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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 5

I’m Up and Down, and Round and Round

Complete Exercise Solutions

Exercise Sets 5.1 – 5.6 · End-of-Chapter Exercises

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IT

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 5

In-Text Questions & Activities

Solutions · 12 items

IN-TEXT 1

Can you recognise the origin of the shapes in Fig. 5.1?



ANSWER

The three photographs show raindrops falling on water, the cross-section of a plant stem (tree trunk) and the inflorescence of a sunflower.

Rain on water: ripples spread out as circles

Cross-section of a plant stem: nearly circular with rings

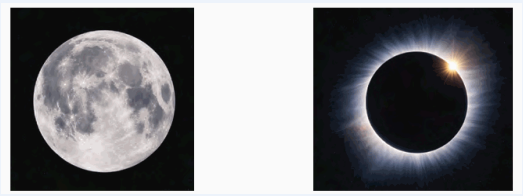
Sunflower head: circular arrangement of florets

Therefore, all three shapes come from circles found in nature: ripples of raindrops, the cross-section of a plant stem and a sunflower.

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ACTIVITY

Activity: List some objects from nature that resemble a circle.



ANSWER

Some natural objects that look like circles:

The full moon and the sun

Ripples formed when a raindrop falls on water

The cross-section of a plant stem or tree trunk

The inflorescence (head) of a sunflower

The pupil of the eye, a seed such as a pea, a bubble (circular in cross-section)

Therefore, natural objects such as the moon, the sun, water ripples and the sunflower resemble circles.

IN-TEXT 2

Think and Reflect

Jamuna has a circular piece of paper. She is trying to locate its centre. Amina gives her a suggestion. She follows the instructions and is thrilled to find that it works. Can you guess what Amina told her?

ANSWER

Amina’s suggestion uses the fact that every fold that makes the boundary overlap gives a line of reflection symmetry, and every such line is a diameter (it passes through the centre).

Step 1: Fold the circular paper so that the boundary overlaps itself; open it. The crease is a diameter.

Step 2: Fold it again in a different direction so that the boundary overlaps; open it. The second crease is another diameter.

Step 3: The two creases meet at exactly one point.

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Therefore, the point where the two creases (diameters) meet is the centre of the circle.

IN-TEXT 3

Think and Reflect

1. What are the rotational symmetries of a square? How many lines of reflection symmetry does it have? What about a regular pentagon? A regular hexagon?

2. What is the length of the longest chord in a circle of radius 5 units? Is there a smallest chord?

3. The locus of points at a given distance from a given point is a circle. What can we say about the locus of points equidistant from two given points? (Hint: We know that any point that is equidistant from two given points A and B lies on the perpendicular bisector of AB. Does this make the perpendicular bisector the locus? For this, we have to show that all the points on the perpendicular bisector are equidistant from A and B.)

ANSWER

1 Symmetries of regular polygons

Square: rotations by 90°, 180°, 270° (and 360°); 4 lines of reflection symmetry (2 diagonals and 2 lines through midpoints of opposite sides).

Regular pentagon: rotations by multiples of 72° (72°, 144°, 216°, 288°); 5 lines of reflection symmetry (each through a vertex and the midpoint of the opposite side).

Regular hexagon: rotations by multiples of 60°; 6 lines of reflection symmetry (3 through opposite vertices and 3 through midpoints of opposite sides).

2 Longest and smallest chord

The longest chord passes through the centre, so it is a diameter: $2 \times 5 = 10$ units. There is no smallest chord: a chord can be made as short as we like by moving its endpoints closer together, but it can never have length 0 (its two endpoints are distinct).

3 Locus of points equidistant from two points

Let M be the midpoint of AB and P any point on the perpendicular bisector. In $\triangle PMA$ and $\triangle PMB$: $AM = BM$, $\angle PMA = \angle PMB = 90°$ and PM is common.

$\triangle PMA \cong \triangle PMB$ (SAS) $\Rightarrow PA = PB$

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So every point on the perpendicular bisector is equidistant from A and B. Together with the given fact, this makes the perpendicular bisector the locus.

Answers: 1. square: 4 rotations and 4 reflection lines; pentagon: 5 and 5; hexagon: 6 and 6. 2. The longest chord is 10 units; there is no smallest chord. 3. The locus is the perpendicular bisector of the segment joining the two points.

IN-TEXT 4

Think and Reflect

1. How many circles pass through two points on a plane?

2. Are there circles of all possible radii passing through A and B? What is the radius of the smallest circle passing through A and B? What is the radius of the largest circle passing through A and B?

3. As you move away from segment AB along its perpendicular bisector, do the radii of the circles containing A and B increase or decrease?

4. As you go along the perpendicular bisector, will the circle drawn from that point through A and B appear more curved or less curved?

5. You are given two points A and B on a plane. How many squares can you draw on the same plane with A and B on the boundary? How many squares can you draw on the plane with A and B as the corners of the square?

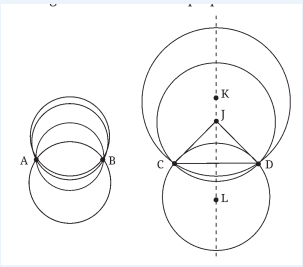


Fig. 5.4

ANSWER

1 Number of circles

The centre of any such circle lies on the perpendicular bisector of AB, which has infinitely many points. So infinitely many circles pass through two given points.

2 Radii

If the centre is a point O at distance x from the midpoint M of AB, then the radius is $OA = \sqrt{(x^2 + (AB/2)^2)}$. This is at least AB/2 and can be made as large as we like.

Smallest radius = AB/2 (centre at M, AB is a diameter)

There is no largest radius

Radii of all values ≥ AB/2 occur; no circle through A and B has a radius smaller than AB/2.

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3 **Radii along the bisector**

As the centre moves away from M, x increases, so the radius $\sqrt{(x^2 + (AB/2)^2)}$ increases.

4 **Curvature**

A larger circle is flatter near AB, so the circle appears less curved as we go away from AB along the perpendicular bisector.

5 **Squares through A and B**

With A and B on the boundary: infinitely many (for example, squares of any side greater than or equal to AB/√2 can be placed with A, B on their sides).
With A and B as corners: 3 squares (2 with AB as a side, one on each side of AB, and 1 with AB as a diagonal).

