

### 1 Rule

At each stage 4 squares are added, starting from 1, so the number of squares at Stage n is 1 + (n – 1) × 4 = 4n – 3.

### 2 Predictions

Stage 5: 4 × 5 – 3 = 17
Stage 6: 4 × 6 – 3 = 21
Stage 10: 37, Stage 11: 41, Stage 12: 45
Stage 20: 4 × 20 – 3 = 77
Stage n: 4n – 3

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**Stages 5 and 6: 17 and 21; Stages 10, 11, 12: 37, 41, 45; Stage 20: 77; any stage: 4n – 3.**

THINK AND REFLECT

**Consider all the sequences we have discussed so far in this chapter. Which ones are arithmetic progressions and which ones are not? Can you justify your claim?**

ANSWER

A sequence is an AP if the difference between consecutive terms is constant.

| Sequence | Differences | AP? |
|---|---|---|
| **Natural numbers 1, 2, 3, …** | 1, 1, 1, … | Yes (d = 1) |
| **Odd numbers 1, 3, 5, …** | 2, 2, 2, … | Yes (d = 2) |
| **Triangular numbers 1, 3, 6, 10, …** | 2, 3, 4, 5, … | No |
| **Square numbers 1, 4, 9, 16, …** | 3, 5, 7, 9, … | No |
| **1, 4, 7, 10, 13, …** | 3, 3, 3, … | Yes (d = 3) |
| **1, 1/2, 1/3, 1/4, …** | –1/2, –1/6, –1/12, … | No |
| **–7, –3, 1, 5, 9, …** | 4, 4, 4, … | Yes (d = 4) |
| **Primes 2, 3, 5, 7, 11, …** | 1, 2, 2, 4, … | No |
| **1, 5, 13, 29, … and 3, 6, 30, 870** | 4, 8, 16 and 3, 24, 840 | No |
| **Virahānka 1, 2, 3, 5, 8, …** | 1, 1, 2, 3, … | No |
| **Squares of Fig. 8.3: 1, 5, 9, 13, …** | 4, 4, 4, … | Yes (d = 4) |
| **11, 7, 3, –1, –5, …** | –4, –4, –4, … | Yes (d = –4) |

**The AP sequences are natural, odd, 1, 4, 7, 10, …, –7, –3, 1, 5, …, 1, 5, 9, 13, … and 11, 7, 3, –1, …; the others have changing differences.**

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EXERCISE

**Verify that the following sequences are arithmetic progressions and write their nth terms. What do you observe when you plot the ordered pairs emerging from them?**

**(i) 2, 5, 8, 11, …**

**(ii) –5, –1, 3, 7, …**

ANSWER

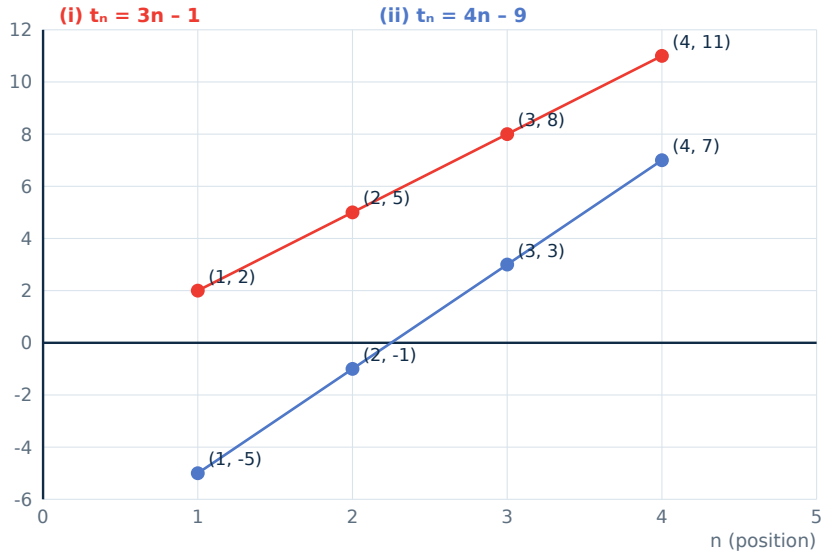
**(i)** **2, 5, 8, 11, …**

Differences: 5 – 2 = 3, 8 – 5 = 3, 11 – 8 = 3, so it is an AP with a = 2, d = 3
$t_n = 2 + (n - 1) \times 3 = 3n - 1$

**(ii)** **–5, –1, 3, 7, …**

Differences: –1 – (–5) = 4, 3 – (–1) = 4, 7 – 3 = 4, so it is an AP with a = –5, d = 4
$t_n = -5 + (n - 1) \times 4 = 4n - 9$

**3** **Plotting the ordered pairs $(n, t_n)$**




The points of each sequence lie on a straight line. The slope of the line is the common difference (3 for (i) and 4 for (ii)).

**(i) $t_n = 3n - 1$; (ii) $t_n = 4n - 9$. The ordered pairs of an AP lie on a straight line.**

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EXERCISE

**Using the formula $t_n = a + (n - 1) \times d$, find the nth term of the following arithmetic progressions.**

**(i) 1/2, 5/2, 9/2, 13/2, …**

**(ii) 1.5, 3.5, 5.5, 7.5, …**

ANSWER

**(i) 1/2, 5/2, 9/2, 13/2, …**

$a = 1/2$, $d = 5/2 - 1/2 = 2$
$t_n = 1/2 + (n - 1) \times 2 = 2n - 3/2 = (4n - 3)/2$

**(ii) 1.5, 3.5, 5.5, 7.5, …**

$a = 1.5$, $d = 3.5 - 1.5 = 2$
$t_n = 1.5 + (n - 1) \times 2 = 2n - 0.5$

**(i) $t_n = 2n - 3/2$; (ii) $t_n = 2n - 0.5$.**

EXERCISE

**Find recursive rules for the APs in the previous exercises.**

ANSWER

For an AP with first term a and common difference d, the recursive rule is $t_1 = a$, $t_n = t_{n-1} + d$ for $n \geq 2$.

