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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 6

Measuring Space: Perimeter and Area

Complete Exercise Solutions

Exercise Sets 6.1 – 6.3 & End-of-Chapter Exercises

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Every question is followed by its answer. Question numbers above are clickable and the PDF bookmarks panel lets you jump to any exercise set or question. In the first section, T = Think and Reflect, E = Example, Act = Activity and I = other in-text questions.

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IT

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 6

In-Text Questions, Examples & Activities

9 Think and Reflect · 7 Examples · 1 Activity · 4 In-text questions

Solutions to the Think and Reflect questions, Examples, the Home Measurement activity and the other questions asked in the text, in the order in which they appear in the chapter.

THINK AND REFLECT 1

Notice that the athletes in Fig. 6.1 are not at the same starting line. Do you think the stagger gives anyone an unfair advantage? Why or why not? On what basis can the organisers work out the length of the stagger between lanes?

In my school, the playground is too small to have a 400 m track, so the school constructed a 200 m track instead. Does this mean that we need a smaller stagger for the race tracks in my school (i.e., smaller than the stagger used in the Olympics), for the same 4 × 100 m relay race?

ANSWER

1 Why the stagger, and is it fair?

Every runner must cover the same distance. On a bend, a runner in an outer lane runs along a circle of larger radius, so the curve is longer. The outer-lane runners therefore start ahead (the stagger) to make up exactly this extra length. The stagger gives nobody an advantage; it removes the disadvantage of the longer curve.

2 On what basis is the stagger worked out?

On the lengths of the curved parts, which are arcs (semicircles) of circles, so on the circumference of a circle. If a lane is w m wide, the bend of the next lane out has radius $r + w$ instead of r. For one semicircular bend:

$$\text{extra distance} = \pi(r + w) - \pi r = \pi w$$

This does not contain r. So the stagger depends only on the lane width and the number of bends run, not on the size of the track.

Therefore, no: if the lanes have the same width and each runner runs one bend, the 200 m track needs the same stagger as the 400 m track (about π × 1.22 ≈ 3.83 m per lane).

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THINK AND REFLECT 2

What is the connection between the question "What is the perimeter of a circle of radius *r*?" and the one about the 400 m athletics track?

ANSWER

The curved parts of a track are semicircles (arcs of circles). To find how much longer the curve in an outer lane is, we must know the length of an arc of a circle of a given radius. That is the same as knowing the perimeter (circumference) of a circle of radius r, since a semicircle is half of it.

Therefore, the stagger question can be answered only once we know how to find the length around a circle.

ACTIVITY

HOME MEASUREMENT. Take a cotton reel with thin thread around it. Measure the diameter *D* of the reel as accurately as possible. Wrap the thread tightly around the reel 20 times, unwrap it, measure its length *L*, and calculate $\frac{L}{20D}$. Do you get a ratio between 3 and 4? Between 3.1 and 3.2?

It is also possible to estimate the *C*/*D* ratio using pure geometry, without any measurements at all! Can you imagine how?

Is the ratio of circumference (*C*) to diameter (*D*) the same for circles of all sizes?

ANSWER

1 The experiment

Thread wrapped 20 times covers 20 circumferences, so $L = 20C$, and $\frac{L}{20D} = \frac{C}{D}$. Sample readings (illustrative): $D = 2.8$ cm and $L = 176$ cm give $\frac{176}{20\times2.8} = \frac{176}{56} \approx 3.14$. A careful measurement gives a value between 3.1 and 3.2.

2 Pure geometry

Trap the circle between polygons: the perimeter of a polygon inscribed in the circle is less than the circumference and that of a circumscribed polygon is greater. For radius 1 and hexagons, $6 < 2\pi < 4\sqrt{3}$, so $3 < \pi < 2\sqrt{3} \approx 3.46$ (Fig. 6.6 and 6.7). Polygons with more sides give better bounds (Archimedes used 96 sides).

3 Same ratio for all circles? YES

Any circle is an enlargement of any other. Enlarging by a factor k multiplies both the circumference and the diameter by k, so $\frac{C}{D}$ does not change.

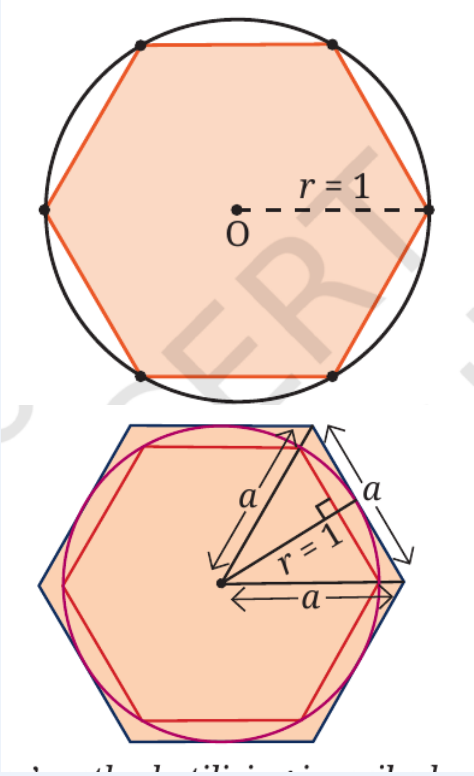
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Therefore, $\frac{C}{D}$ is the same for every circle; it is approximately 3.14, and we call it π.

IN-TEXT QUESTION 1

Fig. 6.6: Mesopotamian hexagon inscribed in a circle of radius 1. Can you see why this shows that π > 3?

Fig. 6.7: Inscribed and circumscribed hexagons. Can you see why this diagram tells us that π is between 3 and 2√3? (Hint: Use the Baudhāyana–Pythagoras Theorem.)



ANSWER

1 Fig. 6.6: π > 3

A regular hexagon inscribed in a circle of radius 1 is made of 6 equilateral triangles of side 1, so its perimeter is 6. A straight segment is the shortest path between two points, so each side of the hexagon is shorter than the arc it cuts off. Hence 6 < circumference = 2π, i.e. π > 3.

2 Fig. 6.7: π < 2√3

Let the circumscribed regular hexagon have side a; the circle (radius 1) touches each side at its midpoint. The half-side $a/2$, the radius to the touching point (1), and the line from the centre to the vertex (equal to a) form a right triangle:

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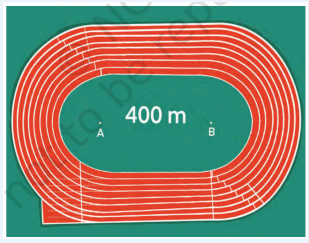
$a^2 = (\frac{a}{2})^2 + 1^2$
$\frac{3a^2}{4} = 1$, so $a = \frac{2}{\sqrt{3}}$
perimeter $= 6a = \frac{12}{\sqrt{3}} = 4\sqrt{3}$

The circle lies inside the circumscribed hexagon, so $2\pi < 4\sqrt{3}$, i.e. $\pi < 2\sqrt{3} \approx 3.46$.

Therefore, $3 < \pi < 2\sqrt{3} \approx 3.46$.

THINK AND REFLECT 3

What is the difference in radius between the first and second lanes? Use Fig. 6.11 to find the stagger needed by the runner in the second lane. Will an equal stagger be needed between the third and second lanes?



ANSWER

The width of each lane is 1.22 m, so the radius of the bend increases by 1.22 m from one lane to the next.

Lane 1 bends (full circle): $2\pi \times 36.8 = 231.22$ m
Lane 2 bends: $2\pi \times (36.8 + 1.22) = 2\pi \times 38.02 \approx 238.89$ m
Extra distance $= 2\pi \times 1.22 \approx 7.67$ m

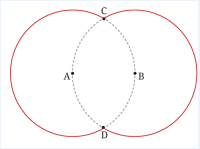
The extra distance depends only on the lane width, $2\pi \times 1.22$, so it is the same for lane 3 versus lane 2. (For a relay leg that has only one bend, the stagger is half of this, $\pi \times 1.22 \approx 3.83$ m.)

Therefore, the radius difference is 1.22 m; the second-lane runner needs a stagger of about 7.67 m in a one-lap race; and yes, the same stagger is needed between the third and second lanes.

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EXAMPLE 1

Two circles of equal radius are located such that each circle passes through the centre of the other circle (Fig. 6.12). Given that the radius of each circle is *r* units, find the perimeter of the shape formed by the two circles in terms of *r*. (Ignore the dotted portions that lie within the circles.)



ANSWER

A and B are the centres, AB = r, and C, D are the points where the circles meet. AC = AB = BC = r, so ΔABC is equilateral and ∠CAB = ∠CBA = 60°. Similarly ∠BAD = ∠ABD = 60°. So ∠CAD = ∠CBD = 120°.

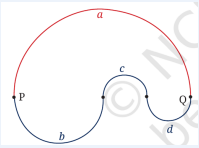
Each dotted arc subtends 120° at its centre, which is $\frac{1}{3}$ of the circumference, so each red arc is $\frac{2}{3}$ of a circumference.

$$\text{Perimeter} = 2 \times \frac{2}{3} \times 2\pi r = \frac{8}{3}\pi r$$

Therefore, the perimeter of the shape is $\frac{8}{3}\pi r$ units.

EXAMPLE 2

In Fig. 6.13, points P and Q are joined by two paths. The first path is the semicircle *a*. The other path is made up of three semicircles (*b*, *c* and *d*). Which path is longer? (i) Path *a* is longer (ii) Path *b* + *c* + *d* is longer (iii) The two paths have equal length.



ANSWER

Let the radii of the semicircles a, b, c, d be a', b', c', d'. The length of semicircle a is $\frac{1}{2}(2\pi)a' = \pi a'$, and the others are $\pi b'$, $\pi c'$, $\pi d'$. The second path has length $\pi(b' + c' + d')$.

PQ = $2a'$ (diameter of a) and also PQ = $2b' + 2c' + 2d'$. Hence $a' = b' + c' + d'$.

Therefore, option (iii): the two paths have equal length.

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THINK AND REFLECT 4

What happens if the parallelogram is 'thin' (Fig. 6.18) and the foot of the perpendicular from C to AD does not lie on side AD? The construction then does not seem to work. How do we fix this 'gap'?

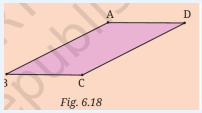


Fig. 6.18

ANSWER

Choose D′ on DA close to D and A′ on DA extended with A′A = D′D. Then A′BCD′ is a parallelogram with the same base BC and the same height, and since ΔCDD′ ≅ ΔBAA′, its area equals the area of ABCD. Each such step moves the top side along its own line without changing the base, height or area. Repeat the step until the foot of the perpendicular from C falls on the top side; then cut off the triangle on one side and attach it to the other to get a rectangle of base BC and the same height.