Answers: 1. infinitely many; 2. no, only radii ≥ AB/2; smallest AB/2, no largest; 3. increase; 4. less curved; 5. infinitely many; 3 squares.

IN-TEXT 5

What if A, B and C lie on a straight line, i.e., are collinear? Can you explain why, in this case, there is no circle through A, B and C?

ANSWER

Suppose a circle passes through collinear points A, B and C with centre O. Then OA = OB = OC, so O lies on the perpendicular bisector of AB and on the perpendicular bisector of BC.

Both bisectors are perpendicular to the same line ABC, so they are parallel. They are distinct, because AB and BC have different midpoints. Two distinct parallel lines have no common point, so no such O exists.

Therefore, no circle can pass through three collinear points (a line meets a circle in at most two points).

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ACTIVITY

Activity (Section 5.6): Take a paper circle. Fold the circle from the boundary, inwards. Open the fold. The crease is now a chord (see Fig. 5.13B). Now fold the paper again, so that the end points of the chord meet. Open the fold (see Fig. 5.13C).
Measure the lengths of the parts into which the chord is divided. Measure the angle between the creases. Measure the distance from the centre to the midpoint of the chord. Now draw another chord of the same length. Join the centre to the midpoint of the new chord and measure its length. Is it the same as distance from the centre to the first chord?



Fig. 5.13

ANSWER

The second fold bisects the chord: the two parts of the chord are equal.

The angle between the two creases is 90°: the second crease is along the perpendicular from the centre to the chord.

The distance from the centre to the midpoint of the chord is the distance from the centre to the chord.

A second chord of the same length (obtained by rotating the circle, or by tracing) has the same distance from the centre.

Therefore, a perpendicular from the centre bisects the chord, and chords of equal length are at equal distance from the centre.

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ACTIVITY

Activity (Section 5.6.1): Draw a circle. Draw chords of various lengths. Drop a perpendicular to each chord from the centre. Record the length of the chord and its distance from the centre in a table (Table 1).

ANSWER

Sample readings for a circle of radius 5 cm (distance from the centre = $\sqrt{(25 - (\text{half chord})^2)}$):

Length of chord (cm): 4.0 | 6.0 | 8.0 | 10.0

Distance from centre (cm): 4.58 | 4.0 | 3.0 | 0.0

The distance from the centre decreases as the chord gets longer; the diameter (10 cm) passes through the centre (distance 0).

Therefore, the longer the chord, the closer it is to the centre.

IN-TEXT 6

Exercise: A circle with centre O is drawn, and A, B, C, D are points on the circle (see Fig. 5.19). Measure the angles subtended by arc AKB and arc CLD at the centre O. If the angle at the centre is less than 180°, it is a minor arc. If the angle at the centre is greater than 180°, it is a major arc. State whether arcs AKB and CLD are minor arcs or major arcs.

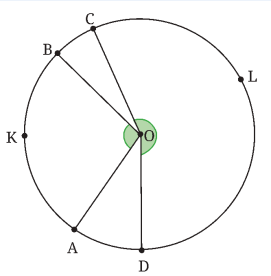


Fig. 5.19

ANSWER

Measuring from the figure (the angle is swept from one radius to the other along the arc):

Arc AKB (via K): $\angle AOB \approx 99° < 180°$

Arc CLD (via L): the angle swept from OC to OD through L $\approx 204° > 180°$

Therefore, arc AKB (≈ 99°) is a minor arc and arc CLD (≈ 204°) is a major arc. (Measured values are approximate.)

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ACTIVITY

Activity (Section 5.7.1): Draw a circle and a chord AB. Fix an arc AKB formed by AB and a point K between A, B on the circle. Measure the angle subtended at the centre by arc AKB. Take three points P, Q, R on the circle outside arc AKB. Measure the angles subtended by arc AKB at points P, Q, R. What do you notice? Repeat this activity for a different arc AKB.

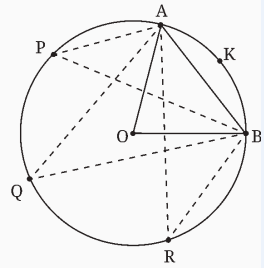


Fig. 5.20

ANSWER

Measuring Fig. 5.20, ∠AOB ≈ 80° and ∠APB ≈ ∠AQB ≈ ∠ARB ≈ 40°.

∠APB = ∠AQB = ∠ARB

Each of them = ½ × ∠AOB

Therefore, the angles subtended by an arc at points P, Q, R on the circle outside the arc are all equal, and each is half the angle subtended by the arc at the centre. (Theorem 9)

IN-TEXT 7

Exercise: A cyclic quadrilateral has angles measuring ∠A = 80°, ∠B = 110°, ∠C = 100°, and ∠D = 70°. Can such a quadrilateral be drawn? Explain why or why not.

ANSWER

Sum of angles = 80° + 110° + 100° + 70° = 360° ✓

∠A + ∠C = 80° + 100° = 180°

∠B + ∠D = 110° + 70° = 180°

In a cyclic quadrilateral the sum of each pair of opposite angles is 180° (Theorem 11), and the converse also holds (Theorem 12): a quadrilateral whose opposite angles add up to 180° is cyclic.

Therefore, yes, such a cyclic quadrilateral can be drawn.

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IN-TEXT 8

Theorem 12: We deal only with the first case and leave the other to the reader. (Case: C lies inside the circle through A, D, B.)

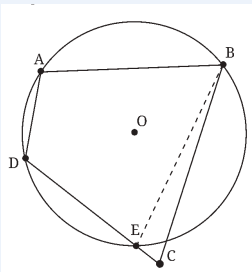


Fig. 5.29

ANSWER

Let the circle through A, D, B not pass through C, and let C lie inside it. Extend DC to meet the circle at E (see Fig. 5.29 for the labelling). Then ABED is a cyclic 4-gon.

$\angle BAD + \angle BED = 180°$ (Theorem 11)

$\angle BAD + \angle BCD = 180°$ (given)

$\Rightarrow \angle BCD = \angle BED$

But C lies between D and E, so $\angle BCD$ is an exterior angle of $\triangle BCE$. Hence $\angle BCD > \angle BEC = \angle BED$, which contradicts $\angle BCD = \angle BED$.

