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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 4

Exploring Algebraic Identities

Complete Exercise Solutions

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4.1

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Exercise Set 4.1

Solutions · 2 questions

QUESTION 1

Using the identity $(a + b)^2 = a^2 + 2ab + b^2$, expand the following:

(i) $(7x + 4y)^2$

(ii) $((7/5)x + (3/2)y)^2$

(iii) $(2.5p + 1.5q)^2$

(iv) $((3/4)s + 8t)^2$

(v) $(x + 1/(2y))^2$

(vi) $(1/x + 1/y)^2$

ANSWER

In each part find *a* and *b* and use $(a + b)^2 = a^2 + 2ab + b^2$.

(i) $(7x + 4y)^2$

$a = 7x$, $b = 4y$

$(7x)^2 + 2(7x)(4y) + (4y)^2 = 49x^2 + 56xy + 16y^2$

(ii) $((7/5)x + (3/2)y)^2$

$a = (7/5)x$, $b = (3/2)y$

$a^2 = 49x^2/25$, $2ab = 2 \times (7/5) \times (3/2)\, xy = 21xy/5$, $b^2 = 9y^2/4$

$(7x/5 + 3y/2)^2 = 49x^2/25 + 21xy/5 + 9y^2/4$

(iii) $(2.5p + 1.5q)^2$

$a = 2.5p$, $b = 1.5q$

$a^2 = 6.25p^2$, $2ab = 2 \times 2.5 \times 1.5\, pq = 7.5pq$, $b^2 = 2.25q^2$

$(2.5p + 1.5q)^2 = 6.25p^2 + 7.5pq + 2.25q^2$

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(iv) **$((3/4)s + 8t)^2$**

$a = 3s/4$, $b = 8t$

$a^2 = 9s^2/16$, $2ab = 2 \times (3/4) \times 8\ st = 12st$, $b^2 = 64t^2$

$(3s/4 + 8t)^2 = 9s^2/16 + 12st + 64t^2$

(v) **$(x + 1/(2y))^2$**

$a = x$, $b = 1/(2y)$

$a^2 = x^2$, $2ab = 2 \times x \times 1/(2y) = x/y$, $b^2 = 1/(4y^2)$

$(x + 1/(2y))^2 = x^2 + x/y + 1/(4y^2)$

(vi) **$(1/x + 1/y)^2$**

$a = 1/x$, $b = 1/y$

$(1/x + 1/y)^2 = 1/x^2 + 2/(xy) + 1/y^2$

The expansions are as above in each part.

QUESTION 2

Using the same identity, find the values of the following:

(i) $(64)^2$

(ii) $(105)^2$

(iii) $(205)^2$

ANSWER

(i) **64^2**

$64^2 = (60 + 4)^2 = 60^2 + 2 \times 60 \times 4 + 4^2 = 3600 + 480 + 16 = 4096$

(ii) **105^2**

$105^2 = (100 + 5)^2 = 10000 + 2 \times 100 \times 5 + 25 = 10000 + 1000 + 25 = 11025$

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(iii) **205^2**

$205^2 = (200 + 5)^2 = 40000 + 2 \times 200 \times 5 + 25 = 40000 + 2000 + 25 = 42025$

(i) 4096 (ii) 11025 (iii) 42025

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4.2

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Exercise Set 4.2

Solutions · 2 questions

QUESTION 1 *

Factor completely:

(i) $9x^2 + 24xy + 16y^2$

(ii) $4s^2 + 20st + 25t^2$

(iii) $49x^2 + 28xy + 4y^2$

(iv) $64p^2 + (32/3)pq + (4/9)q^2$

(v) $3a^2 + 4ab + (4/3)b^2$

(vi) $(9/5)s^2 + 6sv + 5v^2$

(Hint: 2 was taken out as a common factor in Example 7. Is it possible to do something similar in Exercises (v) and (vi) above?**)**

ANSWER

Write each expression in the form $a^2 + 2ab + b^2$ and use $a^2 + 2ab + b^2 = (a + b)^2$.