2, 5, 8, 11, …: $t_1 = 2$, $t_n = t_{n-1} + 3$
–5, –1, 3, 7, …: $t_1 = -5$, $t_n = t_{n-1} + 4$
1/2, 5/2, 9/2, …: $t_1 = 1/2$, $t_n = t_{n-1} + 2$
1.5, 3.5, 5.5, …: $t_1 = 1.5$, $t_n = t_{n-1} + 2$

**Each recursive rule is $t_1 = a$, $t_n = t_{n-1} + d$, with $n \geq 2$.**

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EXAMPLE 5

**A person books a taxi to travel in the city. The taxi company charges a fixed booking fee of ₹200 plus ₹40 per kilometre travelled. Let us write the sequence representing the total fare after travelling 1 km, 2 km, 3 km, and so on. If the person travels 10 km, what will be the total fare?**

ANSWER

**1 The sequence of fares**

After 1 km: 200 + 40 = ₹240
After 2 km: 200 + 80 = ₹280
After 3 km: 200 + 120 = ₹320

The sequence 240, 280, 320, ... is an AP with a = 240 and d = 40.

**2 nth term and the fare for 10 km**

$t_n = 240 + (n - 1) \times 40 = 200 + 40n$
$t_{10} = 200 + 40 \times 10 = ₹600$

**The fare after n km is ₹(200 + 40n); the total fare for 10 km is ₹600.**

IN-TEXT QUESTION

**Can you find the sum of the first ten natural numbers without actually adding all of them?**

ANSWER

**1 Write the sum forwards and backwards**

S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10
S = 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1

**2 Add the two equations**

Each pair adds up to 11: 1 + 10, 2 + 9, ..., 10 + 1
2S = 11 × 10 = 110, so S = 55

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**The sum of the first 10 natural numbers is 55.**

THINK AND REFLECT

**Can the same approach be used to find the sum of 1 + 2 + 3 + … + 100?**

ANSWER

**1** **Yes: write the sum forwards and backwards** YES

$S = 1 + 2 + 3 + \ldots + 99 + 100$
$S = 100 + 99 + 98 + \ldots + 2 + 1$
Each pair adds up to 101, and there are 100 pairs: $2S = 101 \times 100 = 10100$
$S = 5050$

**1 + 2 + 3 + … + 100 = 5050.**

THINK AND REFLECT

**Can you use this formula ($S_n = n(n + 1)/2$) to find $S_{20}$, $S_{50}$ or $S_{1000}$?**

ANSWER

$S_{20} = 20 \times 21 \div 2 = 210$
$S_{50} = 50 \times 51 \div 2 = 1275$
$S_{1000} = 1000 \times 1001 \div 2 = 500500$

**$S_{20} = 210$, $S_{50} = 1275$, $S_{1000} = 500500$.**

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WORKED EXAMPLE

**The formula can be used to find the sum of consecutive numbers such as 25 + 26 + 27 + … + 58.**

ANSWER

**1 Subtract two sums from 1**

$25 + 26 + \ldots + 58 = (1 + 2 + \ldots + 58) - (1 + 2 + \ldots + 24) = S_{58} - S_{24}$

**2 Calculate**

$S_{58} = 58 \times 59 \div 2 = 1711$
$S_{24} = 24 \times 25 \div 2 = 300$
$1711 - 300 = 1411$

Check: there are 34 terms with first term 25 and last term 58, so the sum is $34 \times (25 + 58) \div 2 = 17 \times 83 = 1411$ ✓

**25 + 26 + 27 + … + 58 = 1411.**

THINK AND REFLECT

**The nth term of the sequence of triangular numbers 1, 3, 6, 10, 15, … is the sum of the first n natural numbers, so $t_n = n(n + 1)/2$. Can you use this to find the 10th, 17th and 80th triangular numbers?**

ANSWER

$t_{10} = 10 \times 11 \div 2 = 55$
$t_{17} = 17 \times 18 \div 2 = 153$
$t_{80} = 80 \times 81 \div 2 = 3240$

**The 10th, 17th and 80th triangular numbers are 55, 153 and 3240.**

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8.2

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 8

# Exercise Set 8.2

Solutions · 7 questions

**QUESTION 1**

**Find the 10th and 26th terms of the AP: 3, 8, 13, 18, ….**

**ANSWER**

$a = 3$, $d = 5$, $t_n = 3 + (n - 1) \times 5 = 5n - 2$

$t_{10} = 3 + 9 \times 5 = 48$

$t_{26} = 3 + 25 \times 5 = 128$

**The 10th term is 48 and the 26th term is 128.**

**QUESTION 2**

**Which term of the AP: 21, 18, 15, … is –81? Also, is 0 a term of this AP? Give reasons for your answer.**

**ANSWER**

$a = 21$, $d = -3$, $t_n = 21 + (n - 1)(-3) = 24 - 3n$

**1 Which term is –81?** 35TH TERM

$24 - 3n = -81$

$3n = 105$, so $n = 35$

**2 Is 0 a term?** YES, 8TH

$24 - 3n = 0$

$3n = 24$, so $n = 8$

$n = 8$ is a natural number, so 0 is a term: 21, 18, 15, 12, 9, 6, 3, 0, …

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**–81 is the 35th term; 0 is a term of the AP (the 8th term).**

QUESTION 3

**Find the nth term of the AP: 11, 8, 5, 2 … Write the recursive rule for this AP.**

ANSWER

$a = 11$, $d = 8 - 11 = -3$
$t_n = 11 + (n - 1)(-3) = 14 - 3n$
Recursive rule: $t_1 = 11$, $t_n = t_{n-1} - 3$ for $n \geq 2$