Therefore, C cannot lie inside the circle either, so C lies on the circle and A, B, C, D are concyclic.

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5.1

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Exercise Set 5.1

Solutions · 6 questions

QUESTION 1

Draw ΔABC with AB = 5 cm, ∠A = 70° and ∠B = 60°. Draw the circumcircle of ΔABC. Is the centre inside or outside the triangle?

ANSWER

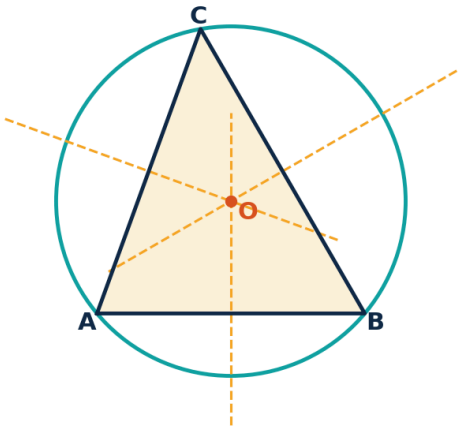
1 Construction

Step 1: Draw AB = 5 cm.

Step 2: At A draw ∠A = 70° and at B draw ∠B = 60°; the arms meet at C. Then ∠C = 180° − 70° − 60° = 50°.

Step 3: Draw the perpendicular bisectors of AB and AC. They meet at O, the circumcentre.

Step 4: With centre O and radius OA, draw the circle; it passes through A, B and C.



Circumcircle of △ABC

2 Position of the centre

All three angles (70°, 60°, 50°) are less than 90°, so the triangle is acute-angled.

Measurements: AC ≈ 5.65 cm, circumradius OA = OB = OC ≈ 3.26 cm

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Therefore, the triangle is acute-angled, so the circumcentre lies inside the triangle.

QUESTION 2

Draw ΔABC with AB = 5 cm, ∠A = 100°, AC = 4 cm. Draw the circumcircle of ΔABC. Is the centre inside or outside the triangle?

ANSWER

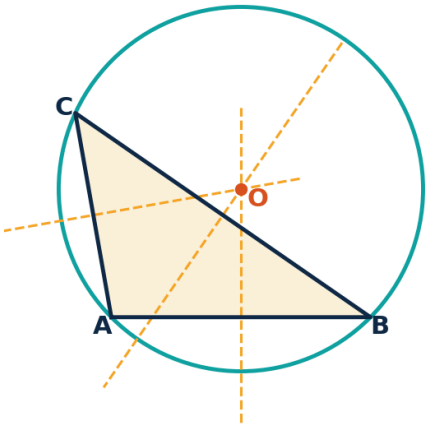
1 Construction

Step 1: Draw AB = 5 cm.

Step 2: At A draw ∠BAC = 100° and mark C on the arm with AC = 4 cm. Join BC.

Step 3: Draw the perpendicular bisectors of AB and AC; they meet at O.

Step 4: With centre O and radius OA, draw the circle through A, B, C.



Circumcircle of △ABC

2 Position of the centre

∠A = 100° is greater than 90°, so the triangle is obtuse-angled.

Measurements: BC ≈ 6.92 cm, circumradius ≈ 3.52 cm

Therefore, the triangle is obtuse-angled, so the circumcentre lies outside the triangle.

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QUESTION 3

Draw ΔABC, with AB = 6 cm, BC = 7 cm and CA = 7 cm. Draw the circumcircle of ΔABC. Let the circumcentre be O. Measure OA, OB, OC.

ANSWER

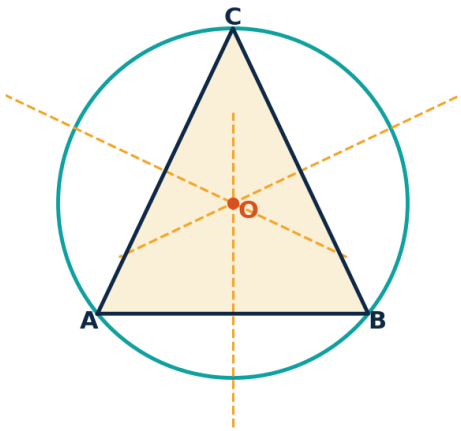
1 Construction

Step 1: Draw AB = 6 cm.

Step 2: With centre B and radius 7 cm, and with centre A and radius 7 cm, draw arcs meeting at C. Join AC and BC.

Step 3: Draw the perpendicular bisectors of AB and AC; they meet at O.

Step 4: With centre O and radius OA, draw the circumcircle.



Circumcircle of △ABC

2 Measurements

OA ≈ 3.87 cm, OB ≈ 3.87 cm, OC ≈ 3.87 cm

O is on the perpendicular bisectors of AB and AC, so OA = OB and OA = OC.

Therefore, OA = OB = OC ≈ 3.87 cm (each is the radius of the circumcircle).

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QUESTION 4

What is the least possible radius of a circle through two points A and B?

ANSWER

The centre of a circle through A and B lies on the perpendicular bisector of AB, at distance $x \geq 0$ from the midpoint M of AB. Its radius is

$$r = \sqrt{x^2 + (AB/2)^2} \geq AB/2$$

Equality holds when $x = 0$, i.e., the centre is M and AB is a diameter.

Therefore, the least possible radius is AB/2 (half the length of AB).

THINK, DRAW AND INFER 1

A, B and C are three collinear points. Can you find a point P such that PA = PB = PC ? What can you say about the perpendicular bisectors of AB and BC? Draw and check. Can you show that for three collinear points A, B and C, the perpendicular bisector of AB and BC are parallel? Is it possible for a circle to pass through collinear points? Can you draw a line that cuts a given circle in three distinct points?

ANSWER

1 A point P with PA = PB = PC?

PA = PB means P is on the perpendicular bisector of AB; PB = PC means P is on the perpendicular bisector of BC. These two lines have to meet at P.

No such point P exists (shown below).