Therefore, the 'gap' is fixed by repeated shifting, and the area of every parallelogram is base × height.

THINK AND REFLECT 5

The area of a rectangle can be found when we know the lengths of its sides. Is the same true for a parallelogram? That is, can we find the area of a parallelogram when we know the lengths of its sides? Why or why not? (Hint: What happens to the area of a parallelogram if we decrease or increase the angle between the adjacent sides while keeping the lengths fixed?)

ANSWER

No. A parallelogram with sides a and b has area base × height = a × (height), and the height changes as the angle between the sides changes, even though the sides stay the same. For a rhombus with all sides 3 the areas are 9 (square), 8.01 and 5.41 for different angles. As the angle shrinks towards 0° the area shrinks towards 0; it is largest (= ab) at 90°.

Therefore, the side lengths alone do not determine the area of a parallelogram; we also need an angle or the height.

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IN-TEXT QUESTION 2

What would we do if angle EFG is obtuse and the triangle were shaped like triangle EFG in Fig. 6.20B? Please work out the answer to this question.

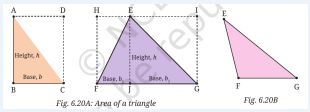


Fig. 6.20A: Area of a triangle Fig. 6.20B

ANSWER

Drop the perpendicular EJ from E to FG extended beyond F (J lies outside the base, with height h = EJ). Triangle EJG is right-angled with base JG, and triangle EJF is right-angled with base JF, both of height h.

$$\text{area}(\Delta EFG) = \text{area}(\Delta EJG) - \text{area}(\Delta EJF) = \frac{1}{2}\cdot JG\cdot h - \frac{1}{2}\cdot JF\cdot h = \frac{1}{2}(JG - JF)h = \frac{1}{2}\cdot FG\cdot h$$

Therefore, even for an obtuse angle the area of the triangle is $\frac{1}{2}$ × base × height.

IN-TEXT QUESTION 3

Do you see why the two triangles in Fig. 6.21 fit together to make a parallelogram? (If you study the angles in the figure (e.g., ∠B′C′A′ and ∠BCA), you will see why this is so. Keep in mind the criterion by which we check whether two lines are parallel.)

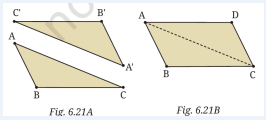


Fig. 6.21A Fig. 6.21B

ANSWER

Triangle A′B′C′ is congruent to ABC, so its angles match: ∠B′C′A′ = ∠BCA. Turn the copy through a half-turn so that it is placed against ABC with the shared diagonal AC. Then ∠BCA and ∠CAD are equal alternate angles, so AD ∥ BC. In the same way ∠BAC = ∠ACD (alternate angles), so AB ∥ DC.

Therefore, both pairs of opposite sides are parallel, so the two triangles form a parallelogram, and each triangle has half its area: $\frac{1}{2}$ × base × height.

THINK AND REFLECT 6

Since ΔABD and ΔACD have equal area, you may wonder — can we divide ΔABD using straight cuts into two or more pieces that we can then rearrange to exactly cover ΔACD? What do you think? Is it possible?

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ANSWER

Yes. ΔABD and ΔACD have equal bases (BD = DC = a) and equal height h. Cut each along the line joining the midpoints of its two slanting sides, and turn the top small triangle through a half-turn about a midpoint: each triangle becomes a parallelogram with base a and height $\frac{h}{2}$. Two parallelograms with the same base and height can be cut into each other by moving a small triangle from one end to the other (through a rectangle if needed). Reversing the steps for ΔACD turns ΔABD into ΔACD.

Therefore, it is possible, using a few straight cuts (the method above uses a small number of pieces; finding the least number is the challenge posed in the text).

THINK AND REFLECT 7

Suppose we are given two polygons P and Q with equal area. Will it always be possible to divide one of them using straight cuts into two or more pieces and then rearrange the pieces to exactly cover the other polygon? Try this out for (1) a square and a non-square rectangle with equal area, (2) two triangles with different shapes but equal area, (3) a triangle and a square with equal area. Formulate a conjecture of your own about this.

Think of various rectangles with perimeter 40 units (the sides do not have to be integers). (1) How many such rectangles are there? (2) Among them, is there one whose area is the largest? What are its dimensions? (3) Among all these rectangles, is there one whose area is the smallest? What are its dimensions? Do either of these answers come as a surprise to you?

ANSWER

1 Cutting and rearranging

(1) A rectangle with sides a, b can be turned into a square of side $\sqrt{(ab)}$ by Baudhāyana's construction (Section 6.9). (2) Each triangle can be turned into a rectangle of base b and height $\frac{h}{2}$ (cut along the midline and move the top piece); rectangles of equal area can then be turned into each other as in (1). (3) Triangle → rectangle → square by (2) and (1).

Conjecture: yes, any two polygons of equal area can be cut into finitely many pieces that rearrange to cover each other (this is true, and is known as the Wallace–Bolyai–Gerwien theorem).

2 Rectangles of perimeter 40

If the sides are l and w, then $l + w = 20$ and the area is $l(20 - l) = 100 - (l - 10)^2$.

(1) l can be any number between 0 and 20, so there are infinitely many such rectangles. (2) The area is largest when $l = 10$: the 10×10 square, area 100. (3) The area gets closer and closer to 0 as l approaches 0 or 20, but never reaches it, so there is no rectangle with the smallest area. (It is a little surprising that the largest one is a square while no smallest one exists.)

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Therefore: infinitely many rectangles; the largest area is 100 sq. units (10 × 10); there is no smallest area.

EXAMPLE 3

Test Heron's formula for an equilateral triangle with side *a* units.

ANSWER

$s = \frac{1}{2}(a + a + a) = \frac{3}{2}a$. Heron's formula gives

$$\text{area} = \sqrt{\left(\frac{3}{2}a \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2}\right)} = \frac{\sqrt{3}}{4}a^2$$

Check with ½ × base × height: the height h satisfies $a^2 - h^2 = \frac{a^2}{4}$, so $h^2 = \frac{3a^2}{4}$ and $h = \frac{\sqrt{3}}{2}a$. Area = $\frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$ ✓.

Therefore, both methods give $\frac{\sqrt{3}}{4}a^2$ sq. units.

EXAMPLE 4

Test Heron's formula for an isosceles triangle with equal sides *a* units and base 2*b* units.

ANSWER

$s = \frac{1}{2}(a + a + 2b) = a + b$, so $s - a = b$ (twice) and $s - 2b = a - b$.

$$\text{area} = \sqrt{(a + b) \cdot b \cdot b \cdot (a - b)} = b\sqrt{(a^2 - b^2)}$$

Check: the height h satisfies $a^2 - h^2 = b^2$, so $h = \sqrt{(a^2 - b^2)}$ and the area is $\frac{1}{2} \times 2b \times h = b\sqrt{(a^2 - b^2)}$ ✓.

Therefore, both methods give $b\sqrt{(a^2 - b^2)}$ sq. units.

EXAMPLE 5

Test Heron's formula for a triangle with sides 3, 4 and 5 units.

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ANSWER

$s = \frac{1}{2}(3 + 4 + 5) = 6$
area $= \sqrt{(6 \times (6 - 3) \times (6 - 4) \times (6 - 5))} = \sqrt{(6 \times 3 \times 2 \times 1)} = \sqrt{36} = 6$

Since $3^2 + 4^2 = 5^2$, the triangle is right-angled, with legs 3 and 4: area $= \frac{1}{2} \times 3 \times 4 = 6$ ✓.

Therefore, the area is 6 sq. units by both methods.

EXAMPLE 6

Verify Brahmagupta's formula for the case of a rectangle with sides *a*, *b*. (All rectangles are cyclic.)

ANSWER

$s = \frac{1}{2}(2a + 2b) = a + b$. So $s - a = b$, $s - b = a$ (each twice).

area $= \sqrt{((a + b - a)(a + b - b)(a + b - a)(a + b - b))} = \sqrt{(b \cdot a \cdot b \cdot a)} = \sqrt{(ab)^2} = ab$

Therefore, the formula gives *ab*, which is the area of the rectangle.

EXAMPLE 7

Verify Brahmagupta's formula for the case of an isosceles trapezium with parallel sides 2*a* and 2*b* and equal slanting sides *c* (Fig. 6.29). (All isosceles trapezia are cyclic.)

ANSWER

Perimeter $= 2a + 2b + 2c$, so $s = a + b + c$. Then $s - 2a = c + b - a$, $s - 2b = c + a - b$, and $s - c = a + b$ (twice).

area $= \sqrt{((c + b - a)(c + a - b)(a + b)^2)} = (a + b)\sqrt{(c^2 - (b - a)^2)}$

The perpendicular from B to the base CD has length $h = \sqrt{(c^2 - (b - a)^2)}$ by the Baudhāyana–Pythagoras theorem, and the area of the trapezium is $\frac{1}{2}(2a + 2b)h = (a + b)h$.

Therefore, Brahmagupta's formula agrees with the usual trapezium area formula.

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THINK AND REFLECT 8

What procedure would you use to square a given triangle? Here, the task is to construct a square whose area is equal to the area of some given triangle. Think carefully. How would you proceed?

ANSWER

Let the triangle have base b and height h; its area is $\frac{1}{2}bh$.

Step 1: turn it into a rectangle with sides b and $\frac{h}{2}$ (cut along the midline joining the midpoints of the slanting sides and turn the top piece through a half-turn).
Step 2: square the rectangle by Baudhāyana's method (Section 6.9): with sides $a > b'$, build the square of side $\sqrt{(ab')}$.

Therefore, the square has side $\sqrt{(\frac{1}{2}bh)}$; equal in area to the given triangle.

THINK AND REFLECT 9

Why were human beings so fond of using circular shapes? Was this only for practical reasons, or could there have been other reasons too? What kinds of uses have human beings found for the circular shape?

ANSWER

Practical reasons: a circle is easy to draw with a rope and a peg; every point is at the same distance from the centre, so a wheel rolls smoothly and a round pot, tower or granary has no weak corners; among all shapes with the same perimeter, a circle encloses the largest area, so it uses the least boundary (wall or fence) for a given area.

Other reasons: perfect symmetry and beauty; the circle resembles the sun, the moon and cycles in nature, so it appears in art, religion and design (mandalas, temples, stupas).

Uses: wheels, coins, plates, pots, clocks, gears, round houses and tents, wells, stadiums, rings and jewellery.

Therefore, circular shapes were chosen for practical, structural and aesthetic reasons alike (an open-ended question).

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IN-TEXT QUESTION 4

As the slices become smaller and smaller, the figure in Fig. 6.37B becomes closer to a parallelogram with base = half the circumference (why?) = π*r*, height = radius *r*.