(i) $9x^2 + 24xy + 16y^2$

$9x^2 = (3x)^2$, $16y^2 = (4y)^2$, $2(3x)(4y) = 24xy$

$9x^2 + 24xy + 16y^2 = (3x + 4y)^2$

(ii) $4s^2 + 20st + 25t^2$

$(2s)^2 + 2(2s)(5t) + (5t)^2 = (2s + 5t)^2$

(iii) $49x^2 + 28xy + 4y^2$

$(7x)^2 + 2(7x)(2y) + (2y)^2 = (7x + 2y)^2$

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(iv) **$64p^2 + (32/3)pq + (4/9)q^2$**

$64p^2 = (8p)^2$, $4q^2/9 = (2q/3)^2$, $2(8p)(2q/3) = 32pq/3$
$64p^2 + (32/3)pq + (4/9)q^2 = (8p + 2q/3)^2$

(v) **$3a^2 + 4ab + (4/3)b^2$** COMMON FACTOR 3

$3a^2$ is not the square of a simple term, so take 3 out as a common factor (as 2 was taken out in Example 7):

$3a^2 + 4ab + (4/3)b^2 = 3[a^2 + (4/3)ab + (4/9)b^2]$
$= 3[a^2 + 2(a)(2b/3) + (2b/3)^2] = 3(a + 2b/3)^2$

(vi) **$(9/5)s^2 + 6sv + 5v^2$** COMMON FACTOR 1/5

Take out 1/5 as a common factor:

$(9/5)s^2 + 6sv + 5v^2 = (1/5)(9s^2 + 30sv + 25v^2)$
$= (1/5)[(3s)^2 + 2(3s)(5v) + (5v)^2] = (1/5)(3s + 5v)^2$

(i) $(3x + 4y)^2$ (ii) $(2s + 5t)^2$ (iii) $(7x + 2y)^2$ (iv) $(8p + 2q/3)^2$ (v) $3(a + 2b/3)^2$ (vi) $(1/5)(3s + 5v)^2$

QUESTION 2

Find the values of the following using the identity $(a - b)^2 = a^2 - 2ab + b^2$.

(i) $(79)^2$

(ii) $(193)^2$

(iii) $(299)^2$

ANSWER

(i) **79^2**

$79^2 = (80 - 1)^2 = 6400 - 2 \times 80 \times 1 + 1 = 6400 - 160 + 1 = 6241$

(ii) **193^2**

$193^2 = (200 - 7)^2 = 40000 - 2 \times 200 \times 7 + 49 = 40000 - 2800 + 49 = 37249$

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(iii) 299^2

$299^2 = (300 - 1)^2 = 90000 - 600 + 1 = 89401$

(i) 6241 (ii) 37249 (iii) 89401

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4.3

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Exercise Set 4.3

Solutions · 4 questions

QUESTION 1

Find the following squares using one of the above identities. Determine which of these identities will make these calculations easier.

(i) 117^2 (ii) 78^2 (iii) 198^2

(iv) 214^2 (v) 1104^2 (vi) 1120^2

ANSWER

The three identities are $(a + b)^2 = a^2 + 2ab + b^2$, $(a – b)^2 = a^2 – 2ab + b^2$ and $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$. The easiest choice is to write the number as a sum or difference of a round number and a small number.

(i) 117^2 $(a - b)^2$

$117^2 = (120 – 3)^2 = 14400 – 720 + 9 = 13689$

(check with $(a + b + c)^2$: $(100 + 10 + 7)^2 = 10000 + 100 + 49 + 2000 + 1400 + 140 = 13689$)

(ii) 78^2 $(a - b)^2$

$78^2 = (80 – 2)^2 = 6400 – 320 + 4 = 6084$

(iii) 198^2 $(a - b)^2$

$198^2 = (200 – 2)^2 = 40000 – 800 + 4 = 39204$

(iv) 214^2 $(a + b)^2$

$214^2 = (200 + 14)^2 = 40000 + 5600 + 196 = 45796$

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(v) **1104^2** $(a + b)^2$

$1104^2 = (1100 + 4)^2 = 1210000 + 8800 + 16 = 1218816$

(vi) **1120^2** $(a + b)^2$

$1120^2 = (1100 + 20)^2 = 1210000 + 44000 + 400 = 1254400$

(i) 13689 (ii) 6084 (iii) $198^2 = 39204$ (iv) 45796 (v) 1218816 (vi) 1254400. Writing the number as (round number ± small number) and using $(a + b)^2$ or $(a – b)^2$ is easier than using the three-term identity.