2 The two perpendicular bisectors are parallel

Both perpendicular bisectors are perpendicular to the line ABC. Two lines perpendicular to the same line are parallel (co-interior angles 90° + 90° = 180°). They are distinct, since the midpoints of AB and BC are different points. So they never meet.

3 Circle through collinear points?

The centre of such a circle would be equidistant from A, B and C, i.e., a point P as above. No such point exists, so no circle passes through three collinear points.

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4 **A line cutting a circle in three points?**

If a line met a circle in three distinct points, these three collinear points would lie on a circle, which is impossible.

Answers: no point P; the perpendicular bisectors are parallel; no circle through three collinear points; no line cuts a circle in three distinct points (at most two).

THINK, DRAW AND INFER 2

The circumcircle of a given ΔABC is drawn. Can there be other triangles congruent to ΔABC that share the same circumcircle?

ANSWER

Yes. Take the circumcircle with centre O and rotate △ABC about O through any angle (or reflect it in a diameter). The rotated triangle A′B′C′ is congruent to △ABC, since a rotation preserves lengths and angles, and its vertices are on the same circle because rotation about O keeps every point at the same distance from O.

Therefore, yes: there are infinitely many triangles congruent to △ABC with the same circumcircle (the rotations of △ABC about O).

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5.2

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Exercise Set 5.2

Solutions · 2 questions

QUESTION 1

Show that the triangle formed by a chord and the centre of the circle is isosceles.

ANSWER

Let AB be a chord of a circle with centre O. Join OA and OB.

OA = OB (both are radii of the circle)

Therefore, △OAB has two equal sides, so it is isosceles (with base AB and ∠OAB = ∠OBA).

QUESTION 2

Show that if two such isosceles triangles (occurring in the previous question) have equal base length, they are congruent to each other.

ANSWER

Let AB and DE be two chords of the same circle (centre O) with AB = DE. Consider △OAB and △ODE.

OA = OD (radii)

OB = OE (radii)

AB = DE (given)

△OAB ≅ △ODE (SSS)

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Therefore, the two isosceles triangles with equal base length are congruent. (If the triangles are in circles of equal radius, the same SSS argument applies.)

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5.3

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Exercise Set 5.3

Solutions · 3 questions

QUESTION 1

Can you explain why the converse to Theorem 4 is true, i.e., why does the perpendicular from the centre of a circle to a chord of the circle bisect the chord?
(Hint: Use Fig. 5.12. You are told that ∠CMA = ∠CMB = 90°. You need to show that AM = BM.)

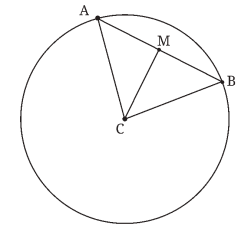


Fig. 5.12

ANSWER

Given: C is the centre, AB a chord and CM ⊥ AB with M on AB. To show: AM = BM.

In △CMA and △CMB:

CA = CB (radii)

∠CMA = ∠CMB = 90°

CM = CM (common)

△CMA ≅ △CMB (RHS)

⇒ AM = BM

Therefore, the perpendicular from the centre to a chord bisects the chord.

QUESTION 2

An isosceles triangle ABC is inscribed in a circle, with AB = AC. Show that the altitude from A to BC passes through the centre of the circle.

ANSWER

Let AD be the altitude from A to BC (D on BC). In △ABD and △ACD:

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AB = AC (given)

∠ADB = ∠ADC = 90°

AD = AD (common)

△ABD ≅ △ACD (RHS) ⇒ BD = DC

So AD is perpendicular to BC and passes through the midpoint D of BC: AD lies on the perpendicular bisector of the chord BC. The centre of the circle is equidistant from B and C, so it lies on the perpendicular bisector of BC (the line AD).

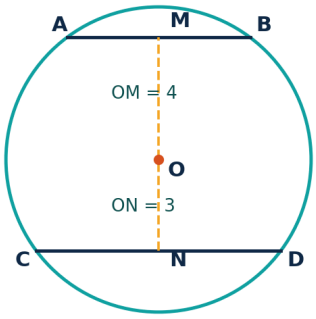
Therefore, the altitude from A to BC passes through the centre of the circle.

QUESTION 3

Two parallel chords of lengths 6 cm and 8 cm are on opposite sides of the centre of a circle. If the radius of the circle is 5 cm, find the distance between the midpoints of the chords.

ANSWER

Let AB = 6 cm and CD = 8 cm, with midpoints M and N and centre O. The line from O perpendicular to AB is also perpendicular to CD (the chords are parallel), so M, O, N are on one line, with O between M and N.



Parallel chords on opposite sides of the centre

$OM^2 = OA^2 - AM^2 = 5^2 - 3^2 = 16 \Rightarrow OM = 4$ cm

$ON^2 = OC^2 - CN^2 = 5^2 - 4^2 = 9 \Rightarrow ON = 3$ cm

$MN = OM + ON = 4 + 3 = 7$ cm

Therefore, the distance between the midpoints of the chords is 7 cm.

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5.4

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Exercise Set 5.4

Solutions · 3 questions

QUESTION 1

Use the Baudhāyana–Pythagoras theorem to show why Theorem 6 must be true.

ANSWER

Theorem 6: Chords of the same length are at the same distance from the centre. Let AB = FG be chords of a circle with centre C, and let E and H be their midpoints. Then CE ⊥ AB and CH ⊥ FG (Theorem 4).

$AE = AB/2 = FG/2 = FH$

$CE^2 = CA^2 - AE^2$ (right-angled △CEA)

$CH^2 = CF^2 - FH^2$ (right-angled △CHF)

$CA = CF$ = radius and $AE = FH \Rightarrow CE^2 = CH^2 \Rightarrow CE = CH$

Therefore, equal chords are at equal distance from the centre (Theorem 6).

QUESTION 2

Consider Fig. 5.15. If CE is perpendicular to AB, CH is perpendicular to GH, and CE = CH, show that AB = GF.

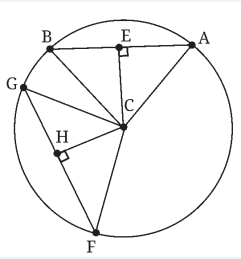


Fig. 5.15

ANSWER

(The perpendicular from C meets GF at H, i.e., CH ⊥ GF.) Join CA and CF; both are radii.