ANSWER

The circle is cut into equal thin slices; half of them are placed pointing up and the other half pointing down, side by side. The curved edges (arcs) of the slices pointing up make the top edge, and the arcs of those pointing down make the bottom edge. The total length of all arcs is the circumference $2\pi r$, so each of the two edges has length half of it, πr.

$$\text{Area of circle} = \text{base} \times \text{height} = \pi r \times r = \pi r^2$$

Therefore, the base is half the circumference, π*r*, and the area of the circle is πr^2.

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6.1

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Exercise Set 6.1

Solutions · 8 questions

Unless stated otherwise, use the approximation $\frac{22}{7}$ for π.

QUESTION 1

The perimeter of a circle is 44 cm. What is its radius?

ANSWER

$2\pi r = 44$

$2 \times \frac{22}{7} \times r = 44$, so $r = \frac{44 \times 7}{2 \times 22} = 7$

Therefore, the radius is 7 cm.

QUESTION 2

Calculate, correct to 3 significant figures, the circumference of a circle with: (i) radius 7 cm (ii) radius 10 cm (iii) radius 12 cm.

ANSWER

Circumference = $2\pi r$, with $\pi = \frac{22}{7}$.

(i) **r = 7 cm**

$2 \times \frac{22}{7} \times 7 = 44 \approx 44.0$ cm

(ii) **r = 10 cm**

$2 \times \frac{22}{7} \times 10 = \frac{440}{7} \approx 62.857 \approx 62.9$ cm

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(iii) $r = 12$ cm

$$2 \times \frac{22}{7} \times 12 = \frac{528}{7} \approx 75.43 \approx 75.4 \text{ cm}$$

Therefore, the circumferences are 44.0 cm, 62.9 cm and 75.4 cm.

QUESTION 3

Calculate the length of the arc of a circle if: (i) the radius is 3.5 cm and the angle at the centre is 60°, and (ii) the radius is 6.3 m and the angle at the centre is 120°.

ANSWER

Arc length $= 2\pi r \times \frac{\theta^\circ}{360^\circ}$.

(i) $r = 3.5$ cm, $\theta = 60°$

$$2 \times \frac{22}{7} \times 3.5 \times \frac{60}{360} = 22 \times \frac{1}{6} = \frac{11}{3} \approx 3.67 \text{ cm}$$

(ii) $r = 6.3$ m, $\theta = 120°$

$$2 \times \frac{22}{7} \times 6.3 \times \frac{120}{360} = 39.6 \times \frac{1}{3} = 13.2 \text{ m}$$

Therefore, the arc lengths are $\frac{11}{3} \approx 3.67$ cm and 13.2 m.

QUESTION 4

Find the perimeter of a sector (i.e., the curved portion as well as the two straight portions) of a circle of radius 14 cm and sector angle 75°.

ANSWER

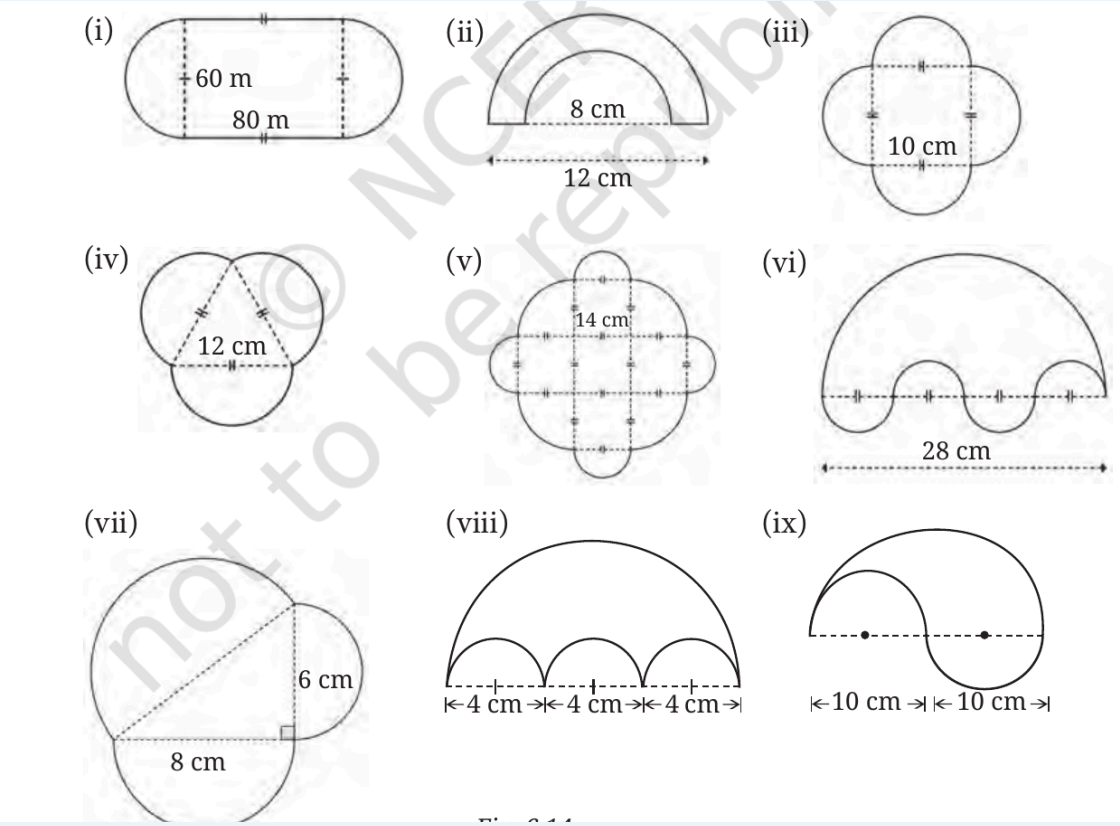
$$\text{Arc} = 2 \times \frac{22}{7} \times 14 \times \frac{75}{360} = 88 \times \frac{5}{24} = \frac{55}{3} \approx 18.33 \text{ cm}$$
$$\text{Perimeter} = \text{arc} + 2r = \frac{55}{3} + 28 = \frac{139}{3} \approx 46.33 \text{ cm}$$

Therefore, the perimeter of the sector is $\frac{139}{3} \approx 46.3$ cm.

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QUESTION 5

Find the perimeters of the following shapes (taking the arcs to be quarter or half or three-quarters of a circle, as appropriate) (Fig. 6.14i to 6.14ix):



ANSWER

Take $\pi = \frac{22}{7}$. Only the outer boundary lines drawn solid are counted (dotted lines are not part of the perimeter).

(i) Stadium: 80 m straight sides, semicircles of diameter 60 m

$$2 \times 80 + 2 \times (\pi \times 30) = 160 + 60\pi = 160 + \frac{1320}{7} = \frac{2440}{7} \approx 348.57 \text{ m}$$

(ii) Semicircular ring: outer diameter 12 cm, inner diameter 8 cm

Outer arc $= \pi \times 6 = 6\pi$, inner arc $= \pi \times 4 = 4\pi$, and the two straight ends are each $\frac{12 - 8}{2} = 2$ cm.

$$6\pi + 4\pi + 2 + 2 = 10\pi + 4 = \frac{220}{7} + 4 = \frac{248}{7} \approx 35.43 \text{ cm}$$

(iii) Four semicircles on the sides of a square of side 10 cm

$$4 \times (\pi \times 5) = 20\pi = \frac{440}{7} \approx 62.86 \text{ cm}$$

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(iv) Three semicircles on the sides of an equilateral triangle of side 12 cm

The outer boundary is three semicircles, each with a side of the triangle (12 cm) as diameter.

$$3 \times (\pi \times 6) = 18\pi = \frac{396}{7} \approx 56.57 \text{ cm}$$

(v) Grid of 3 × 3 squares of side 14 cm

The boundary has 4 semicircles of diameter 14 cm (one on each side) and 4 quarter circles of radius 14 cm (at the corners).

$$4 \times 7\pi + 4 \times (\frac{1}{4} \times 2\pi \times 14) = 28\pi + 28\pi = 56\pi = 176 \text{ cm}$$

(vi) Big semicircle of diameter 28 cm and four small semicircles

The base of 28 cm is divided into 4 equal parts of 7 cm; each part carries a semicircle of diameter 7 cm.

$$\pi \times 14 + 4 \times (\pi \times 3.5) = 14\pi + 14\pi = 28\pi = 88 \text{ cm}$$

(vii) Right triangle with legs 8 cm, 6 cm; semicircles on its three sides

The hypotenuse is $\sqrt{(8^2 + 6^2)} = 10$ cm. The boundary is three semicircles of diameters 8, 6 and 10 cm.

$$\pi(4 + 3 + 5) = 12\pi = \frac{264}{7} \approx 37.71 \text{ cm}$$

(viii) Big semicircle of diameter 12 cm and three small semicircles of diameter 4 cm

$$\pi \times 6 + 3 \times (\pi \times 2) = 12\pi = \frac{264}{7} \approx 37.71 \text{ cm}$$

(ix) Semicircle of diameter 20 cm, with a semicircle of diameter 10 cm on each side of the base

$$\pi \times 10 + \pi \times 5 + \pi \times 5 = 20\pi = \frac{440}{7} \approx 62.86 \text{ cm}$$

Therefore, the perimeters are: (i) ≈ 348.57 m, (ii) ≈ 35.43 cm, (iii) ≈ 62.86 cm, (iv) ≈ 56.57 cm, (v) 176 cm, (vi) 88 cm, (vii) ≈ 37.71 cm, (viii) ≈ 37.71 cm, (ix) ≈ 62.86 cm.

QUESTION 6

If the diameter of a car tyre is 56 cm, then: (i) How far does the car need to travel for the tyre to complete one revolution? (ii) How many revolutions does the tyre make if the car travels 10 km?

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ANSWER

(i) One revolution

$$\text{distance} = \text{circumference} = \pi \times 56 = \frac{22}{7} \times 56 = 176 \text{ cm}$$

(ii) 10 km

$$10 \text{ km} = 10 \times 1000 \times 100 = 1\,000\,000 \text{ cm}$$
$$\text{revolutions} = \frac{1000000}{176} \approx 5681.8$$

Therefore, the car travels 176 cm per revolution, and the tyre makes about 5682 revolutions in 10 km.

QUESTION 7

Find the total perimeter of all the petals in each of the given flowers.