QUESTION 2

Factor using suitable identities:

(i) $16y^2 – 24y + 9$

(ii) $(9/4)s^2 + 6st + 4t^2$

(iii) $m^2/9 + mk/3 + k^2/4 + 3nk + 2mn + 9n^2$

(iv) $p^2/16 – 2 + 16/p^2$

(v) $9a^2 + 4b^2 + c^2 – 12ab + 6ac – 4bc$

ANSWER

(i) **$16y^2 – 24y + 9$**

$(4y)^2 – 2(4y)(3) + 3^2 = (4y – 3)^2$

(ii) **$(9/4)s^2 + 6st + 4t^2$**

$(3s/2)^2 + 2(3s/2)(2t) + (2t)^2 = (3s/2 + 2t)^2$

(iii) **$m^2/9 + mk/3 + k^2/4 + 3nk + 2mn + 9n^2$**

Take $a = m/3$, $b = k/2$, $c = 3n$ and use $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$. Then $a^2 = m^2/9$, $b^2 = k^2/4$, $c^2 = 9n^2$, $2ab = mk/3$, $2bc = 3nk$, $2ca = 2mn$, which are exactly the terms.

$m^2/9 + mk/3 + k^2/4 + 3nk + 2mn + 9n^2 = (m/3 + k/2 + 3n)^2$

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(iv) **$p^2/16 - 2 + 16/p^2$**

$(p/4)^2 - 2(p/4)(4/p) + (4/p)^2 = (p/4 - 4/p)^2$
(the middle term: $2 \times (p/4) \times (4/p) = 2$)

(v) **$9a^2 + 4b^2 + c^2 - 12ab + 6ac - 4bc$**

Take the three terms as 3a, –2b and c:

$(3a)^2 + (-2b)^2 + c^2 + 2(3a)(-2b) + 2(3a)(c) + 2(-2b)(c)$
$= (3a - 2b + c)^2$

(i) $(4y - 3)^2$ (ii) $(3s/2 + 2t)^2$ (iii) $(m/3 + k/2 + 3n)^2$ (iv) $(p/4 - 4/p)^2$ (v) $(3a - 2b + c)^2$

QUESTION 3

Expand the following using the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$:

(i) $(p + 3q + 7r)^2$

(ii) $(3x - 2y + 4z)^2$

ANSWER

(i) **$(p + 3q + 7r)^2$**

$a = p, b = 3q, c = 7r$
$a^2 + b^2 + c^2 = p^2 + 9q^2 + 49r^2$
$2ab = 6pq, 2bc = 42qr, 2ca = 14pr$
$(p + 3q + 7r)^2 = p^2 + 9q^2 + 49r^2 + 6pq + 42qr + 14pr$

(ii) **$(3x - 2y + 4z)^2$**

$a = 3x, b = -2y, c = 4z$
$a^2 + b^2 + c^2 = 9x^2 + 4y^2 + 16z^2$
$2ab = -12xy, 2bc = -16yz, 2ca = 24zx$
$(3x - 2y + 4z)^2 = 9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24zx$

(i) $p^2 + 9q^2 + 49r^2 + 6pq + 42qr + 14pr$ (ii) $9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24zx$

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QUESTION 4

Is this an identity?

$(a + b - c)^2 + (a - b + c)^2 + (a - b - c)^2 = 2a^2 + 2b^2 + 2c^2$.

ANSWER

An identity must hold for all values of the variables, so we expand the left side.

$$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2ac - 2bc$$
$$(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$$
$$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$$
$$\text{Sum} = 3a^2 + 3b^2 + 3c^2 - 2ab - 2ac - 2bc$$

This is not $2a^2 + 2b^2 + 2c^2$. A quick check: take $a = b = c = 1$. The left side is $(1)^2 + (1)^2 + (-1)^2 = 3$, while the right side is $2 + 2 + 2 = 6$.

No, it is not an identity (it fails for $a = b = c = 1$). The left side equals $3a^2 + 3b^2 + 3c^2 - 2ab - 2ac - 2bc$. For comparison, with four squares we do get an identity: $(a + b + c)^2 + (a + b - c)^2 + (a - b + c)^2 + (a - b - c)^2 = 4a^2 + 4b^2 + 4c^2$.