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In right-angled △CEA and △CHF:

∠CEA = ∠CHF = 90°

CA = CF (radii)

CE = CH (given)

△CEA ≅ △CHF (RHS) ⇒ AE = FH

The perpendicular from the centre bisects the chord (Theorem 5), so AB = 2AE and GF = 2FH.

AB = 2AE = 2FH = GF

Therefore, AB = GF: chords equidistant from the centre are equal (Theorem 7).

QUESTION 3

Solve the previous question using the Baudhāyana–Pythagoras theorem.

ANSWER

$AE^2 = CA^2 - CE^2$ (right-angled △CEA)

$FH^2 = CF^2 - CH^2$ (right-angled △CHF)

CA = CF (radii) and CE = CH (given) ⇒ $AE^2 = FH^2$ ⇒ AE = FH

AB = 2AE = 2FH = GF (the perpendicular from the centre bisects the chord)

Therefore, AB = GF.

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5.5

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 5

Exercise Set 5.5

Solutions · 3 questions

QUESTION 1

Find the length of the chord of a circle where the radius is 7 cm and perpendicular distance is 6 cm.

ANSWER

The perpendicular from the centre bisects the chord. Half the chord is h:

$h^2 = r^2 - d^2 = 7^2 - 6^2 = 49 - 36 = 13$

$h = \sqrt{13}$

Chord $= 2h = 2\sqrt{13} \approx 7.21$ cm

Therefore, the length of the chord is $2\sqrt{13}$ cm $\approx$ 7.21 cm.

QUESTION 2

Explain why the following statement is true: If the perpendicular distance of a chord from the centre is d and the radius is r, then the chord length is $2\sqrt{(r^2 - d^2)}$.

ANSWER

Let AB be the chord, O the centre and M the foot of the perpendicular from O to AB, so OM = d and OA = r. By Theorem 5, M is the midpoint of AB, so AB = 2AM.

In right-angled $\triangle OMA$: $AM^2 = OA^2 - OM^2 = r^2 - d^2$

$AM = \sqrt{(r^2 - d^2)}$

$AB = 2AM = 2\sqrt{(r^2 - d^2)}$

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Therefore, the chord length is $2\sqrt{(r^2 - d^2)}$.

QUESTION 3 *

In a circle, if the distance of chord AB from the centre is twice the distance of another chord CD from the centre, then can we conclude that CD = 2 AB? Give reasons for your answer.

ANSWER

Let the radius be r, and let the distance of CD from the centre be d, so the distance of AB is $2d$.

$CD = 2\sqrt{(r^2 - d^2)}$

$AB = 2\sqrt{(r^2 - 4d^2)}$

CD = 2AB would need $\sqrt{(r^2 - d^2)} = 2\sqrt{(r^2 - 4d^2)}$, i.e., $r^2 - d^2 = 4r^2 - 16d^2$, i.e., $r^2 = 5d^2$. This holds only for one special value of d, so it is not true in general.

Counter-example: $r = 5$, $d = 1$: $CD = 2\sqrt{24} \approx 9.80$ and $AB = 2\sqrt{21} \approx 9.17$, but $2AB \approx 18.33 \neq CD$.

(Special case $r^2 = 5d^2$: $r = \sqrt{5}$, $d = 1$ gives $AB = 2$, $CD = 4 = 2AB$.)

Therefore, no, we cannot conclude that CD = 2AB. We can only say that AB is shorter than CD (the chord farther from the centre is shorter).

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5.6

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 5

Exercise Set 5.6

Solutions · 3 questions

QUESTION 1

In a circle with centre O, the central angle AOB is 60°. If the radius of the circle is 12 cm, what is the length of the chord AB?

ANSWER

In △OAB, OA = OB = 12 cm, so ∠OAB = ∠OBA = (180° − 60°) ÷ 2 = 60°.

All three angles are 60°, so △OAB is equilateral.

AB = OA = OB = 12 cm

Therefore, the length of the chord AB is 12 cm.

QUESTION 2

Let A and B be two points on a circle with centre O.

(i) Are there points X, Y on the circle, on the same side of AB, such that ∠AXB is different from ∠AYB?

(ii) Is it true that if ∠AXB = ∠AYB, then X and Y lie on the same side of the circle?

(iii) If ∠AXB = ∠AYB, and X and Y do not lie on the circle, does the circle through A, B and X also pass through Y?

ANSWER

(i) **Different angles on the same side?**

By Theorem 9, the angle subtended by an arc at any point on the circle outside the arc is half the central angle. So all points on the same side of AB (on the same arc) give the same angle.

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Answer: No, ∠AXB = ∠AYB for all X, Y on the circle on the same side of AB.

(ii) Equal angles ⇒ same side?

If X and Y are on opposite sides of AB, then A, X, B, Y form a cyclic quadrilateral and ∠AXB + ∠AYB = 180°. The angles can be equal only when each is 90°, i.e., when AB is a diameter.

Answer: Not always. If AB is a diameter, points on opposite sides both give 90°. Otherwise (AB not a diameter), equal angles do force X and Y to lie on the same side.

(iii) Does the circle through A, B, X pass through Y?

If X and Y are on the same side of AB and ∠AXB = ∠AYB, then by Theorem 10 the points A, B, X, Y are concyclic: the circle through A, B, X passes through Y.

Answer: Yes, provided X and Y lie on the same side of AB. If they lie on opposite sides, it need not be so (for example, X and Y mirror images of each other in AB).

QUESTION 3

Find x in Fig. 5.26.

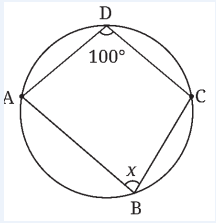


Fig. 5.26

ANSWER

A, B, C, D lie on a circle, so ABCD is a cyclic quadrilateral. In the figure ∠ADC = 100° and ∠ABC = x are opposite angles.

∠ADC + ∠ABC = 180° (opposite angles of a cyclic quadrilateral)

100° + x = 180°

x = 80°

Therefore, x = 80°.