(i)

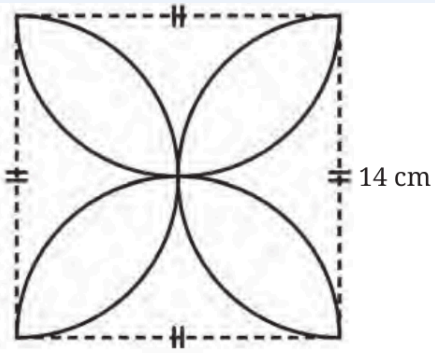


Fig. 6.15A: The centres of the arcs are the midpoints of the sides of the square

(ii)

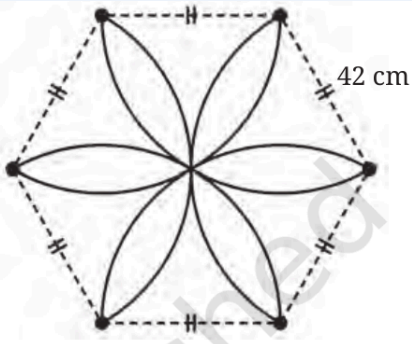


Fig. 6.15B: The centres of the arcs are the vertices of the hexagon

ANSWER

(i) Square of side 14 cm

The arcs have centres at the midpoints of the sides and radius 7 cm; each arc runs from a corner to the centre of the square, so it is a quarter circle. Each of the 4 petals has 2 such arcs, so there are 8 quarter circles = 2 full circles.

$$2 \times (2\pi \times 7) = 28\pi = 88 \text{ cm}$$

(ii) Regular hexagon of side 42 cm

Each petal runs from the centre O to a vertex, bounded by two arcs of radius 42 cm. Each arc, e.g. arc AO of the circle with centre B, subtends $\angle ABO = 60°$ (ΔABO is equilateral). So each arc is $\frac{1}{6}$ of a

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circle: $\frac{1}{6} \times 2\pi \times 42 = 14\pi = 44$ cm. Six petals have 12 arcs.

$$12 \times 44 = 528 \text{ cm}$$

Therefore, the total perimeters of the petals are (i) 88 cm and (ii) 528 cm.

QUESTION 8

The ratio of the perimeters of two circles is 5:4. What is the ratio of their radii?

ANSWER

Perimeters $2\pi r_1$ and $2\pi r_2$, so $\frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{5}{4}$.

Therefore, the ratio of the radii is 5 : 4.

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6.2

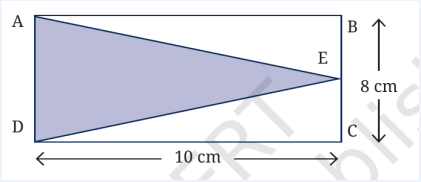
CLASS 9 MATHS · GANITA MANJARI · CHAPTER 6

Exercise Set 6.2

Solutions · 11 questions

QUESTION 1

Find the area of triangle ADE in Fig. 6.31.



ANSWER

AD = BC = 8 cm is taken as the base. E lies on BC, so the height of ΔADE from E to AD equals the width of the rectangle, 10 cm.

$$\text{area} = \frac{1}{2} \times 8 \times 10 = 40 \text{ cm}^2$$

Therefore, the area of triangle ADE is 40 cm².

QUESTION 2

The parallel sides of a trapezium are 40 cm and 20 cm. If its non-parallel sides are both equal, each being 26 cm, find the area of the trapezium.

ANSWER

Each slanting side cuts off a right triangle at the longer base; the base of that triangle is $\frac{40 - 20}{2} = 10$ cm.

$$\text{height} = \sqrt{(26^2 - 10^2)} = \sqrt{(676 - 100)} = \sqrt{576} = 24 \text{ cm}$$
$$\text{area} = \frac{1}{2}(40 + 20) \times 24 = 720 \text{ cm}^2$$

Therefore, the area of the trapezium is 720 cm².

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QUESTION 3

Find the area of a triangle, given that its sides are 8 cm and 11 cm long, and its perimeter is 32 cm.

ANSWER

third side = 32 – 8 – 11 = 13 cm

$s = 16$

area = $\sqrt{(16 \times (16 - 8)(16 - 11)(16 - 13))} = \sqrt{(16 \times 8 \times 5 \times 3)} = \sqrt{1920} = 8\sqrt{30} \approx 43.82$

Therefore, the area is $8\sqrt{30} \approx 43.8$ cm².

QUESTION 4

The sides of a triangular plot are in the ratio 3: 5: 7; its perimeter is 300 m. Find its area.

ANSWER

sides = $300 \times \frac{3}{15}, 300 \times \frac{5}{15}, 300 \times \frac{7}{15}$ = 60 m, 100 m, 140 m

$s = 150$

area = $\sqrt{(150 \times 90 \times 50 \times 10)} = \sqrt{6\,750\,000} = 1500\sqrt{3} \approx 2598$ m²

Therefore, the area of the plot is $1500\sqrt{3} \approx 2598$ m².

QUESTION 5

One diagonal of a rhombus is twice as long as the other diagonal. If the rhombus has area 128 cm², find the length of the shorter diagonal.

ANSWER

Let the diagonals be d and $2d$. The area of a rhombus is half the product of the diagonals.

$\frac{1}{2} \times d \times 2d = d^2 = 128$

$d = \sqrt{128} = 8\sqrt{2} \approx 11.31$

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Therefore, the shorter diagonal is $8\sqrt{2} \approx 11.3$ cm.

QUESTION 6

ABCD is a parallelogram. P and Q are any two points on side AB. What can you say about the ratio area (ΔPCD): area (ΔQCD)?

ANSWER

ΔPCD and ΔQCD have the same base CD. Their heights are the distance between the parallel lines AB and CD, which is the same for all points on AB.

Therefore, the two triangles have equal area; the ratio is 1 : 1.

QUESTION 7

O is any point on the diagonal PR of a parallelogram PQRS. Prove that the areas of triangles PSO and PQO are equal.

ANSWER

The diagonal PR divides the parallelogram into two congruent triangles PQR and RSP, which have the same base PR and equal area, so their heights from Q and S to PR are equal; call it h.

$$\text{area}(\Delta PQO) = \frac{1}{2} \times PO \times h$$
$$\text{area}(\Delta PSO) = \frac{1}{2} \times PO \times h$$

Therefore, area(ΔPSO) = area(ΔPQO).

QUESTION 8

If the mid-points of the sides of a 4-gon are joined in order, prove that the area of the parallelogram thus formed will be half of the area of the given 4-gon.

ANSWER

Let ABCD be the 4-gon (convex) and P, Q, R, S the midpoints of AB, BC, CD, DA.

Join AQ. A median divides a triangle into two equal areas: AQ is a median of ΔABC, so area(ΔABQ) = $\frac{1}{2}$ area(ΔABC); PQ is a median of ΔABQ, so area(ΔBPQ) = $\frac{1}{2}$ area(ΔABQ) = $\frac{1}{4}$ area(ΔABC). Similarly area(ΔDRS) = $\frac{1}{4}$ area(ΔADC).

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$$\text{area}(\Delta BPQ) + \text{area}(\Delta DRS) = \frac{1}{4}(\text{area } \Delta ABC + \text{area } \Delta ADC) = \frac{1}{4} \text{ area}(ABCD)$$

In the same way, using the diagonal BD: area(ΔAPS) + area(ΔCQR) = $\frac{1}{4}$ area(ABCD).

$$\text{area}(PQRS) = \text{area}(ABCD) - \frac{1}{4} \text{ area}(ABCD) - \frac{1}{4} \text{ area}(ABCD) = \frac{1}{2} \text{ area}(ABCD)$$

Therefore, the parallelogram formed by joining the midpoints has half the area of the 4-gon.

QUESTION 9

In ΔABC, the midpoint of BC is D (Fig. 6.32). Median AD is drawn. P is any point on AD. Show that area (ΔABP) = area (ΔACP).

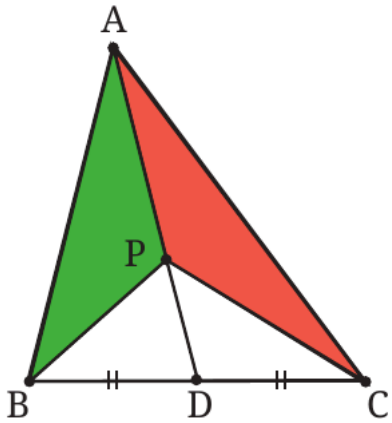


Fig. 6.32

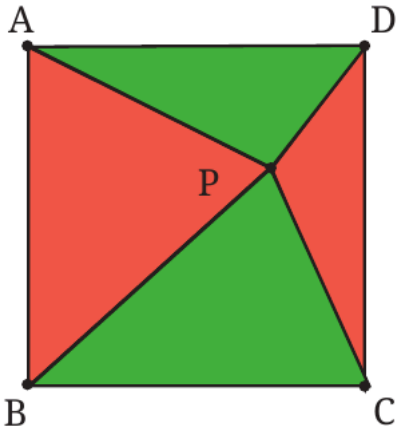


Fig. 6.33

ANSWER

AD is a median of ΔABC, so area(ΔABD) = area(ΔACD). PD is a median of ΔPBC, so area(ΔPBD) = area(ΔPCD).

$$\text{area}(\Delta ABP) = \text{area}(\Delta ABD) - \text{area}(\Delta PBD)$$
$$= \text{area}(\Delta ACD) - \text{area}(\Delta PCD) = \text{area}(\Delta ACP)$$

Therefore, area(ΔABP) = area(ΔACP).

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QUESTION 10

Given a square ABCD, let P be a point within it. Join PA, PB, PC, PD (Fig. 6.33). What is the ratio of the areas of the red region (ΔPAB and ΔPCD) and the green region (ΔPBC and ΔPDA)?

ANSWER

Let the side of the square be s. Let the distances of P from AB and CD be h_1 and h_2; then $h_1 + h_2 = s$. Similarly the distances from BC and DA, $k_1 + k_2 = s$.

$$\text{red} = \frac{1}{2}sh_1 + \frac{1}{2}sh_2 = \frac{1}{2}s(h_1 + h_2) = \frac{1}{2}s^2$$
$$\text{green} = \frac{1}{2}sk_1 + \frac{1}{2}sk_2 = \frac{1}{2}s^2$$

Therefore, the red and green regions each have half the area of the square; the ratio is 1 : 1.

QUESTION 11

In ΔABC, D is the midpoint of AB. P is any point on BC, and Q is a point on AB such that CQ || PD. PQ is joined (Fig. 6.34). Prove that Area (ΔBPQ) = $\frac{1}{2}$ Area (ΔABC).

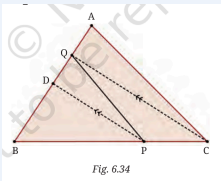


Fig. 6.34

ANSWER

ΔDPQ and ΔDPC have the same base DP and lie between the parallel lines DP and CQ, so they have equal area.

$$\text{area}(\Delta BPQ) = \text{area}(\Delta BPD) + \text{area}(\Delta DPQ) = \text{area}(\Delta BPD) + \text{area}(\Delta DPC) = \text{area}(\Delta BDC)$$

CD is a median of ΔABC (D is the midpoint of AB), so area(ΔBDC) = $\frac{1}{2}$ area(ΔABC).