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4.4

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Exercise Set 4.4

Solutions · 3 questions

QUESTION 1

Fill in the blanks to complete the following identities:

(i) $s^2 - 11s + 24 =$ (_____) (_____)

(ii) (_____) $(x + 1) = (3x^2 - 4x - 7)$

(iii) $10x^2 - 11x - 6 = (2x -$ ___) (___ + 2)

(iv) $6x^2 + 7x + 2 =$ (_____) (_____)

ANSWER

(i) $s^2 - 11s + 24$

Compare with $s^2 + (a + b)s + ab$: $a + b = -11$ and $ab = 24$, so $a = -3$, $b = -8$.

$$s^2 - 11s + 24 = s^2 - 3s - 8s + 24 = s(s - 3) - 8(s - 3) = (s - 3)(s - 8)$$

(ii) $3x^2 - 4x - 7$

The factor $x + 1$ divides $3x^2 - 4x - 7$; split $-4x$ as $-7x + 3x$:

$$3x^2 - 7x + 3x - 7 = x(3x - 7) + 1(3x - 7) = (3x - 7)(x + 1)$$

(iii) $10x^2 - 11x - 6$

The first factor is $2x -$ ___ and the second is ___ + 2 (a term in x, then + 2). The product of the x^2 terms is $10x^2$, so the second factor starts with $5x$; the product of the constants is -6, so the first constant is 3 (since $3 \times 2 = 6$):

$$(2x - 3)(5x + 2) = 10x^2 + 4x - 15x - 6 = 10x^2 - 11x - 6 \square$$

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(iv) **$6x^2 + 7x + 2$**

Split 7x so that the two parts have product 6 × 2 = 12: 7x = 3x + 4x.

$6x^2 + 3x + 4x + 2 = 3x(2x + 1) + 2(2x + 1) = (2x + 1)(3x + 2)$

(i) (s – 3)(s – 8) (ii) (3x – 7)(x + 1) (iii) (2x – 3)(5x + 2) (iv) (2x + 1)(3x + 2)

QUESTION 2

Select and use the identity that will help you to find the following products without multiplying directly:

(i) $(41)^2$ (ii) $(27)^2$ (iii) (23 × 17) (iv) $(135)^2$

(v) $(97)^2$ (vi) (18 × 29) (vii) (34 × 43) (viii) $(205)^2$

ANSWER

(i) **41^2** $(a + b)^2$

$41^2 = (40 + 1)^2 = 1600 + 80 + 1 = 1681$

(ii) **27^2** $(a - b)^2$

$27^2 = (30 - 3)^2 = 900 - 180 + 9 = 729$

(iii) **23 × 17** $(a + b)(a - b)$

$23 \times 17 = (20 + 3)(20 - 3) = 20^2 - 3^2 = 400 - 9 = 391$

(iv) **135^2** $a^2 = (a + b)(a - b) + b^2$

$135^2 = (135 + 5)(135 - 5) + 5^2 = 140 \times 130 + 25 = 18200 + 25 = 18225$
(or $(100 + 35)^2 = 10000 + 7000 + 1225 = 18225$)

(v) **97^2** $(a - b)^2$

$97^2 = (100 - 3)^2 = 10000 - 600 + 9 = 9409$

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(vi) **18 × 29** $(x + a)(x + b)$

$18 \times 29 = (20 - 2)(20 + 9) = 20^2 + (-2 + 9)(20) + (-2)(9) = 400 + 140 - 18 = 522$

(vii) **34 × 43** $(x + a)(x + b)$

$34 \times 43 = (40 - 6)(40 + 3) = 40^2 + (-6 + 3)(40) + (-6)(3) = 1600 - 120 - 18 = 1462$

(viii) **205^2** $(a + b)^2$

$205^2 = (200 + 5)^2 = 40000 + 2000 + 25 = 42025$

(i) 1681 (ii) 729 (iii) 391 (iv) 18225 (v) 9409 (vi) 522 (vii) 1462 (viii) 42025

QUESTION 3

Factor the following:

(i) $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$

(ii) $16s^2 + 25t^2 - 40st$

(iii) $r^2 - r - 42$

(iv) $49g^2 + 14gh + h^2$

(v) $64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$

ANSWER

(i) **$9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$**

Take 3a, –b, 2c: $2(3a)(-b) = -6ab$, $2(3a)(2c) = 12ac$, $2(-b)(2c) = -4bc$.