**$t_n = 14 - 3n$; recursive rule $t_1 = 11$, $t_n = t_{n-1} - 3$.**

QUESTION 4

**An AP consists of 50 terms in which the 3rd term is 12 and the last term is 106. Find the 29th term. (Hint: If 'a' is the first term and 'd' the common difference, then we arrive at the equations a + 2d = 12 and a + 49d = 106. Solve this pair of linear equations for 'a' and 'd'.)**

ANSWER

**1 Solve for a and d**

$a + 49d = 106$
$a + 2d = 12$
Subtract: $47d = 94$, so $d = 2$
$a = 12 - 2 \times 2 = 8$

**2 The 29th term**

$t_{29} = a + 28d = 8 + 28 \times 2 = 64$

**The 29th term is 64 (a = 8, d = 2).**

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QUESTION 5

**How many 2-digit numbers are divisible by 3? What is the sum of all these 2-digit numbers?**

ANSWER

**1 The AP**

The 2-digit multiples of 3 are 12, 15, 18, ..., 99, an AP with a = 12, d = 3 and last term 99.

**2 How many?** 30

$12 + (n - 1) \times 3 = 99$
$(n - 1) \times 3 = 87$, so $n - 1 = 29$ and $n = 30$

**3 Their sum** 1665

$\text{Sum} = (\text{number of terms}) \times (\text{first} + \text{last}) \div 2 = 30 \times (12 + 99) \div 2 = 30 \times 55.5 = 1665$

Check using $S_{33} - S_3$: the multiples of 3 up to 99 are $3 \times (1 + 2 + \ldots + 33) = 3 \times 561 = 1683$, minus $3 + 6 + 9 = 18$, gives 1665 ✓

**There are 30 two-digit numbers divisible by 3, and their sum is 1665.**

QUESTION 6

**Harish started work at an annual salary of ₹5,00,000 and received an increment of ₹20,000 each year. After how many years did his income reach ₹7,00,000?**

ANSWER

**1 The salaries form an AP**

The yearly salaries 5,00,000, 5,20,000, 5,40,000, ... form an AP with a = 5,00,000 and d = 20,000.

**2 When does it reach ₹7,00,000?**

$5{,}00{,}000 + (n - 1) \times 20{,}000 = 7{,}00{,}000$
$(n - 1) \times 20{,}000 = 2{,}00{,}000$, so $n - 1 = 10$ and $n = 11$

So ₹7,00,000 is the salary in the 11th year, that is, after 10 yearly increments.

**His income reached ₹7,00,000 after 10 years (in his 11th year of work).**

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**QUESTION 7**

**A child arranges marbles in rows so that the first row has 1 marble, the second has 2 marbles, the third has 3, and so on up to 25 rows. How many marbles does the child use in all?**

**ANSWER**

Total = $1 + 2 + 3 + \ldots + 25 = S_{25} = 25 \times 26 \div 2 = 325$

**The child uses 325 marbles.**

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§8.6

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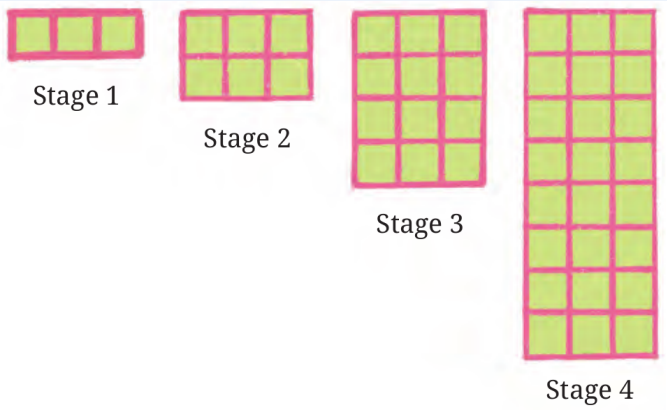
# Geometric Progressions

Examples, exercises & in-text questions · 10 items

**Note:** Solutions of the Examples, in-text Exercises, Think and Reflect prompts and in-text questions of Section 8.6 (Geometric Progressions, Fun with Fractals, Visualising a GP), in the order in which they appear in the chapter.

THINK AND REFLECT

**Fig. 8.6 shows a growing pattern of squares. The total number of green squares in the four stages is 3, 6, 12, 24.**




**Can you predict the number of squares in Stages 5 and 6 of the pattern? In Stages 10, 11 and 12? In Stage 20? At any stage? How is this different from the growing pattern in Fig. 8.3?**

ANSWER

**1 Rule**

The number of squares doubles at every stage: 3, 6, 12, 24, ..., so Stage n has $3 \times 2^{n-1}$ squares.

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**2** **Predictions**

Stage 5: $24 \times 2 = 48$
Stage 6: $48 \times 2 = 96$
Stage 10: $3 \times 2^9 = 3 \times 512 = 1536$
Stage 11: $3 \times 2^{10} = 3072$
Stage 12: $3 \times 2^{11} = 6144$
Stage 20: $3 \times 2^{19} = 3 \times 524288 = 1{,}572{,}864$
Stage n: $3 \times 2^{n-1}$

**3** **Difference from Fig. 8.3**

In Fig. 8.3 the same number (4) of squares is added at each stage, so the counts grow by a constant difference (an AP). Here each count is multiplied by the same number, 2 (a GP), so the counts grow much faster.