Therefore, area(ΔBPQ) = $\frac{1}{2}$ area(ΔABC).

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6.3

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 6

Exercise Set 6.3

Solutions · 10 questions

Unless stated otherwise, use the approximation $\frac{22}{7}$ for π.

QUESTION 1

Find the area of a sector of a circle with radius 7 cm if the angle of the sector is 60°.

ANSWER

$$\text{area} = \pi r^2 \times \frac{\theta^\circ}{360^\circ} = \frac{22}{7} \times 49 \times \frac{60}{360} = 154 \times \frac{1}{6} = \frac{77}{3} \approx 25.67 \text{ cm}^2$$

Therefore, the area of the sector is $\frac{77}{3} \approx 25.7$ cm².

QUESTION 2

Find the area of a quadrant of a circle whose circumference is 44 cm.

ANSWER

$2\pi r = 44$, so $r = 7$ cm

$$\text{area} = \frac{1}{4} \times \frac{22}{7} \times 7^2 = 38.5 \text{ cm}^2$$

Therefore, the area of the quadrant is 38.5 cm².

QUESTION 3

The length of the minute hand of a clock is 7 cm. Find the area swept by the minute hand in 10 minutes.

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ANSWER

The minute hand turns 360° in 60 minutes, so in 10 minutes it turns 60°. The swept region is a sector of radius 7 cm and angle 60°.

$$\text{area} = \frac{22}{7} \times 49 \times \frac{60}{360} = \frac{77}{3} \approx 25.67 \text{ cm}^2$$

Therefore, the area swept is $\frac{77}{3} \approx 25.7$ cm².

QUESTION 4

A chord of a circle of radius 10 cm subtends 90° at the centre. Find the area of the corresponding: (i) minor sector (that subtends 90° at the centre), and (ii) major sector (that subtends 270° at the centre). (Use π ≈ 3.14.)

ANSWER

(i) **Minor sector**

$$3.14 \times 10^2 \times \frac{90}{360} = 314 \times \frac{1}{4} = 78.5 \text{ cm}^2$$

(ii) **Major sector**

$$3.14 \times 10^2 \times \frac{270}{360} = 314 \times \frac{3}{4} = 235.5 \text{ cm}^2$$

Therefore, the areas are 78.5 cm² and 235.5 cm².

QUESTION 5

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre of the circle. Find the areas of the corresponding minor and major segments of the circle. (Use π ≈ 3.14 and √3 ≈ 1.73.)

ANSWER

Minor segment = area of sector – area of triangle formed by the two radii and the chord. The triangle has two sides 15 cm and the angle between them 60°, so it is equilateral with side 15 cm.

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sector = $3.14 \times 225 \times \frac{60}{360} = 117.75$ cm²
triangle = $\frac{\sqrt{3}}{4} \times 15^2 = \frac{1.73 \times 225}{4} = 97.3125$ cm²
minor segment = $117.75 - 97.3125 = 20.4375 \approx 20.44$ cm²
circle = $3.14 \times 225 = 706.5$ cm²
major segment = $706.5 - 20.4375 = 686.0625 \approx 686.06$ cm²

Therefore, the minor segment has area ≈ 20.44 cm² and the major segment ≈ 686.06 cm².

QUESTION 6

A car has two wipers which do not overlap. Each wiper has a blade of length 28 cm and sweeps through an angle of 120°. Find the total area cleaned at each sweep of the blades.

ANSWER

Each wiper sweeps a sector of radius 28 cm and angle 120°.

one wiper: $\frac{22}{7} \times 28^2 \times \frac{120}{360} = \frac{22}{7} \times 784 \times \frac{1}{3} = \frac{2464}{3} \approx 821.33$ cm²
two wipers: $2 \times \frac{2464}{3} = \frac{4928}{3} \approx 1642.67$ cm²

Therefore, the total area cleaned at each sweep is $\frac{4928}{3} \approx 1642.7$ cm².

QUESTION 7 *

A chord of a circle of radius r subtends an angle of 60° at the centre of the circle. Show that the area of the corresponding minor segment of the circle is equal to $\pi r^2\left(\frac{1}{6} - \frac{\sqrt{3}}{4}\right)$.

ANSWER

The sector has area $\pi r^2 \times \frac{60}{360} = \frac{\pi r^2}{6}$. The triangle formed by the two radii and the chord has two sides r and the angle 60° between them, so it is equilateral with side r; its area is $\frac{\sqrt{3}}{4} r^2$.

segment = sector − triangle = $\frac{\pi r^2}{6} - \frac{\sqrt{3}}{4} r^2 = r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

Note: the factor outside the bracket in the printed statement should be r^2, not πr^2, with π inside the bracket: $r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$. Check with $r = 15$: $225(0.5236 - 0.4330) \approx 20.4$, which matches Question 5.

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Therefore, the area of the minor segment is $r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \frac{r^2}{12}(2\pi - 3\sqrt{3})$.

QUESTION 8 *

An equilateral triangle is inscribed in a circle of radius r. Show that the ratio of the area of the triangle to the area of the circle is equal to $\frac{3\sqrt{3}}{4\pi} \approx 0.413$.

ANSWER

In an equilateral triangle ABC the centre O of the circumscribed circle lies on the altitude AD and divides it in the ratio 2 : 1, so AO = r and OD = $\frac{r}{2}$, giving AD = $\frac{3r}{2}$.

$\mathrm{AD}^2 + \left(\frac{a}{2}\right)^2 = a^2$ (side a)
$\frac{9r^2}{4} = \frac{3a^2}{4}$, so $a^2 = 3r^2$
area of triangle $= \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}r^2}{4}$
ratio $= \frac{3\sqrt{3}r^2/4}{\pi r^2} = \frac{3\sqrt{3}}{4\pi} \approx \frac{3 \times 1.732}{12.566} \approx 0.413$

Therefore, the ratio of the areas is $\frac{3\sqrt{3}}{4\pi} \approx 0.413$.

QUESTION 9 *

A square is inscribed in a circle of radius r. Show that the ratio of the area of the square to the area of the circle is equal to $\frac{2}{\pi} \approx 0.637$.

ANSWER

The diagonal of the square is a diameter of the circle, so it is $2r$. If the side is a, then $a^2 + a^2 = (2r)^2$, so $a^2 = 2r^2$.

area of the square $= a^2 = 2r^2$
ratio $= \frac{2r^2}{\pi r^2} = \frac{2}{\pi} \approx 0.637$

Therefore, the ratio is $\frac{2}{\pi} \approx 0.637$.

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QUESTION 10 *

A hexagon is inscribed in a circle of radius r. Show that the ratio of the area of the hexagon to the area of the circle is equal to $\frac{3\sqrt{3}}{2\pi} \approx 0.827$. Can you see why the answer is exactly twice the answer to Question 8?

ANSWER

A regular hexagon inscribed in a circle of radius r consists of 6 equilateral triangles of side r.

$$\text{area} = 6 \times \frac{\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{2} r^2$$
$$\text{ratio} = \frac{3\sqrt{3}r^2/2}{\pi r^2} = \frac{3\sqrt{3}}{2\pi} \approx 0.827$$

Why twice the answer to Q8: joining alternate vertices V1, V3, V5 gives the inscribed equilateral triangle. If O is the centre, ΔOV1V3 is half of the rhombus OV1V2V3 (all sides r), which is made of 2 of the 6 small equilateral triangles, so ΔOV1V3 equals one small triangle. Hence the inscribed triangle = 3 small triangles = half of the hexagon, i.e. the hexagon has twice the area of the inscribed triangle, and so twice the ratio.

Therefore, the ratio is $\frac{3\sqrt{3}}{2\pi} \approx 0.827$, exactly twice $\frac{3\sqrt{3}}{4\pi}$.

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EC

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 6

End-of-Chapter Exercises

Solutions · 27 questions

Unless stated otherwise, use the approximation $\frac{22}{7}$ for π.

QUESTION 1

Identities in algebra can sometimes be shown as area relationships. For example, the figure shown corresponds to the identity $(a + b)^2 = a^2 + 2ab + b^2$. Do you see how? Draw figures corresponding to the identities $(a + b)(a - b) = a^2 - b^2$ and $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

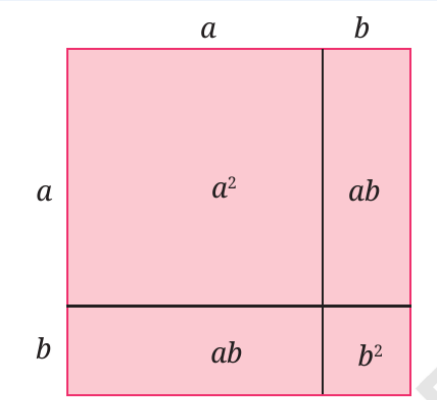


Fig. 6.41: Area model of an identity

ANSWER

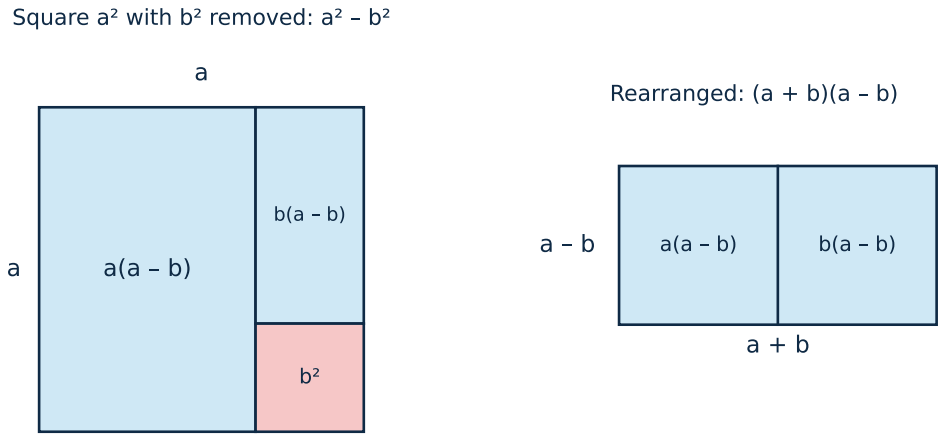
0 The given figure

A square of side $a + b$ is divided into a square a^2, a square b^2 and two rectangles ab, so its area $(a + b)^2$ equals $a^2 + 2ab + b^2$.