$= (3a - b + 2c)^2$

(ii) **$16s^2 + 25t^2 - 40st$**

$(4s)^2 - 2(4s)(5t) + (5t)^2 = (4s - 5t)^2$

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(iii) **$r^2 - r - 42$**

We need a + b = –1 and ab = –42: a = –7, b = 6.

$$r^2 - 7r + 6r - 42 = r(r - 7) + 6(r - 7) = (r - 7)(r + 6)$$

(iv) **$49g^2 + 14gh + h^2$**

$$(7g)^2 + 2(7g)(h) + h^2 = (7g + h)^2$$

(v) **$64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$**

Take 8u, –11v, –2w: 2(8u)(–11v) = –176uv, 2(8u)(–2w) = –32uw, 2(–11v)(–2w) = 44vw.

$$= (8u - 11v - 2w)^2$$

(i) $(3a - b + 2c)^2$ (ii) $(4s - 5t)^2$ (iii) $(r - 7)(r + 6)$ (iv) $(7g + h)^2$ (v) $(8u - 11v - 2w)^2$

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4.5

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 4

Exercise Set 4.5

Solutions · 1 question

QUESTION 1

Simplify the following rational expressions assuming that the expressions in the denominators are not equal to zero:

(i) $(3p^2 - 3pq - 18q^2)/(p^2 + 3pq - 10q^2)$

(ii) $(n^3 - 3n^2m + 3nm^2 - m^3)/(5m^2 - 10mn + 5n^2)$

(iii) $(w^3 - v^3 + x^3 + 3wvx)/(w^2 + v^2 + x^2 - 2wv - 2vx + 2wx)$

(iv) $(4y^2 - 20yz + 25z^2)/(25z^2 - 4y^2)$

(v) $(x^2 + x - 6)(x^2 - 7x + 12)/((x^2 - 6x + 8)(x^2 - 9))$

(vi) $(p^4 - 16)/(p^2 - 4p + 4)$

ANSWER

(i) **$(3p^2 - 3pq - 18q^2)/(p^2 + 3pq - 10q^2)$**

Numerator: $3p^2 - 3pq - 18q^2 = 3(p^2 - pq - 6q^2) = 3(p - 3q)(p + 2q)$ [$a + b = -1$, $ab = -6$: $a = -3$, $b = 2$]

Denominator: $p^2 + 3pq - 10q^2 = (p + 5q)(p - 2q)$ [$a + b = 3$, $ab = -10$: $a = 5$, $b = -2$]

The numerator and denominator have no common factor, so no cancellation is possible.

$= 3(p - 3q)(p + 2q) / ((p + 5q)(p - 2q))$

(ii) **$(n^3 - 3n^2m + 3nm^2 - m^3)/(5m^2 - 10mn + 5n^2)$**

Numerator $= (n - m)^3$ (identity for $(a - b)^3$)

Denominator $= 5(m^2 - 2mn + n^2) = 5(m - n)^2 = 5(n - m)^2$

Expression $= (n - m)^3 / (5(n - m)^2) = (n - m)/5$

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(iii) **$(w^3 - v^3 + x^3 + 3wvx)/(w^2 + v^2 + x^2 - 2wv - 2vx + 2wx)$**

Put $a = w$, $b = -v$, $c = x$. Then the numerator is $a^3 + b^3 + c^3 - 3abc$ (since $-3abc = -3(w)(-v)(x) = 3wvx$), which equals $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$:

Numerator = $(w - v + x)(w^2 + v^2 + x^2 + wv + vx - wx)$
Denominator = $(w - v + x)^2$ (the terms $-2wv$, $+2wx$, $-2vx$ are twice the cross-products of w, $-v$, x)