**Stages 5, 6: 48, 96; Stages 10, 11, 12: 1536, 3072, 6144; Stage 20: 1,572,864; any stage: $3 \times 2^{n-1}$. Fig. 8.3 adds a fixed number; Fig. 8.6 multiplies by a fixed number.**

**EXAMPLE 6**

**Is 1, 2, 4, 8, 16, … a geometric progression? If so, what is the common ratio?**

**ANSWER**

$2 \div 1 = 2$, $4 \div 2 = 2$, $8 \div 4 = 2$, $16 \div 8 = 2$

**Yes, it is a GP with common ratio 2 ($t_n = 2^{n-1}$).**

**EXAMPLE 7**

**Is 1, 3, 9, 27, 81, … a geometric progression? If so, what is the common ratio?**

**ANSWER**

$3 \div 1 = 3$, $9 \div 3 = 3$, $27 \div 9 = 3$, $81 \div 27 = 3$

**Yes, it is a GP with common ratio 3 ($t_n = 3^{n-1}$).**

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**EXAMPLE 8**

**Is 1, –1, 1, –1, 1, … a geometric progression? If so, what is the common ratio?**

**ANSWER**

$-1 \div 1 = -1,\ 1 \div (-1) = -1,\ -1 \div 1 = -1,\ 1 \div (-1) = -1$

The ratio of consecutive terms is always –1.

**Yes, it is a GP with common ratio –1 ($t_n = (-1)^{n-1}$).**

**EXAMPLE 9**

**Check whether the sequence 5, 15/4, 45/16, 135/64, … is a geometric progression and find its nth term.**

**ANSWER**

**1 Ratios of consecutive terms**

$t_2 \div t_1 = (15/4) \div 5 = 15/4 \times 1/5 = 3/4$
$t_3 \div t_2 = (45/16) \div (15/4) = 45/16 \times 4/15 = 3/4$
$t_4 \div t_3 = (135/64) \div (45/16) = 135/64 \times 16/45 = 3/4$

The ratio is the constant 3/4, so the sequence is a GP with a = 5 and r = 3/4.

**2 nth term**

$t_n = a \times r^{n-1} = 5 \times (3/4)^{n-1}$
Check: $t_2 = 5 \times 3/4 = 15/4$, $t_3 = 5 \times 9/16 = 45/16$, $t_4 = 5 \times 27/64 = 135/64$ ✓

**Yes, it is a GP with a = 5, r = 3/4 and $t_n = 5 \times (3/4)^{n-1}$.**

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EXERCISE

**Check whether the following sequences are geometric progressions and find their nth terms.**

**(i) 2, 10, 50, 250, …**

**(ii) 4, 8/3, 16/9, 32/27, …**

**(iii) 3, –3/2, 3/4, –3/8, …**

ANSWER

**(i) 2, 10, 50, 250, …** — GP, r = 5

$10 \div 2 = 5,\ 50 \div 10 = 5,\ 250 \div 50 = 5$

$t_n = 2 \times 5^{n-1}$

**(ii) 4, 8/3, 16/9, 32/27, …** — GP, r = 2/3

$(8/3) \div 4 = 2/3$

$(16/9) \div (8/3) = 16/9 \times 3/8 = 2/3$

$(32/27) \div (16/9) = 32/27 \times 9/16 = 2/3$

$t_n = 4 \times (2/3)^{n-1}$

**(iii) 3, –3/2, 3/4, –3/8, …** — GP, r = -1/2

$(-3/2) \div 3 = -1/2$

$(3/4) \div (-3/2) = -1/2$

$(-3/8) \div (3/4) = -1/2$

$t_n = 3 \times (-1/2)^{n-1}$

**All three are GPs: (i) $t_n = 2 \times 5^{n-1}$; (ii) $t_n = 4 \times (2/3)^{n-1}$; (iii) $t_n = 3 \times (-1/2)^{n-1}$.**

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EXERCISE

**Can you find a recursive rule for the formula $t_n = 3 \times 10^{n-1}$ that generates the geometric progression 3, 30, 300, 3000, … ?**

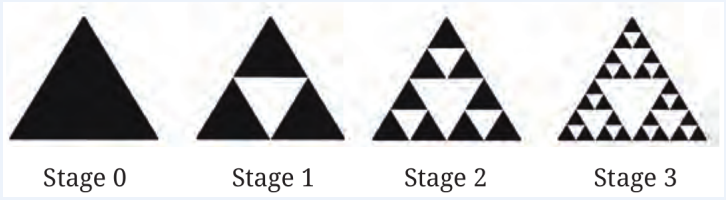
ANSWER

$t_1 = 3 \times 10^0 = 3$
$t_n \div t_{n-1} = 3 \times 10^{n-1} \div (3 \times 10^{n-2}) = 10$

**Recursive rule: $t_1 = 3$, $t_n = 10 \times t_{n-1}$ for $n \geq 2$.**

THINK AND REFLECT

**Observe the Sierpiński triangle (Fig. 8.7) and try to answer the following questions.**




**(a) How many black triangles are there in Stages 0 to 3 of Fig. 8.7?**

**(b) Can you predict the number of black triangles at Stages 4 and 5?**

**(c) Can you find a rule for the number of black triangles at the nth stage?**

**(d) Suppose the area of the triangle (that is, the black region) in Stage 0 is 1 square unit. What is the area of the black region in Stages 1, 2 and 3? What will be the area of the black region in Stages 4 and 5? Find a rule for the area of the black region at the nth stage. What happens to this area as n, the number of stages, goes on increasing?**

ANSWER

(a) **Black triangles in Stages 0 to 3** — 1, 3, 9, 27

Each black triangle is replaced by three smaller black triangles at the next stage, so the number is multiplied by 3 each time.

(b) **Stages 4 and 5**

Stage 4: $27 \times 3 = 81$
Stage 5: $81 \times 3 = 243$

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### (c) Rule for the nth stage

$1 = 3^0$, $3 = 3^1$, $9 = 3^2$, $27 = 3^3$, ...
Number of black triangles = $3^n$

### (d) Area of the black region

At each stage the middle one of the 4 equal parts of every triangle is removed, so 3/4 of the previous black area is left.