1 $(a + b)(a - b) = a^2 - b^2$

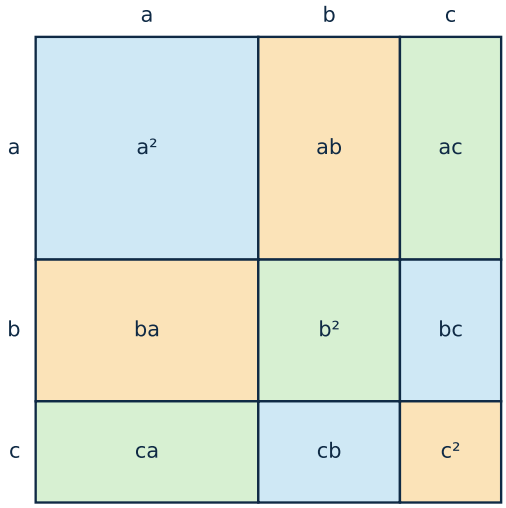
Start with a square of side a (area a^2) and remove a square of side b from a corner: the area left is $a^2 - b^2$. Cut the remaining region into the rectangle $a \times (a - b)$ and the rectangle $b \times (a - b)$, and place them side by side to get a rectangle of sides $(a + b)$ and $(a - b)$.

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2 **$(a + b + c)^2$**

Divide a square of side $a + b + c$ by lines at distances a, b, c along both sides into 9 pieces: three squares a^2, b^2, c^2 and six rectangles, two each of ab, bc, ca.



Therefore, the two figures show $(a + b)(a - b) = a^2 - b^2$ and $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

QUESTION 2

An isosceles triangle has perimeter 40 cm; the equal sides are 15 cm each. Find the area of the triangle.

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ANSWER

base = 40 – 15 – 15 = 10 cm
$s = 20$
area = $\sqrt{(20 \times 5 \times 5 \times 10)} = \sqrt{5000} = 50\sqrt{2} \approx 70.71$

Check: height = $\sqrt{(15^2 - 5^2)} = \sqrt{200} = 10\sqrt{2}$, area = $\frac{1}{2} \times 10 \times 10\sqrt{2} = 50\sqrt{2}$ ✓.

Therefore, the area is $50\sqrt{2} \approx 70.7$ cm².

QUESTION 3

An isosceles triangle has base 10 cm, and its area is 60 cm². What are the lengths of the equal sides?

ANSWER

$\frac{1}{2} \times 10 \times h = 60$, so $h = 12$ cm
equal side = $\sqrt{(12^2 + 5^2)} = \sqrt{169} = 13$

Therefore, the equal sides are 13 cm each.

QUESTION 4

The area of a right-angled triangle is 54 sq. cm. One of its legs has length 12 cm. Find its perimeter.

ANSWER

$\frac{1}{2} \times 12 \times b = 54$, so the other leg $b = 9$ cm
hypotenuse = $\sqrt{(12^2 + 9^2)} = \sqrt{225} = 15$ cm
perimeter = 12 + 9 + 15 = 36 cm

Therefore, the perimeter is 36 cm.

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QUESTION 5

The sides of a triangle are in the ratio 2: 3: 4, and its perimeter is 45 cm. Find its area.

ANSWER

$\text{sides} = 45 \times \frac{2}{9}, 45 \times \frac{3}{9}, 45 \times \frac{4}{9} = 10, 15, 20 \text{ cm}$

$s = 22.5$

$\text{area} = \sqrt{(22.5 \times 12.5 \times 7.5 \times 2.5)} = \frac{75\sqrt{15}}{4} \approx 72.62$

Therefore, the area is $\frac{75\sqrt{15}}{4} \approx 72.6$ cm².

QUESTION 6

The sides of a triangle have lengths 7 cm, 24 cm, 25 cm. Find the area of the triangle in two different ways.

ANSWER

1 Right-angled triangle

$7^2 + 24^2 = 49 + 576 = 625 = 25^2$, so the triangle is right-angled with legs 7 and 24.

$\text{area} = \frac{1}{2} \times 7 \times 24 = 84$

2 Heron's formula

$s = \frac{7 + 24 + 25}{2} = 28$

$\text{area} = \sqrt{(28 \times 21 \times 4 \times 3)} = \sqrt{7056} = 84$

Therefore, the area is 84 cm² by both methods.

QUESTION 7

If the wheel of a bicycle has a diameter of 60 cm, find how far a cyclist will have travelled after the wheel has rotated 100 times.

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ANSWER

circumference $= \frac{22}{7} \times 60 = \frac{1320}{7} \approx 188.57$ cm
distance $= 100 \times \frac{1320}{7} = \frac{132000}{7} \approx 18\,857.14$ cm ≈ 188.57 m

Therefore, the cyclist travels about 188.57 m.

QUESTION 8

Find the area of a quadrant of a circle whose circumference is 66 cm.

ANSWER

$2 \times \frac{22}{7} \times r = 66$, so $r = 10.5$ cm
area $= \frac{1}{4} \times \frac{22}{7} \times 10.5^2 = \frac{1}{4} \times 346.5 = 86.625$

Therefore, the area of the quadrant is 86.625 ≈ 86.63 cm².

QUESTION 9

The wheel of a car has an outer radius of 28 cm. Calculate how far the car travels after one complete turn of the wheel, and how many times the wheel turns during a journey of 1 km.

ANSWER

one turn $= 2 \times \frac{22}{7} \times 28 = 176$ cm
1 km = 100 000 cm; turns $= \frac{100000}{176} \approx 568.18$

Therefore, the car travels 176 cm per turn, and the wheel turns about 568 times in 1 km.

QUESTION 10 *

Two rectangles have the same area and the same perimeter. Does this mean that they are congruent to each other?

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ANSWER

Let the sides of a rectangle be l and w, with $l + w = s$ (half the perimeter) and $lw = A$ (the area). Then $(l - w)^2 = (l + w)^2 - 4lw = s^2 - 4A$, so $|l - w|$ is fixed by the given area and perimeter. Together with $l + w = s$, this fixes the pair of sides (the longer is $(s + \sqrt{(s^2 - 4A)}) \div 2$, the shorter is $(s - \sqrt{(s^2 - 4A)}) \div 2$).

Therefore, yes: two rectangles with the same area and perimeter have the same pair of side lengths, so they are congruent.

QUESTION 11

You know that the area of a parallelogram is base × height. Using this and the figure, show that the area of a trapezium is half the sum of the parallel sides × height, i.e., $\frac{1}{2}(a + b)h$.

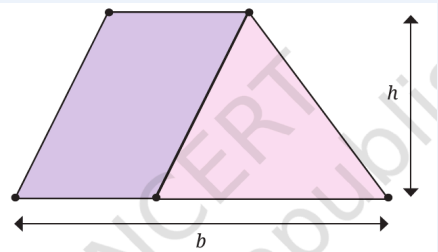


Fig. 6.42: Trapezium: sides a and b, height h

ANSWER

The line from the end of the top side a parallel to the left slanting side cuts the trapezium into a parallelogram (purple) and a triangle (pink), each of height h.

parallelogram: base a, height h: area ah
triangle: base $(b - a)$, height h: area $\frac{1}{2}(b - a)h$
total $= ah + \frac{1}{2}(b - a)h = \frac{1}{2}h(2a + b - a) = \frac{1}{2}(a + b)h$

Therefore, the area of the trapezium is $\frac{1}{2}(a + b)h$.

QUESTION 12

By dividing a trapezium into two triangles show that its area is half the sum of the parallel sides multiplied by the height (the same formula as the one given above).

ANSWER

Draw a diagonal of the trapezium with parallel sides a and b and height h. It divides the trapezium into two triangles: one has base a, the other has base b, and both have height h (the distance

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between the parallel sides).

$$\text{area} = \frac{1}{2}ah + \frac{1}{2}bh = \frac{1}{2}(a + b)h$$

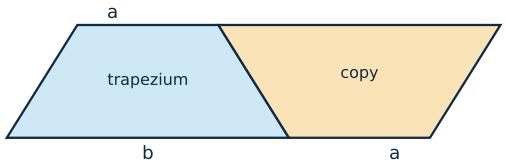
Therefore, the area of the trapezium is $\frac{1}{2}(a + b)h$.

QUESTION 13

Show how we can use two identical copies of a trapezium to make a parallelogram. How will this give us the formula for the area of a trapezium?

ANSWER

Turn a second copy through a half-turn about the midpoint of one of the slanting sides and join it to the first copy along that side. The two slanting sides fit exactly, and the figure has top and bottom sides of length $a + b$ each, which are parallel; the other two sides are the remaining slanting sides, also parallel. So it is a parallelogram with base $(a + b)$ and height h.



$$\text{area of parallelogram} = (a + b)h$$
$$\text{area of one trapezium} = \frac{1}{2}(a + b)h$$

Therefore, the area of a trapezium is half the sum of its parallel sides times the height.

QUESTION 14

Show that the area of a kite is half the product of its diagonals. Show this: (i) using algebra, and (ii) using geometry.

ANSWER

Let ABCD be a kite with AB = AD and CB = CD. The diagonal AC is the line of symmetry; it is the perpendicular bisector of the other diagonal BD. Let AC and BD meet at O, with AC = p and BD = q.

(i) Algebra

Let AO = x and OC = y, so $x + y = p$; also BO = OD = $\frac{q}{2}$.

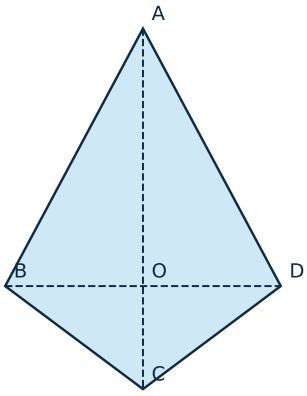
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$$\text{area} = 2 \times \frac{1}{2} \times \frac{q}{2} \times x + 2 \times \frac{1}{2} \times \frac{q}{2} \times y = \frac{q}{2}(x + y) = \frac{pq}{2}$$

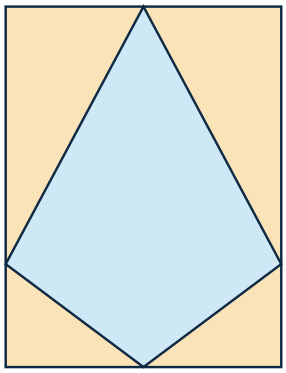
(ii) Geometry

Draw the rectangle through A and C parallel to BD and through B and D parallel to AC. Its sides are p and q. Each of the four small rectangles around O is cut by a side of the kite into two equal triangles, one inside the kite and one outside. So the kite is half of the rectangle.

Kite ABCD, diagonals AC ⊥ BD



Enclosing rectangle (AC × BD)



Therefore, area of the kite = $\frac{1}{2} \times p \times q$, half the product of its diagonals.