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CLASS 9 MATHS · GANITA MANJARI · CHAPTER 5

End-of-Chapter Exercises

Solutions · 26 questions

QUESTION 1

In a circle, a chord is 5 cm away from the centre. If the radius of the circle is 13 cm, what is the length of the chord?

ANSWER

Half chord = $\sqrt{(13^2 - 5^2)} = \sqrt{(169 - 25)} = \sqrt{144} = 12$ cm

Chord = $2 \times 12 = 24$ cm

Therefore, the length of the chord is 24 cm.

QUESTION 2

An arc of a circle subtends an angle of 70° at the centre. What is the measure of the angle subtended by the arc at a point on the circle?

ANSWER

The angle subtended by an arc at the centre is double the angle subtended at any point on the circle outside the arc (Theorem 9).

Angle at the circle = ½ × 70° = 35°

Therefore, the angle subtended at a point on the circle is 35°.

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QUESTION 3

The diameter of a circle is 26 cm. A chord of length 24 cm is drawn in the circle. Find the distance from the centre of the circle to the chord.

ANSWER

Radius = 26 ÷ 2 = 13 cm; half chord = 24 ÷ 2 = 12 cm

Distance² = 13² − 12² = 169 − 144 = 25

Distance = 5 cm

Therefore, the distance from the centre to the chord is 5 cm.

QUESTION 4

A circle has a radius of 15 cm. A chord is drawn. The distance from the centre of the circle to the chord is 9 cm. What is the length of the chord?

ANSWER

Half chord = $\sqrt{(15^2 - 9^2)} = \sqrt{(225 - 81)} = \sqrt{144} = 12$ cm

Chord = 24 cm

Therefore, the length of the chord is 24 cm.

QUESTION 5

Prove that the perpendicular bisector of a chord passes through the centre of the circle.

ANSWER

Let AB be a chord of a circle with centre O. Then OA = OB, because both are radii.

Every point that is equidistant from A and B lies on the perpendicular bisector of AB. Since O is equidistant from A and B, O lies on the perpendicular bisector of AB.

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Therefore, the perpendicular bisector of a chord passes through the centre of the circle.

QUESTION 6

The diameter of a circle is AB. Point C is on the circumference. What is the measure of the ∠ACB? Explain your reasoning.

ANSWER

The arc AB not containing C is a semicircle, so it subtends a straight angle, 180°, at the centre. By Theorem 9, the angle at C is half of it.

$$\angle ACB = \tfrac{1}{2} \times 180^\circ = 90^\circ$$

Therefore, ∠ACB = 90° (the angle in a semicircle is a right angle).

QUESTION 7

ABCD is a cyclic quadrilateral inscribed in a circle. If ∠A measures 75°, what is the measure of ∠C? If ∠B measures 110°, what is the measure of ∠D?

ANSWER

$$\angle C = 180^\circ - \angle A = 180^\circ - 75^\circ = 105^\circ$$
$$\angle D = 180^\circ - \angle B = 180^\circ - 110^\circ = 70^\circ$$

Therefore, ∠C = 105° and ∠D = 70° (opposite angles of a cyclic quadrilateral are supplementary).

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QUESTION 8

Quadrilateral PQRS is inscribed in a circle. If ∠P = (2x + 10)° and ∠R = (3x − 20)°, find the value of x and the measures of ∠P and ∠R.

ANSWER

∠P and ∠R are opposite angles of a cyclic quadrilateral, so their sum is 180°.

$(2x + 10) + (3x − 20) = 180$

$5x − 10 = 180 \Rightarrow 5x = 190 \Rightarrow x = 38$

$\angle P = 2(38) + 10 = 86°$

$\angle R = 3(38) − 20 = 94°$ (check: 86° + 94° = 180°)

Therefore, x = 38, ∠P = 86° and ∠R = 94°.

QUESTION 9

The distance of a chord of length 16 cm from the centre of a circle is 6 cm. Find the radius of the circle.

ANSWER

Half chord = 16 ÷ 2 = 8 cm

$r^2 = 8^2 + 6^2 = 64 + 36 = 100$

r = 10 cm

Therefore, the radius of the circle is 10 cm.

QUESTION 10

A cyclic quadrilateral has sides 5, 5, 12, 12 units. Find its area.

ANSWER

The sides can be taken in two orders. In both cases the area is the same.

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1 **Sides in the order 5, 5, 12, 12 (a kite)**

Let AB = BC = 5 and CD = DA = 12. Then △ABD ≅ △CBD (SSS), so ∠A = ∠C. As ∠A + ∠C = 180° (cyclic), each is 90°. So BD is a diameter and

$BD^2 = 5^2 + 12^2 = 169 \Rightarrow BD = 13$

Area = 2 × (½ × 5 × 12) = 60 square units

2 **Sides in the order 5, 12, 5, 12**

Opposite sides are equal, so it is a parallelogram. A cyclic parallelogram is a rectangle (see Question 14).

Area = 5 × 12 = 60 square units

Therefore, the area of the cyclic quadrilateral is 60 square units.

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QUESTION 11 *

Consider a cyclic quadrilateral. Without drawing its circumcircle, how can we find out whether the centre of the circumcircle lies inside the quadrilateral or outside? What is the best way of finding out?

ANSWER

The centre lies inside the quadrilateral ABCD exactly when it is on the same side as the quadrilateral of every side. For the side AB, the centre is on the same side as C and D exactly when the arc AB cut off by this side is less than a semicircle, i.e., when the angle that AB subtends at C (equal to ∠ADB) is acute (less than 90°), because that angle is half the arc.

Find the angle that each side subtends at the other vertices:

∠ACB (= ∠ADB), ∠BDC (= ∠BAC), ∠CAD (= ∠CBD), ∠DBA (= ∠DCA)

If all four are acute (< 90°): the centre is inside.

If any one of them is obtuse (> 90°): the centre is outside.

If one is exactly 90°: that side is a diameter and the centre lies on that side.

Therefore, check the angles subtended by each side at the opposite vertices: the centre is inside the quadrilateral when all of them are acute, and outside as soon as one is obtuse.

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QUESTION 12 *

When two chords intersect, each of them is divided into two line segments. Show that if the intersecting chords are of equal length, then the line segments of one chord are equal to the corresponding line segments of the other chord.