Stage 1: 3/4
Stage 2: $3/4 \times 3/4 = (3/4)^2 = 9/16$
Stage 3: $(3/4)^3 = 27/64$
Stage 4: $(3/4)^4 = 81/256$
Stage 5: $(3/4)^5 = 243/1024$
Stage n: $(3/4)^n$

Since 3/4 is less than 1, the area becomes smaller at every stage: $(3/4)^n$ gets closer and closer to 0 as n increases.

**(a) 1, 3, 9, 27; (b) 81, 243; (c) $3^n$; (d) 3/4, 9/16, 27/64, then 81/256, 243/1024; area $(3/4)^n$, which tends to 0.**

IN-TEXT QUESTIONS

**The number of black triangles increases very quickly as the stage numbers increase. Can you explain why? Can you explain why the area of the black region at Stage n will be $(3/4)^n$?**

ANSWER

### 1 Why the number increases very quickly

The number of triangles is multiplied by 3 at every stage (a GP with common ratio 3), so the counts go 1, 3, 9, 27, 81, 243, ... and each stage triples the previous count. Repeated multiplication grows much faster than repeated addition.

### 2 Why the area is $(3/4)^n$

Each stage divides every black triangle into 4 equal parts and removes one, leaving 3/4 of its area. Starting from area 1, after one stage the area is 3/4, after two stages $(3/4) \times (3/4) = (3/4)^2$, and after n stages 3/4 has been multiplied n times, giving $(3/4)^n$.

**The count is multiplied by 3 each stage; the area is multiplied by 3/4 each stage, so after n stages it is $(3/4)^n$.**

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**EXAMPLE 10**

**A ball is dropped from a height of 24 feet above the ground. Each time the ball bounces up to (3/4)th of its previous height.**

**(a) Can you write the sequence of numbers obtained from the heights attained by the ball in five successive bounces?**

**(b) How many bounces are required for the ball to remain below a height of 1/6 of the original height from which it was dropped?**

**ANSWER**

**(a) Heights after five bounces**

1st bounce: $24 \times 0.75 = 18$ feet
2nd bounce: $18 \times 0.75 = 13.5$ feet
3rd bounce: $13.5 \times 0.75 = 10.125$ feet
4th bounce: $10.125 \times 0.75 = 7.59375$ feet
5th bounce: $7.59375 \times 0.75 \approx 5.695$ feet

The heights form a GP with a = 18 and r = 3/4: 18, 13.5, 10.125, 7.594, 5.695, ...

**(b) Remaining below 1/6 of 24 feet** — **7 BOUNCES**

1/6 of 24 feet = 4 feet
6th bounce: $5.695 \times 0.75 \approx 4.271$ feet (still above 4)
7th bounce: $4.271 \times 0.75 \approx 3.203$ feet (below 4)

The heights keep decreasing, so after the 7th bounce the ball stays below 4 feet.

**(a) 18, 13.5, 10.125, 7.59375, 5.695 feet; (b) 7 bounces.**

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8.3

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# Exercise Set 8.3

Solutions · 7 questions

**QUESTION 1**

**Find the 12th term of a GP with common ratio 2, whose 8th term is 192.**

**ANSWER**

**1** **Find a**

$t_8 = a \times 2^7 = 128a = 192$
$a = 192 \div 128 = 1.5$

**2** **The 12th term**

$t_{12} = a \times 2^{11} = 1.5 \times 2048 = 3072$
Check: $t_{12} = t_8 \times 2^4 = 192 \times 16 = 3072$ ✓

**The 12th term is 3072.**

**QUESTION 2**

**Find the 10th and nth terms of the GP: 5, 25, 125, … .**

**ANSWER**

$a = 5$, $r = 25 \div 5 = 5$
$t_n = 5 \times 5^{n-1} = 5^n$
$t_{10} = 5^{10} = 9{,}765{,}625$

**$t_n = 5^n$ and the 10th term is $5^{10} = 9{,}765{,}625$.**

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**QUESTION 3***

**A sequence is given by the recursive rule $t_1 = 2$, $t_{n+1} = 3t_n - 2$ for $n \geq 1$. Which term of the sequence is 730?**

**ANSWER**

**1 Generate the terms**

$t_1 = 2$
$t_2 = 3 \times 2 - 2 = 4$
$t_3 = 3 \times 4 - 2 = 10$
$t_4 = 3 \times 10 - 2 = 28$
$t_5 = 3 \times 28 - 2 = 82$
$t_6 = 3 \times 82 - 2 = 244$
$t_7 = 3 \times 244 - 2 = 730$

**2 Explicit formula (check)**

Subtracting 1 from each term gives 1, 3, 9, 27, … a GP with ratio 3, so $t_n = 3^{n-1} + 1$. Then $3^{n-1} + 1 = 730$ gives $3^{n-1} = 729 = 3^6$, so $n = 7$.

**730 is the 7th term of the sequence.**

**QUESTION 4**

**Which term of the GP: 2, 6, 18, … is 4374? Write the explicit formula as well as the recursive formula for the nth term.**

**ANSWER**

$a = 2$, $r = 3$, $t_n = 2 \times 3^{n-1}$
$2 \times 3^{n-1} = 4374$, so $3^{n-1} = 2187 = 3^7$
$n - 1 = 7$, so $n = 8$

Recursive formula: $t_1 = 2$, $t_n = 3 \times t_{n-1}$ for $n \geq 2$.

**4374 is the 8th term; explicit formula $t_n = 2 \times 3^{n-1}$; recursive formula $t_1 = 2$, $t_n = 3t_{n-1}$.**

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**QUESTION 5**

**A ball is dropped from a height of 80 metres. After hitting the ground, it bounces back to 60% of the height from which it fell. It continues bouncing in this way — each time rising to 60% of the previous height.**

**(i) What height does the ball reach after the 5th bounce?**

**(ii) What is the total vertical distance the ball has travelled by the time it hits the ground for the 6th time?**

**ANSWER**

**(i) Height after the 5th bounce**

Heights after bounces form a GP: $80 \times 0.6 = 48, 28.8, 17.28, 10.368, \ldots$
After the 5th bounce: $80 \times 0.6^5 = 80 \times 0.07776 = 6.2208$ m

**(ii) Total distance when it hits the ground for the 6th time**

The ball hits the ground the 1st time after falling 80 m. After each bounce it rises and falls the same height, and hits the ground again. So the 6th hit comes after the fall of the 5th bounce.