Expression = $(w^2 + v^2 + x^2 + wv + vx - wx) / (w - v + x)$

(iv) **$(4y^2 - 20yz + 25z^2)/(25z^2 - 4y^2)$**

Numerator = $(2y - 5z)^2 = (5z - 2y)^2$
Denominator = $(5z)^2 - (2y)^2 = (5z - 2y)(5z + 2y)$
Expression = $(5z - 2y)/(5z + 2y)$

(v) **$(x^2 + x - 6)(x^2 - 7x + 12)/((x^2 - 6x + 8)(x^2 - 9))$**

$x^2 + x - 6 = (x + 3)(x - 2)$, $x^2 - 7x + 12 = (x - 3)(x - 4)$
$x^2 - 6x + 8 = (x - 2)(x - 4)$, $x^2 - 9 = (x - 3)(x + 3)$
Expression = $(x + 3)(x - 2)(x - 3)(x - 4) / ((x - 2)(x - 4)(x - 3)(x + 3)) = 1$

(vi) **$(p^4 - 16)/(p^2 - 4p + 4)$**

$p^4 - 16 = (p^2)^2 - 4^2 = (p^2 - 4)(p^2 + 4) = (p - 2)(p + 2)(p^2 + 4)$
$p^2 - 4p + 4 = (p - 2)^2$
Expression = $(p + 2)(p^2 + 4)/(p - 2)$

(i) $3(p - 3q)(p + 2q)/((p + 5q)(p - 2q))$, no common factor (ii) $(n - m)/5$ (iii) $(w^2 + v^2 + x^2 + wv + vx - wx)/(w - v + x)$ (iv) $(5z - 2y)/(5z + 2y)$ (v) 1 (vi) $(p + 2)(p^2 + 4)/(p - 2)$

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EOC

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 4

End-of-Chapter Exercises

Solutions · 13 questions

QUESTION 1

Use suitable identities to find the following products:

(i) $(-3x + 4)^2$ (ii) $(2s + 7)(2s - 7)$

(iii) $(p^2 + 1/2)(p^2 - 1/2)$ (iv) $(2n + 7)(2n - 7)$

(v) $(s - 2t)(s^2 + 2st + 4t^2)$ (vi) $(1/(2r) - 4r)^2$

(vii) $(-3m + 4k - l)^2$ (viii) $(x - y/3)^3$ (ix) $((7/2)k - (2/3)m)^3$

ANSWER

(i) $(-3x + 4)^2$

$(-3x)^2 + 2(-3x)(4) + 4^2 = 9x^2 - 24x + 16$

(ii) $(2s + 7)(2s - 7)$

$(a + b)(a - b) = a^2 - b^2$: $(2s)^2 - 7^2 = 4s^2 - 49$

(iii) $(p^2 + 1/2)(p^2 - 1/2)$

$(p^2)^2 - (1/2)^2 = p^4 - 1/4$

(iv) $(2n + 7)(2n - 7)$

$(2n)^2 - 7^2 = 4n^2 - 49$

(v) $(s - 2t)(s^2 + 2st + 4t^2)$

$(x - y)(x^2 + xy + y^2) = x^3 - y^3$ with $x = s$, $y = 2t$:
$= s^3 - (2t)^3 = s^3 - 8t^3$

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(vi) **$(1/(2r) - 4r)^2$**

$(1/(2r))^2 - 2(1/(2r))(4r) + (4r)^2 = 1/(4r^2) - 4 + 16r^2$

(vii) **$(-3m + 4k - l)^2$**

$a = -3m$, $b = 4k$, $c = -l$

$a^2 + b^2 + c^2 = 9m^2 + 16k^2 + l^2$

$2ab = -24mk$, $2bc = -8kl$, $2ca = 6ml$

$= 9m^2 + 16k^2 + l^2 - 24mk - 8kl + 6ml$

(viii) **$(x - y/3)^3$**

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ with $a = x$, $b = y/3$:

$x^3 - 3x^2(y/3) + 3x(y/3)^2 - (y/3)^3 = x^3 - x^2y + xy^2/3 - y^3/27$

(ix) **$((7/2)k - (2/3)m)^3$**

$a = 7k/2$, $b = 2m/3$

$a^3 = 343k^3/8$

$3a^2b = 3 \times (49k^2/4) \times (2m/3) = 49k^2m/2$

$3ab^2 = 3 \times (7k/2) \times (4m^2/9) = 14km^2/3$

$b^3 = 8m^3/27$

$= 343k^3/8 - 49k^2m/2 + 14km^2/3 - 8m^3/27$

Each product is expanded as shown above.