ANSWER

Let AB and CD be chords of equal length intersecting at P, with centre O and midpoints M and N of AB and CD. Then OM ⊥ AB and ON ⊥ CD.

AB = CD ⇒ OM = ON (equal chords are equidistant from the centre)

In right-angled △OMP and △ONP: OP common, OM = ON ⇒ △OMP ≅ △ONP (RHS) ⇒ MP = NP

Let half the chord be h (AM = MB = CN = ND = h) and MP = NP = t. Then P divides each chord into segments of lengths $h + t$ and $h - t$.

AB: segments h + t and h − t

CD: segments h + t and h − t

Therefore, the segments of one chord are equal to the corresponding segments of the other chord.

QUESTION 13 *

Draw a circle in which a chord of 6 cm length stands at a distance of 3 cm from the centre.
(Hint: Is it a circumcircle of a suitable triangle?)

ANSWER

The chord AB = 6 cm has midpoint M, and the centre O is on the perpendicular bisector of AB with OM = 3 cm. So A, B and O form a triangle, and the required circle is the circle with centre O that passes through A and B.

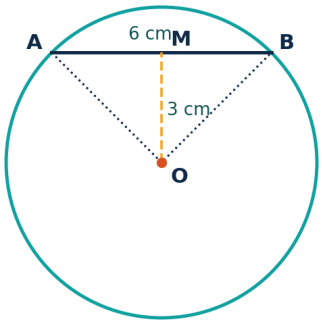
Step 1: Draw AB = 6 cm; mark its midpoint M.

Step 2: Draw the perpendicular bisector of AB and mark O on it with OM = 3 cm.

Step 3: Join OA; with centre O and radius OA draw the circle.

$OA^2 = OM^2 + AM^2 = 9 + 9 = 18 \Rightarrow OA = 3\sqrt{2} \approx 4.24$ cm

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Circle with chord AB = 6 cm at distance 3 cm from O

Therefore, the required circle has centre O and radius $3\sqrt{2} \approx 4.24$ cm (here $\angle AOB = 90°$, since OM = AM = MB).

QUESTION 14 *

Show that rectangle is the only parallelogram that can be inscribed in a circle.

ANSWER

Let ABCD be a parallelogram inscribed in a circle. In a parallelogram, opposite angles are equal: $\angle A = \angle C$. In a cyclic quadrilateral, opposite angles are supplementary: $\angle A + \angle C = 180°$.

$\angle A = \angle C$ and $\angle A + \angle C = 180° \Rightarrow \angle A = \angle C = 90°$

Similarly $\angle B = \angle D = 90°$

Therefore, all angles are right angles, so the parallelogram is a rectangle. (Conversely, every rectangle is cyclic.)

QUESTION 15 *

Show that if a rectangle is inscribed in a circle, then the point of intersection of its diagonals must lie at the centre of the circle.

ANSWER

Let ABCD be a rectangle inscribed in a circle. Then $\angle ABC = 90°$.

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The angle at B subtended by the arc ADC (not containing B) is 90°, so the angle that this arc subtends at the centre is $2 \times 90° = 180°$.

So AC is a straight line through the centre: AC is a diameter.

Similarly $\angle BAD = 90° \Rightarrow$ BD is a diameter.

Both diagonals are diameters, so both pass through the centre. Two diagonals meet in exactly one point.

Therefore, the point of intersection of the diagonals is the centre of the circle.

QUESTION 16 *

Consider all chords of a circle of a fixed length. What is the shape formed by the midpoints of all these chords?

ANSWER

Let the radius be *r* and the fixed chord length be *l*. For each such chord with midpoint M, OM ⊥ chord (Theorem 4).

$OM^2 = r^2 - (l/2)^2$ (same for every such chord)

$OM = \sqrt{(r^2 - l^2/4)}$ = constant

Therefore, the midpoints lie on a circle with the same centre O and radius $\sqrt{(r^2 - l^2/4)}$.

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QUESTION 17 *

In a circle with centre O, chords AB and AC are congruent. Explain why this statement is true: "The centre of the circle lies on the angle bisector of ∠BAC".

ANSWER

In △OAB and △OAC:

OA = OA (common)

OB = OC (radii)

AB = AC (given)

△OAB ≅ △OAC (SSS)

⇒ ∠OAB = ∠OAC

Therefore, AO bisects ∠BAC, so the centre O lies on the angle bisector of ∠BAC.

QUESTION 18

Two parallel chords of lengths 10 cm and 24 cm are on the same side of the centre of a circle. The distance between the chords is 7 cm. Find the radius of the circle.

ANSWER

Let the distances of the chords (length 10 and length 24) from the centre be a and b. The shorter chord is farther from the centre, and both are on the same side, so $a - b = 7$.

$a^2 = r^2 - 5^2$ and $b^2 = r^2 - 12^2$

$a^2 - b^2 = 144 - 25 = 119$

$(a - b)(a + b) = 119 \Rightarrow 7(a + b) = 119 \Rightarrow a + b = 17$

$a = 12, b = 5$

$r^2 = 12^2 + 5^2 = 169 \Rightarrow r = 13$

Therefore, the radius of the circle is 13 cm.

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QUESTION 19 *

A regular hexagon is inscribed in a circle of radius r. Find the length of the sides of the hexagon and the distance of each side from the centre of the circle.

ANSWER

Each side of the regular hexagon subtends 360° ÷ 6 = 60° at the centre O. In △OAB, OA = OB = r and ∠AOB = 60°, so the other two angles are also 60°: △OAB is equilateral.

Side AB = r

The distance of the side from the centre is the perpendicular OM, where M is the midpoint of AB:

$OM^2 = r^2 - (r/2)^2 = 3r^2/4$

$OM = (\sqrt{3}/2)\, r$

Therefore, each side of the hexagon is r, and each side is at distance (√3/2) r from the centre.

QUESTION 20

A quadrilateral MNOP is inscribed in a circle. If MN is a diameter, what can you say about ∠MOP and ∠MNP? Explain your reasoning.