Fall from 80 m: 80
Bounce 1: up and down $2 \times 48 = 96$
Bounce 2: $2 \times 28.8 = 57.6$
Bounce 3: $2 \times 17.28 = 34.56$
Bounce 4: $2 \times 10.368 = 20.736$
Bounce 5: $2 \times 6.2208 = 12.4416$
Total $= 80 + 96 + 57.6 + 34.56 + 20.736 + 12.4416 = 301.3376$ m

**(i) 6.2208 m; (ii) about 301.34 m.**

**QUESTION 6**

**Which term of the sequence 2, $2\sqrt{2}$, 4, … is 128?**

**ANSWER**

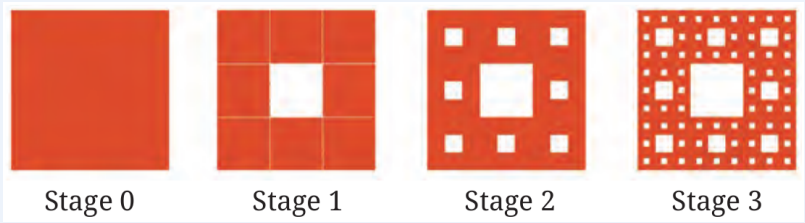
$a = 2$, $r = 2\sqrt{2} \div 2 = \sqrt{2}$ (and $4 \div 2\sqrt{2} = \sqrt{2}$)
$t_n = 2 \times (\sqrt{2})^{n-1}$
$2 \times (\sqrt{2})^{n-1} = 128$, so $(\sqrt{2})^{n-1} = 64 = 2^6 = (\sqrt{2})^{12}$
$n - 1 = 12$, so $n = 13$

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**128 is the 13th term.**

QUESTION 7

**Fig. 8.12 shows Stages 0 to 3 of the Sierpiński square carpet. Stage 0 of this fractal is a square sheet of paper. To construct Stage 1, each side of the square is trisected and the points of trisection of opposite sides are joined to obtain nine smaller squares. The centre square is then removed and the 8 smaller squares are retained, leaving a square hole in the centre. The same process is repeated on the eight smaller shaded squares to obtain Stage 2 and so on.**




**Look at Fig. 8.12 and try to answer the following questions.**

**(i) How many red squares are there in Stages 0 to 3?**

**(ii) Can you predict the number of red squares in Stages 4 and 5?**

**(iii) Can you find a rule for the number of red squares at the nth stage? Write the explicit formula as well as the recursive formula for the number of red squares at any stage.**

**(iv) Suppose the area of the square in Stage 0 is 1 square unit. What is the area of the red region in Stages 1, 2 and 3? What will be the area of the red region in Stages 4 and 5? Find the explicit as well as the recursive formula for the area of the red region at the nth stage. What happens to this area as n, the number of stages, goes on increasing?**

ANSWER

**(i) Red squares in Stages 0 to 3** 1, 8, 64, 512

Each red square is divided into 9 smaller squares and the centre one is removed, leaving 8. So the number is multiplied by 8 at each stage.

**(ii) Stages 4 and 5**

Stage 4: $512 \times 8 = 4096$
Stage 5: $4096 \times 8 = 32768$

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### (iii) Rule for the nth stage

Explicit formula: $t_n = 8^n$ (n = 0, 1, 2, ...)
Recursive formula: $t_0 = 1$, $t_n = 8 \times t_{n-1}$ for $n \geq 1$

### (iv) Area of the red region

Each retained square has 1/9 of the area of the square it came from, and 8 of the 9 parts are kept, so 8/9 of the previous red area remains at each stage.

Stage 1: 8/9
Stage 2: $(8/9)^2 = 64/81$
Stage 3: $(8/9)^3 = 512/729$
Stage 4: $(8/9)^4 = 4096/6561$
Stage 5: $(8/9)^5 = 32768/59049$
Explicit formula: $s_n = (8/9)^n$
Recursive formula: $s_0 = 1$, $s_n = (8/9) \times s_{n-1}$ for $n \geq 1$

Since 8/9 is less than 1, the area decreases at every stage and gets closer and closer to 0 as n increases (although the number of red squares increases).

**(i) 1, 8, 64, 512; (ii) 4096, 32768; (iii) $t_n = 8^n$, $t_0 = 1$, $t_n = 8t_{n-1}$; (iv) areas 8/9, 64/81, 512/729, 4096/6561, 32768/59049; $s_n = (8/9)^n$; the area tends to 0.**

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EC

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 8

# End-of-Chapter Exercises

Solutions · 15 questions

**QUESTION 1**

**Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.**

**ANSWER**

**1 Find d and a**

$t_{16} - t_{11} = 5d = 73 - 38 = 35$, so $d = 7$

$a + 10d = 38$, so $a = 38 - 70 = -32$

**2 The 31st term**

$t_{31} = a + 30d = -32 + 210 = 178$

Check: $t_{31} = t_{16} + 15d = 73 + 105 = 178$ ✓

**The 31st term is 178.**

**QUESTION 2**

**Determine the AP whose third term is 16 and whose 7th term exceeds the 5th term by 12.**

**ANSWER**

$t_7 - t_5 = 2d = 12$, so $d = 6$

$t_3 = a + 2d = 16$, so $a = 16 - 12 = 4$

The AP is 4, 10, 16, 22, 28, … ($t_n = 6n - 2$).