QUESTION 15

Three problems about fitting congruent shapes together:
(i) Rectangle ABCD has sides *a*, *b*, and rectangle PQRS has sides 2*a*, 2*b*. Show that PQRS has 4 times the area of ABCD. Does this mean that 4 copies of rectangle ABCD will fit into rectangle PQRS? Check and see!
(ii) ΔABC has sides *a*, *b*, *c*, and ΔPQR has sides 2*a*, 2*b*, 2*c*. Show that ΔPQR has 4 times the area of ΔABC. Does this mean that 4 copies of ΔABC will fit into ΔPQR?
(iii) ΔABC has sides *a*, *b*, *c*, and ΔPQR has sides 3*a*, 3*b*, 3*c*. Show that ΔPQR has 9 times the area of ΔABC. Does this mean that 9 copies of ΔABC will fit into ΔPQR?

ANSWER

(i) Rectangles — YES, 4 FIT

Area of PQRS = $2a \times 2b = 4ab = 4 \times$ area of ABCD. Divide PQRS into a 2×2 grid by joining midpoints of opposite sides: the four pieces are rectangles $a \times b$, each congruent to ABCD.

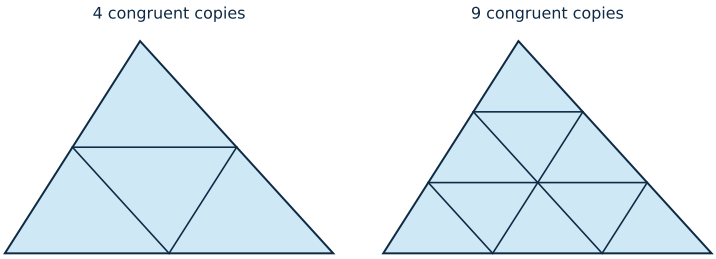
(ii) Triangles, sides doubled — YES, 4 FIT

By Heron's formula, the semi-perimeter becomes $2s$ and each factor doubles: area $= \sqrt{(2s \cdot 2(s-a) \cdot 2(s-b) \cdot 2(s-c)}) = 4 \times$ area(ΔABC). Joining the midpoints of the sides of ΔPQR divides it into 4 triangles with sides a, b, c (by the midpoint theorem), all congruent to ΔABC.

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(iii) **Triangles, sides tripled** YES, 9 FIT

Now the semi-perimeter becomes $3s$ and each factor triples: area = 3^2 × area(ΔABC) = 9 × area(ΔABC). Divide each side of ΔPQR into 3 equal parts and draw lines parallel to the sides through the division points: they cut ΔPQR into 9 triangles, each congruent to ΔABC.



Therefore, doubling or tripling the sides multiplies the area by 4 or 9, and in each case exactly that many copies fit.

QUESTION 16 *

Fig. 6.43: What fraction of the triangle is shaded? Fig. 6.44: What fraction of the square is shaded?

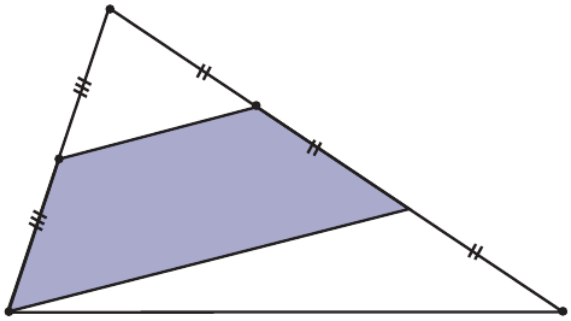


Fig. 6.43: What fraction of the triangle is shaded?

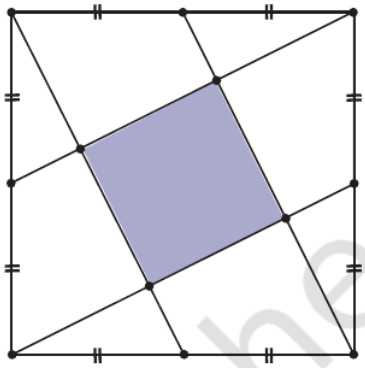


Fig. 6.44: What fraction of the square is shaded?

ANSWER

1 **Fig. 6.43**

Let ΔABC (A at the top) have area Δ. U is the midpoint of AB, and V, W trisect AC (V nearer A). The shaded part is the quadrilateral BUVW.

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area(ΔABW) = $\frac{2}{3}\Delta$ (base AW = $\frac{2}{3}$AC, same vertex B)
area(ΔUAW) = $\frac{1}{2}$ area(ΔABW) = $\frac{1}{3}\Delta$ (U midpoint of AB)
area(ΔUAV) = $\frac{1}{2}$ area(ΔUAW) = $\frac{1}{6}\Delta$ (V midpoint of AW)
shaded = area(ΔABW) – area(ΔUAV) = $\frac{2}{3}\Delta - \frac{1}{6}\Delta = \frac{1}{2}\Delta$

2 **Fig. 6.44**

Take the square of side 2 with vertices (0, 0), (2, 0), (2, 2), (0, 2). The four lines are y = 2 – 2x, y = 4 – 2x, y = 1 + $\frac{x}{2}$, y = $\frac{x}{2}$. They meet in pairs at (0.4, 1.2), (0.8, 0.4), (1.6, 0.8), (1.2, 1.6), which are the vertices of the shaded square.

side^2 = $0.4^2 + 0.8^2 = 0.8$, so shaded area = 0.8
square area = 4, fraction = $\frac{0.8}{4} = \frac{1}{5}$

Therefore, $\frac{1}{2}$ of the triangle is shaded in Fig. 6.43 and $\frac{1}{5}$ of the square is shaded in Fig. 6.44.

QUESTION 17

Fig. 6.45 and Fig. 6.46: What fraction of the rectangle is covered by the circles?

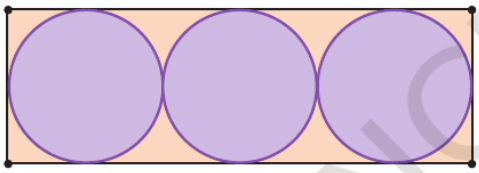


Fig. 6.45: What fraction of the rectangle is covered by the circles?

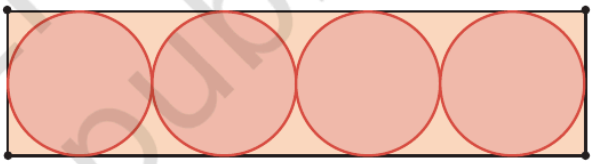


Fig. 6.46: What fraction of the rectangle is covered by the circles?

ANSWER

Let the diameter of each circle be d; the circles touch the sides of the rectangle and each other.

1 **Fig. 6.45 (3 circles)**

rectangle = $3d \times d = 3d^2$
circles = $3 \times \pi(\frac{d}{2})^2 = \frac{3\pi d^2}{4}$
fraction = $\frac{3\pi d^2/4}{3d^2} = \frac{\pi}{4}$

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2 **Fig. 6.46 (4 circles)**

rectangle = $4d \times d = 4d^2$
circles = $4 \times \frac{\pi d^2}{4} = \pi d^2$
fraction = $\frac{\pi d^2}{4d^2} = \frac{\pi}{4}$

Therefore, in both figures the fraction covered is $\frac{\pi}{4} \approx 0.785$ (= $\frac{11}{14}$ with $\pi = \frac{22}{7}$).

QUESTION 18

Use the above to make a conjecture about the area occupied by circles fitted into a rectangle in the manner shown. Test your conjecture for particular cases: 10 circles; 20 circles; 50 circles. Then prove your conjecture!

ANSWER

Conjecture: n equal circles in a row, touching each other and the sides of the rectangle, always cover $\frac{\pi}{4}$ of the rectangle, whatever n is.

Circles n	Rectangle area	Circles area	Fraction
10	$10d^2$	$10 \times \frac{\pi d^2}{4}$	$\frac{\pi}{4}$
20	$20d^2$	$20 \times \frac{\pi d^2}{4}$	$\frac{\pi}{4}$
50	$50d^2$	$50 \times \frac{\pi d^2}{4}$	$\frac{\pi}{4}$

Proof: the rectangle is nd by d, so its area is nd^2. Each circle has area $\pi(\frac{d}{2})^2 = \frac{\pi d^2}{4}$, so the circles cover $\frac{n\pi d^2}{4}$.

fraction = $\frac{n\pi d^2/4}{nd^2} = \frac{\pi}{4}$

Therefore, the circles cover $\frac{\pi}{4}$ of the rectangle for any number of circles.

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QUESTION 19 *

The figure shows nine identical rectangles fitted together to make a large rectangle whose area is 72 cm². Find the perimeter of each small rectangle.

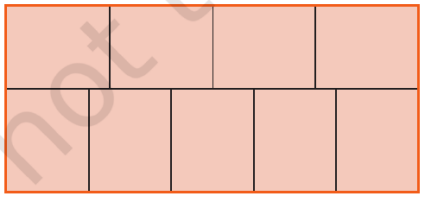


Fig. 6.47: Nine identical rectangles stacked together

ANSWER

Let each small rectangle have length l and width w. The top row has 4 rectangles laid with their lengths along the top, and the bottom row has 5 rectangles standing upright. Equal widths of the two rows give $4l = 5w$, so $l = \frac{5w}{4}$.

The big rectangle is $4l = 5w$ wide and $w + l = \frac{9w}{4}$ high.

$$\text{area} = 5w \times \frac{9w}{4} = \frac{45w^2}{4} = 72$$
$$w^2 = \frac{32}{5}, \text{ so } w = \frac{4\sqrt{2}}{\sqrt{5}} = \frac{4\sqrt{10}}{5} \approx 2.53 \text{ and } l = \frac{5w}{4} = \sqrt{10} \approx 3.16$$

$$\text{perimeter} = 2(l + w) = 2 \times \frac{9w}{4} = \frac{9w}{2} = \frac{18\sqrt{10}}{5} \approx 11.38$$

Check: $lw = 8$ and $9 \times 8 = 72$ ✓.

Therefore, the perimeter of each small rectangle is $\frac{18\sqrt{10}}{5} \approx 11.4$ cm.

QUESTION 20 *

Show that the areas of the shaded blue triangle and the shaded red triangle are equal. Find a way of cutting up the blue triangle into some number of pieces and rearranging the pieces to cover the red triangle.