ANSWER

The quadrilateral has sides MN, NO, OP and PM. Both ∠MOP and ∠MNP are subtended by the chord MP at the points O and N, which are on the same side of MP.

By Theorem 9, angles subtended by the same arc at points of the circle outside the arc are equal:

∠MOP = ∠MNP

Moreover, since MN is a diameter, ∠MON = 90° and ∠MPN = 90° (angle in a semicircle).

Therefore, ∠MOP = ∠MNP (angles in the same segment on chord MP).

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QUESTION 21

Let ABCD be a cyclic quadrilateral. Explain why the exterior angle at any vertex is equal to the interior opposite angle (e.g., ∠CDE = ∠ABC, where E is a point on the extension of side CD).

ANSWER

∠ADC + ∠CDE = 180° (linear pair, since C, D, E are collinear)

∠ADC + ∠ABC = 180° (opposite angles of a cyclic quadrilateral)

⇒ ∠CDE = ∠ABC

Therefore, the exterior angle ∠CDE equals the interior opposite angle ∠ABC.

QUESTION 22 *

"There is no chord of a circle that is longer than its diameter." How do you justify this statement?

ANSWER

Let AB be any chord of a circle with centre O and radius r, at distance $d \geq 0$ from the centre.

$AB = 2\sqrt{(r^2 - d^2)} \leq 2\sqrt{(r^2)} = 2r$ (since $d^2 \geq 0$)

Equality holds only for $d = 0$, i.e., when AB passes through the centre.

(Alternatively, by the triangle inequality, AB ≤ OA + OB = 2r.)

Therefore, no chord is longer than the diameter 2r.

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QUESTION 23 *

Let A be any point within a given circle with centre O. Show that the shortest chord of the circle that passes through point A is the one that is perpendicular to OA.

ANSWER

Take any chord through A, at perpendicular distance d from O. Its length is $2\sqrt{(r^2 - d^2)}$, which is smallest when d is largest.

The perpendicular from O to the chord, with foot F, forms a right-angled triangle OFA, so d = OF ≤ OA.

OF = OA only when F = A, i.e., when OA is perpendicular to the chord.

Therefore, the distance d is largest (= OA) exactly for the chord perpendicular to OA, so that chord is the shortest chord through A.

QUESTION 24

How would you use the following figure to justify the statement that the angle in a semicircle is 90°?

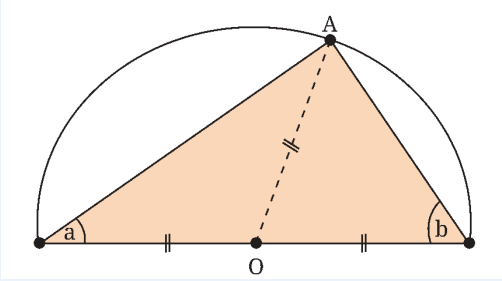


Fig. 5.30

ANSWER

In the figure, O is the centre of the semicircle, the diameter PQ is the base, and A is a point on the semicircle. The marked angles a and b are the base angles of △APQ. Since OA, OP and OQ are all radii, OA = OP = OQ.

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△OAP is isosceles (OA = OP): ∠OAP = ∠OPA = a

△OAQ is isosceles (OA = OQ): ∠OAQ = ∠OQA = b

Angle sum of the large triangle: a + b + (a + b) = 180°

2(a + b) = 180° ⇒ a + b = 90°

∠PAQ = ∠OAP + ∠OAQ = a + b = 90°

Therefore, the angle in a semicircle is 90°.

QUESTION 25 *

In a circle, two chords CC′ and DD′ are drawn perpendicular to a diameter AB. Prove that the segment MM′ joining the midpoints of the chords CD and C′D′ is perpendicular to AB.

ANSWER

Take the diameter AB along the x-axis with the centre O at the origin. A diameter perpendicular to a chord bisects it, so C and C′ are mirror images in AB, and so are D and D′.

C = (p, q), C′ = (p, −q); D = (s, t), D′ = (s, −t)

M = midpoint of CD = ((p + s)/2, (q + t)/2)

M′ = midpoint of C′D′ = ((p + s)/2, −(q + t)/2)

M and M′ have the same x-coordinate, so MM′ is a vertical line, i.e., perpendicular to AB. (In words: reflection in AB maps C to C′ and D to D′, so it maps M to M′; a point and its mirror image are joined by a line perpendicular to the mirror.)

Therefore, MM′ is perpendicular to AB.

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QUESTION 26 *

How would you use the following figure to justify the statement that the sum of the opposite angles of a cyclic quadrilateral is 180°?

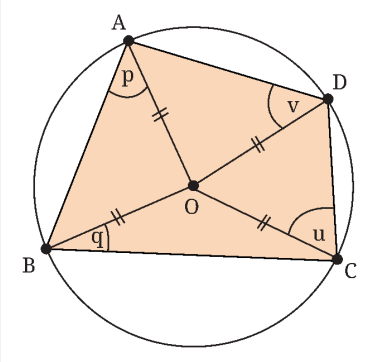


Fig. 5.31

ANSWER

In Fig. 5.31, O is the centre, so OA = OB = OC = OD (radii). Joining O to the four vertices gives four isosceles triangles OAB, OBC, OCD and ODA. In each of them the two base angles are equal. Let

$\angle OAB = \angle OBA = \alpha$

$\angle OBC = \angle OCB = \beta$

$\angle OCD = \angle ODC = \gamma$

$\angle ODA = \angle OAD = \delta$

(these are the equal angles marked p, q, u, v in the figure)

The angles of the quadrilateral ABCD add up to 360°, and each angle of ABCD is the sum of two of these base angles:

$\angle A = \alpha + \delta$, $\angle B = \alpha + \beta$, $\angle C = \beta + \gamma$, $\angle D = \gamma + \delta$

Sum of angles $= 2(\alpha + \beta + \gamma + \delta) = 360° \Rightarrow \alpha + \beta + \gamma + \delta = 180°$

$\angle A + \angle C = (\alpha + \delta) + (\beta + \gamma) = 180°$

$\angle B + \angle D = (\alpha + \beta) + (\gamma + \delta) = 180°$

Therefore, the sum of the opposite angles of a cyclic quadrilateral is 180°.

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