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QUESTION 2

Find the values using suitable identities:

(i) 17 × 21 (ii) 104 × 96 (iii) 24 × 16 (iv) 147^3

(v) 199^3 (vi) 127^3 (vii) $(-107)^3$ (viii) $(-299)^3$

ANSWER

(i) 17 × 21

$17 \times 21 = (19 - 2)(19 + 2) = 19^2 - 2^2 = 361 - 4 = 357$

(ii) 104 × 96

$(100 + 4)(100 - 4) = 10000 - 16 = 9984$

(iii) 24 × 16

$(20 + 4)(20 - 4) = 400 - 16 = 384$

(iv) 147^3

$147^3 = (150 - 3)^3 = 150^3 - 3(150^2)(3) + 3(150)(3^2) - 3^3$
$= 3375000 - 202500 + 4050 - 27 = 3176523$

(v) 199^3

$(200 - 1)^3 = 8000000 - 3(40000)(1) + 3(200)(1) - 1 = 8000000 - 120000 + 600 - 1 = 7880599$

(vi) 127^3

$127^3 = (130 - 3)^3 = 2197000 - 3(16900)(3) + 3(130)(9) - 27$
$= 2197000 - 152100 + 3510 - 27 = 2048383$

(vii) $(-107)^3$

$(-107)^3 = -(100 + 7)^3 = -(1000000 + 3(10000)(7) + 3(100)(49) + 343)$
$= -(1000000 + 210000 + 14700 + 343) = -1225043$

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(viii) **$(-299)^3$**

$(-299)^3 = -(300 - 1)^3 = -(27000000 - 3(90000)(1) + 3(300)(1) - 1)$
$= -(27000000 - 270000 + 900 - 1) = -26730899$

(i) 357 (ii) 9984 (iii) 384 (iv) 3176523 (v) 7880599 (vi) 2048383 (vii) –1225043 (viii) –26730899

QUESTION 3

Factor the following algebraic expressions:

(i) $4y^2 + 1 + 1/(16y^2)$ (ii) $9m^2 - 1/(25n^2)$

(iii) $27b^3 - 1/(64b^3)$ (iv) $x^2 + 5x/6 + 1/6$

(v) $27u^3 - 1/125 - 27u^2/5 + 9u/25$ (vi) $64y^3 + (1/125)z^3$

(vii) $p^3 + 27q^3 + r^3 - 9pqr$ (viii) $9m^2 - 12m + 4$

(ix) $9x^3 - (8/3)y^3 + z^3/3 + 6xyz$

(x) $4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$

(xi) $27u^3 - 1/216 - 9u^2/2 + u/4$

ANSWER

(i) **$4y^2 + 1 + 1/(16y^2)$**

$(2y)^2 + 2(2y)(1/(4y)) + (1/(4y))^2 = (2y + 1/(4y))^2$

(ii) **$9m^2 - 1/(25n^2)$**

$(3m)^2 - (1/(5n))^2 = (3m - 1/(5n))(3m + 1/(5n))$

(iii) **$27b^3 - 1/(64b^3)$**

$(3b)^3 - (1/(4b))^3 = (3b - 1/(4b))(9b^2 + 3/4 + 1/(16b^2))$
$[x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ with $x = 3b$, $y = 1/(4b)$; $xy = 3/4]$

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(iv) **$x^2 + 5x/6 + 1/6$**

$a + b = 5/6$ and $ab = 1/6$: $a = 1/2$, $b = 1/3$.