**The AP is 4, 10, 16, 22, … with a = 4 and d = 6.**

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**QUESTION 3***

**How many three-digit numbers are divisible by 7? (Hint: All three-digit numbers divisible by 7 form an AP. Find the smallest and largest such three-digit numbers.)**

**ANSWER**

Smallest: $7 \times 15 = 105$ (since $7 \times 14 = 98$ has two digits)
Largest: $7 \times 142 = 994$ (since $7 \times 143 = 1001$ has four digits)
AP: 105, 112, ..., 994 with $d = 7$
$105 + (n - 1) \times 7 = 994$, so $n - 1 = 127$ and $n = 128$

**There are 128 three-digit numbers divisible by 7.**

**QUESTION 4***

**How many multiples of 4 lie between 10 and 250? (Hint: All multiples of 4 form an AP. Find the smallest and largest multiples of 4 between 10 and 250.)**

**ANSWER**

Smallest multiple of 4 above 10: 12
Largest multiple of 4 below 250: 248
$12 + (n - 1) \times 4 = 248$, so $(n - 1) \times 4 = 236$, $n - 1 = 59$ and $n = 60$

**There are 60 multiples of 4 between 10 and 250.**

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**QUESTION 5***

**Find a GP for which the sum of the first two terms is –4 and the fifth term is 4 times the third term.**

**ANSWER**

**1 Use the conditions**

$a + ar = -4$ …(1)
$ar^4 = 4 \times ar^2$, so $r^2 = 4$ and $r = 2$ or $r = -2$

**2 Case r = 2** GP 1

$a(1 + 2) = -4$, so $a = -4/3$
GP: –4/3, –8/3, –16/3, –32/3, …

**3 Case r = –2** GP 2

$a(1 - 2) = -4$, so $a = 4$
GP: 4, –8, 16, –32, …

Check (second GP): 4 + (–8) = –4 ✓; fifth term 64 = 4 × 16 (third term) ✓. Check (first GP): –4/3 + (–8/3) = –4 ✓; fifth term –64/3 = 4 × (–16/3) ✓.

**Two GPs are possible: –4/3, –8/3, –16/3, … (r = 2) and 4, –8, 16, –32, … (r = –2).**

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**QUESTION 6***

**Find all possible ways of expressing 100 as the sum of consecutive natural numbers.**

**ANSWER**

**1 Set up**

Let there be k consecutive natural numbers starting from m. Their sum is $k(2m + k - 1) \div 2 = 100$, so $k(2m + k - 1) = 200$. The two factors k and $2m + k - 1$ have opposite parity (one even, one odd), and $2m + k - 1$ is larger than k.

**2** **Factor pairs of 200 = $2^3 \times 5^2$ with one factor odd and one even**

1 × 200, 5 × 40, 8 × 25 (the pairs 2 × 100, 4 × 50, 10 × 20 have both factors even)

k = 5: 2m + 4 = 40, so m = 18: 18 + 19 + 20 + 21 + 22 = 100
k = 8: 2m + 7 = 25, so m = 9: 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 = 100
k = 1: the number 100 itself (a single term)

Check: 18 + ... + 22 = 5 × 20 = 100 ✓; 9 + ... + 16 = 8 × 25 ÷ 2 = 100 ✓.

**100 = 18 + 19 + 20 + 21 + 22 = 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 (apart from the single term 100).**

QUESTION 7*

**The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2nd hour, 4th hour and nth hour?**

ANSWER

The numbers 30, 60, 120, 240, ... form a GP with first term 30 and common ratio 2; the number after n hours is $30 \times 2^n$.

End of 2nd hour: $30 \times 2^2 = 120$
End of 4th hour: $30 \times 2^4 = 480$
End of nth hour: $30 \times 2^n$

**120 bacteria, 480 bacteria, and $30 \times 2^n$ bacteria.**

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**QUESTION 8***

**The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.**

**ANSWER**

$t_4 + t_8 = (a + 3d) + (a + 7d) = 2a + 10d = 24$, so $a + 5d = 12$ ...(1)
$t_6 + t_{10} = (a + 5d) + (a + 9d) = 2a + 14d = 44$, so $a + 7d = 22$ ...(2)
(2) – (1): $2d = 10$, so $d = 5$
$a = 12 – 25 = –13$

The AP is –13, –8, –3, 2, 7, ... Check: $t_4 = 2$, $t_8 = 22$ (sum 24 ✓); $t_6 = 12$, $t_{10} = 32$ (sum 44 ✓).

**The first three terms are –13, –8, –3.**

**QUESTION 9***

**Find the smallest value of n such that the sum of the first n natural numbers is greater than 1,000.**

**ANSWER**

$S_n = n(n + 1) \div 2 > 1000$, so $n(n + 1) > 2000$
$S_{44} = 44 \times 45 \div 2 = 990$ (not greater than 1000)
$S_{45} = 45 \times 46 \div 2 = 1035$ (greater than 1000)

**The smallest value of n is 45.**

**QUESTION 10***

**Which term of the GP: 2, 8, 32, ... is 131072? Write the explicit formula as well as the recursive formula for the nth term.**

**ANSWER**

$a = 2$, $r = 4$, $t_n = 2 \times 4^{n-1}$
$2 \times 4^{n-1} = 131072$, so $4^{n-1} = 65536 = 4^8$
$n – 1 = 8$, so $n = 9$

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Recursive formula: $t_1 = 2$, $t_n = 4 \times t_{n-1}$ for $n \geq 2$.