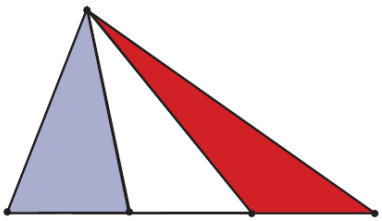


Fig. 6.48: Lines from a vertex to the points of trisection of the opposite side

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ANSWER

The lines from the vertex to the points of trisection of the opposite side divide it into three equal parts. All the triangles so formed have the same height (the distance of the vertex from that side) and equal bases ($b/3$).

$$\text{area(blue)} = \frac{1}{2} \times \frac{b}{3} \times h = \text{area(red)}$$

Cutting: cut the blue triangle along the line joining the midpoints of its two slanting sides and turn the small top triangle through a half-turn about one of those midpoints; this gives a parallelogram of base $\frac{b}{3}$ and height $\frac{h}{2}$. Move a triangle from one end of this parallelogram to the other to make it fit the parallelogram obtained in the same way from the red triangle (cut along its own midline); reversing the cuts on the red triangle then covers it exactly.

Therefore, the two triangles have equal area, and the blue one can be cut into a few pieces that rearrange to cover the red one.

QUESTION 21 *

The figure shows a quarter circle in a square. Its centre is at one vertex, and it passes through two adjacent vertices. There are two semicircles on two adjacent sides as diameters. They create the shaded regions A and B. Show that A and B have equal area.

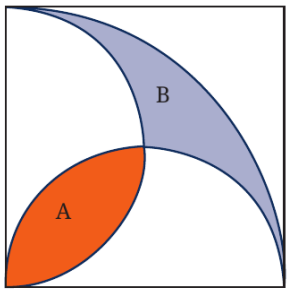


Fig. 6.49: A quarter circle and two semicircles

ANSWER

Let the side of the square be s. The quarter circle has radius s; each semicircle has diameter s.

$$\text{quarter circle} = \frac{1}{4}\pi s^2$$
$$\text{each semicircle} = \frac{1}{2}\pi\left(\frac{s}{2}\right)^2 = \frac{1}{8}\pi s^2$$

The two semicircles overlap in region A. The area covered by the quarter circle is the union of the semicircles together with region B, so

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quarter circle = (semicircle + semicircle – A) + B
$\frac{1}{4}\pi s^2 = \frac{1}{4}\pi s^2 - A + B$

Therefore, A = B.

QUESTION 22 *

In Fig. 6.50, four semicircles have been drawn within the given square whose side is 2 units. The centres of these semicircles are the midpoints of the sides. They create a 4-petalled flower (shown in blue). Find the perimeter and the area of this flower.

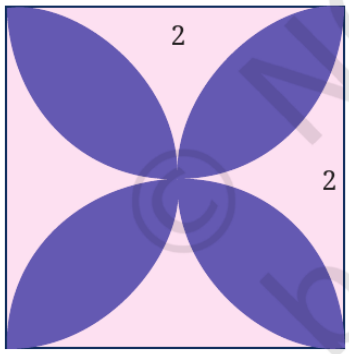


Fig. 6.50

ANSWER

Each semicircle has radius 1 and diameter along a side of the square; it passes through the centre of the square. Each petal is bounded by two quarter circles of radius 1, and adjacent petals share only the centre point.

1 Perimeter

The four petals have 8 quarter circles in all, i.e. 2 full circles of radius 1 (equivalently, the four semicircles).

perimeter = $4 \times \pi \times 1 = 4\pi \approx 12.57$ units

2 Area

A petal is the overlap of two semicircles: two quarter circles minus the square they lie in (side 1).

petal = $2 \times (\frac{1}{4}\pi \times 1^2) - 1^2 = \frac{\pi}{2} - 1$
4 petals = $4(\frac{\pi}{2} - 1) = 2\pi - 4 \approx 2.28$ sq. units

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Therefore, the perimeter of the flower is $4\pi \approx 12.57$ units and its area is $2\pi - 4 \approx 2.28$ sq. units.

QUESTION 23 *

In Fig. 6.51 we see two concentric circles with a common centre O. A chord BC of the larger circle is drawn, touching the smaller circle at A. The length of BC is l. Show that the area of the green region enclosed between the two circles is $\frac{1}{4}\pi l^2$.

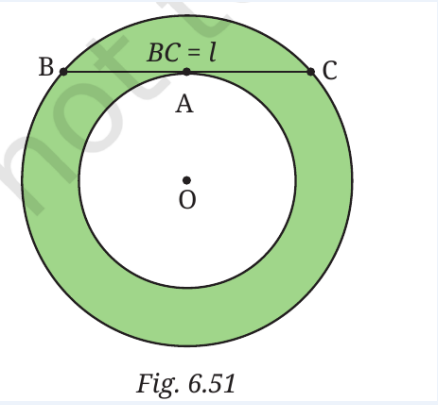


Fig. 6.51

ANSWER

Let the radii of the larger and smaller circles be R and r. BC touches the smaller circle at A, so OA $\perp$ BC, OA = r. The perpendicular from the centre bisects the chord, so AB = $\frac{l}{2}$.

In right ΔOAB: $R^2 = r^2 + (\frac{l}{2})^2$

$R^2 - r^2 = \frac{l^2}{4}$

area of green region = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \frac{1}{4}\pi l^2$

Therefore, the area of the green region is $\frac{1}{4}\pi l^2$.

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QUESTION 24 *

In Fig. 6.52, semicircles have been drawn on all the sides of a right-angled triangle as shown. Show that Area (A) + Area (B) = Area (C).

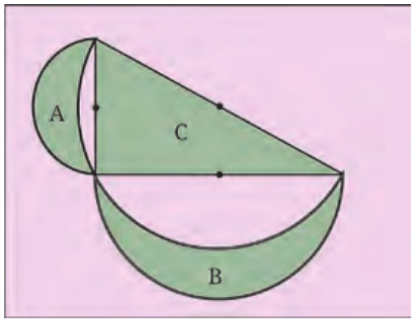


Fig. 6.52

ANSWER

Let the legs of the right triangle (C) be a and b and the hypotenuse c, with $a^2 + b^2 = c^2$. The semicircle on the hypotenuse passes through the right-angle vertex. Let T = area of the triangle C and let S_a, S_b, S_c be the areas of the semicircles on the legs and hypotenuse.

$$S_a + S_b = \frac{\pi}{8}(a^2 + b^2) = \frac{\pi}{8}c^2 = S_c$$

The hypotenuse semicircle consists of the triangle C and the two circular segments cut off by the legs. The lunes A and B are the leg semicircles minus those segments:

$$A + B = (S_a + S_b) - (\text{segments}) = S_c - (S_c - T) = T$$

Therefore, Area (A) + Area (B) = Area (C) (the lunes of Hippocrates).

QUESTION 25 *

Fig. 6.53 shows two circles passing through each other's centres. Find the area of the region enclosed by the two circles in terms of the common radius *r*.

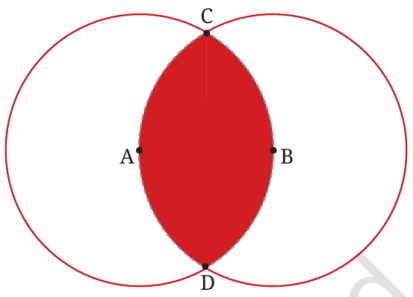


Fig. 6.53: Two congruent circles, radius r

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ANSWER

With centres A and B (AB = r) and the circles meeting at C and D, ΔABC and ΔABD are equilateral, so $\angle CAD = 120°$ and $\angle CBD = 120°$. The common region is two circular segments cut off by the chord CD, one from each circle.

sector of $120° = \frac{1}{3}\pi r^2$
CD $= 2 \times \frac{\sqrt{3}}{2} r = \sqrt{3}r$; ΔACD has base $\sqrt{3}r$ and height AM $= \frac{r}{2}$: area $= \frac{\sqrt{3}}{4} r^2$
one segment $= \frac{\pi r^2}{3} - \frac{\sqrt{3}r^2}{4}$
region $= 2 \times (\frac{\pi r^2}{3} - \frac{\sqrt{3}r^2}{4}) = r^2(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}) \approx 1.228r^2$

Therefore, the area of the region enclosed by both circles is $r^2(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}) \approx 1.23r^2$.

QUESTION 26 *

In Fig. 6.54, we see three triangles within a rectangle. The areas of the triangles are A, B, C, as marked. Show that the area of the rectangle is 2(A + C)(B + C) ÷ C.

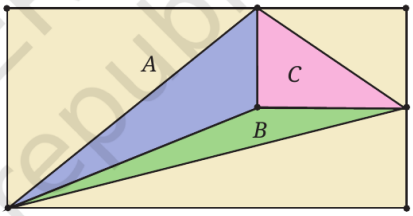


Fig. 6.54

ANSWER

Let the rectangle have width W and height H. Let the vertex at the bottom-left be O, the top point be U at distance p from the left side, the right point be V at height q, and let X be the point at the corner of triangle C (X is p from the left and q from the bottom). Then UX $= H - q$ and XV $= W - p$.

A $= \frac{1}{2}p(H - q)$
C $= \frac{1}{2}(H - q)(W - p)$
B $= \frac{1}{2}(W - p)q$
A + C $= \frac{1}{2}(H - q)W$, B + C $= \frac{1}{2}(W - p)H$

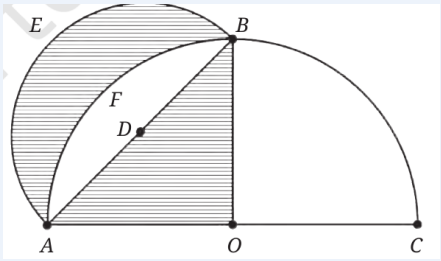
$2(A + C)(B + C) \div C = 2 \times [\frac{1}{2}(H - q)W] \times [\frac{1}{2}(W - p)H] \div [\frac{1}{2}(H - q)(W - p)] = WH$

Therefore, the area of the rectangle is 2(A + C)(B + C) ÷ C.

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QUESTION 27 *

In the figure we see two shaded regions formed by a quarter circle, a semicircle, and a triangle. Show that the areas of the two shaded regions are equal.



ANSWER

Let OA = OB = r. The arc AB (centre O) is a quarter circle, ΔAOB is right-angled, and AB = $\sqrt{2}r$. The semicircle on AB as diameter has radius $\frac{r}{\sqrt{2}}$.

lune (E, between the semicircle and the arc AB) = semicircle on AB – segment

semicircle on AB = $\frac{1}{2}\pi(\frac{r}{\sqrt{2}})^2 = \frac{\pi r^2}{4}$

segment = quarter circle – ΔAOB = $\frac{\pi r^2}{4} - \frac{r^2}{2}$

lune = $\frac{\pi r^2}{4} - (\frac{\pi r^2}{4} - \frac{r^2}{2}) = \frac{r^2}{2}$

area of ΔAOB = $\frac{1}{2} \times r \times r = \frac{r^2}{2}$

Therefore, the lune has the same area ($\frac{r^2}{2}$) as the triangle AOB, so the two shaded regions are equal.

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