**131072 is the 9th term; $t_n = 2 \times 4^{n-1}$; $t_1 = 2$, $t_n = 4t_{n-1}$.**

QUESTION 11*

**The sum of the first three terms of a GP is 13/12 and their product is –1. Find the common ratio and the terms.**

ANSWER

**1 Take the terms as a/r, a, ar**

Product: $(a/r) \times a \times (ar) = a^3 = -1$, so $a = -1$

**2 Use the sum**

$-1/r - 1 - r = 13/12$
Multiply by $-12r$: $12 + 12r + 12r^2 = -13r$
$12r^2 + 25r + 12 = 0$
$r = (-25 \pm \sqrt{(625 - 576)}) \div 24 = (-25 \pm 7) \div 24$
$r = -3/4$ or $r = -4/3$

**3 The terms**

$r = -3/4$: terms $4/3, -1, 3/4$
$r = -4/3$: terms $3/4, -1, 4/3$

Check: $4/3 - 1 + 3/4 = (16 - 12 + 9)/12 = 13/12$ ✓; product $= (4/3)(-1)(3/4) = -1$ ✓.

**Common ratio –3/4 with terms 4/3, –1, 3/4 (or common ratio –4/3 with terms 3/4, –1, 4/3).**

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**QUESTION 12***

**If the 4th, 10th and 16th terms of a GP are x, y and z respectively, prove that x, y, z are in GP.**

**ANSWER**

Let the GP have first term a and common ratio r.
$x = ar^3$, $y = ar^9$, $z = ar^{15}$
$y^2 = a^2r^{18}$
$xz = ar^3 \times ar^{15} = a^2r^{18}$
So $y^2 = xz$, that is, $y \div x = z \div y$

Indeed $y \div x = r^6$ and $z \div y = r^6$, so the ratio of consecutive terms is the same constant $r^6$.

**Hence x, y, z are in GP, with common ratio $r^6$.**

**QUESTION 13***

**The sum of the first three terms of a geometric progression is 26, and the sum of their squares is 364. Find the terms of the GP.**

**ANSWER**

**1 Equations for a and r**

$a + ar + ar^2 = 26$, so $a(1 + r + r^2) = 26$ ...(1)
$a^2 + a^2r^2 + a^2r^4 = 364$, so $a^2(1 + r^2 + r^4) = 364$ ...(2)

**2 Simplify using $1 + r^2 + r^4 = (1 + r + r^2)(1 - r + r^2)$**

$a^2(1 + r + r^2)(1 - r + r^2) = 364$
Using (1): $26 \times a(1 - r + r^2) = 364$, so $a(1 - r + r^2) = 14$ ...(3)
(1) ÷ (3): $(1 + r + r^2) \div (1 - r + r^2) = 26/14 = 13/7$
$7 + 7r + 7r^2 = 13 - 13r + 13r^2$
$6r^2 - 20r + 6 = 0$, i.e. $3r^2 - 10r + 3 = 0$
$(3r - 1)(r - 3) = 0$, so $r = 3$ or $r = 1/3$

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**3** **The terms**

r = 3: a(1 + 3 + 9) = 26, so a = 2: terms 2, 6, 18
r = 1/3: a(1 + 1/3 + 1/9) = 26, so a = 18: terms 18, 6, 2

Check: 2 + 6 + 18 = 26 ✓ and 4 + 36 + 324 = 364 ✓.

**The terms are 2, 6, 18 (r = 3) or in reverse order 18, 6, 2 (r = 1/3).**

**QUESTION 14***

**Suppose $P_1 = 1$, $P_2 = 2$ and for $n > 2$, $P_n = P_1 + P_2 + \ldots + P_{n-1} + 1$. Find the values of $P_1, P_2, \ldots, P_8$. Can you find a simpler recursive formula for $P_n$? Can you give an explicit formula?**

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**ANSWER**

**1** **The values**

$P_3 = 1 + 2 + 1 = 4$
$P_4 = 1 + 2 + 4 + 1 = 8$
$P_5 = 1 + 2 + 4 + 8 + 1 = 16$
$P_6 = 1 + 2 + 4 + 8 + 16 + 1 = 32$
$P_7 = 64$
$P_8 = 128$

So $P_1, \ldots, P_8 = 1, 2, 4, 8, 16, 32, 64, 128$.

**2** **A simpler recursive formula**

$P_n = (P_1 + \ldots + P_{n-2}) + P_{n-1} + 1 = P_{n-1} + P_{n-1} = 2P_{n-1}$ (for $n > 2$),
because $P_1 + \ldots + P_{n-2} + 1 = P_{n-1}$ (the rule applied to the previous term).

So $P_1 = 1$, $P_n = 2P_{n-1}$ for $n \geq 2$ (this also gives $P_2 = 2$).

**3** **Explicit formula**

This is a GP with a = 1 and r = 2, so $P_n = 2^{n-1}$

**$P_1, \ldots, P_8 = 1, 2, 4, 8, 16, 32, 64, 128$; $P_n = 2P_{n-1}$; $P_n = 2^{n-1}$.**

**QUESTION 15***

**Suppose $W_1 = 1$, $W_2 = 2$ and for $n > 2$, $W_n = W_1 + W_2 + \ldots + W_{n-2} + 2$. Find the values of $W_1$, $W_2$, ..., $W_8$. Do you recognise this sequence?**

**ANSWER**

$W_3 = W_1 + 2 = 1 + 2 = 3$
$W_4 = W_1 + W_2 + 2 = 1 + 2 + 2 = 5$
$W_5 = W_1 + W_2 + W_3 + 2 = 1 + 2 + 3 + 2 = 8$
$W_6 = W_1 + \ldots + W_4 + 2 = 1 + 2 + 3 + 5 + 2 = 13$
$W_7 = W_1 + \ldots + W_5 + 2 = 1 + 2 + 3 + 5 + 8 + 2 = 21$
$W_8 = W_1 + \ldots + W_6 + 2 = 1 + 2 + 3 + 5 + 8 + 13 + 2 = 34$

The sequence is 1, 2, 3, 5, 8, 13, 21, 34, ...; each term is the sum of the previous two (3 = 2 + 1, 5 = 3 + 2, 8 = 5 + 3, ...).

**$W_1$, ..., $W_8$ = 1, 2, 3, 5, 8, 13, 21, 34, the Virahānka–Fibonacci sequence.**

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