$$x^2 + (1/2 + 1/3)x + (1/2)(1/3) = (x + 1/2)(x + 1/3)$$

(v) **$27u^3 - 1/125 - 27u^2/5 + 9u/25$**

$$(3u)^3 - 3(3u)^2(1/5) + 3(3u)(1/5)^2 - (1/5)^3 = (3u - 1/5)^3$$

(vi) **$64y^3 + (1/125)z^3$**

$$(4y)^3 + (z/5)^3 = (4y + z/5)(16y^2 - 4yz/5 + z^2/25)$$

$[x^3 + y^3 = (x + y)(x^2 - xy + y^2)]$

(vii) **$p^3 + 27q^3 + r^3 - 9pqr$**

With $a = p$, $b = 3q$, $c = r$: $a^3 + b^3 + c^3 - 3abc$, since $3abc = 9pqr$

$$= (p + 3q + r)(p^2 + 9q^2 + r^2 - 3pq - 3qr - pr)$$

(viii) **$9m^2 - 12m + 4$**

$$(3m)^2 - 2(3m)(2) + 2^2 = (3m - 2)^2$$

(ix) **$9x^3 - (8/3)y^3 + z^3/3 + 6xyz$**

Take out 1/3: the expression is $(1/3)(27x^3 - 8y^3 + z^3 + 18xyz) = (1/3)[(3x)^3 + (-2y)^3 + z^3 - 3(3x)(-2y)(z)]$.

$$= (1/3)(3x - 2y + z)(9x^2 + 4y^2 + z^2 + 6xy + 2yz - 3xz)$$

(x) **$4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$** **AS PRINTED: NOT A SQUARE**

The squares $4x^2$, $9y^2$, $36z^2$ suggest $2x$, $3y$, $6z$. For $(2x + 3y + 6z)^2$ the cross-terms would be $2(2x)(3y) = 12xy$, $2(2x)(6z) = 24xz$, $2(3y)(6z) = 36yz$. In the printed expression the coefficients of xy and xz are interchanged ($24xy$ and $12xz$), so as printed it is not a perfect square and cannot be factored using this identity. The intended expression is almost certainly $4x^2 + 9y^2 + 36z^2 + 12xy + 36yz + 24xz$.

Intended: $4x^2 + 9y^2 + 36z^2 + 12xy + 36yz + 24xz = (2x + 3y + 6z)^2$

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(xi) **$27u^3 - 1/216 - 9u^2/2 + u/4$**

$(3u)^3 - 3(3u)^2(1/6) + 3(3u)(1/6)^2 - (1/6)^3 = (3u - 1/6)^3$

(x): the printed expression has no such factorisation; the intended one gives $(2x + 3y + 6z)^2$. All other parts are factored above.

QUESTION 4

Simplify the following:

(i) $(4x^2 + 4x + 1)/(4x^2 - 1)$ (ii) $9(3a^3 - 24b^3)/(9a^2 - 36b^2)$ (iii) $(s^3 + 125t^3)/(s^2 - 2st - 35t^2)$

Note: Assume that the denominators are not equal to 0.

ANSWER

(i) **$(4x^2 + 4x + 1)/(4x^2 - 1)$**

Numerator = $(2x + 1)^2$, Denominator = $(2x - 1)(2x + 1)$

Expression = $(2x + 1)/(2x - 1)$

(ii) **$9(3a^3 - 24b^3)/(9a^2 - 36b^2)$**

$3a^3 - 24b^3 = 3(a^3 - 8b^3) = 3(a - 2b)(a^2 + 2ab + 4b^2)$

$9a^2 - 36b^2 = 9(a - 2b)(a + 2b)$

Expression = $9 \times 3(a - 2b)(a^2 + 2ab + 4b^2) / (9(a - 2b)(a + 2b)) = 3(a^2 + 2ab + 4b^2)/(a + 2b)$

(iii) **$(s^3 + 125t^3)/(s^2 - 2st - 35t^2)$**

$s^3 + 125t^3 = (s + 5t)(s^2 - 5st + 25t^2)$

$s^2 - 2st - 35t^2 = (s - 7t)(s + 5t)$

Expression = $(s^2 - 5st + 25t^2)/(s - 7t)$

(i) $(2x + 1)/(2x - 1)$ (ii) $3(a^2 + 2ab + 4b^2)/(a + 2b)$ (iii) $(s^2 - 5st + 25t^2)/(s - 7t)$

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QUESTION 5

Find possible expressions for the length and breadth of each of the following rectangles whose areas are given by the following expressions in square units.

(i) $25a^2 - 30ab + 9b^2$ (ii) $36s^2 - 49t^2$

ANSWER

(i) **$25a^2 - 30ab + 9b^2$**

$(5a)^2 - 2(5a)(3b) + (3b)^2 = (5a - 3b)^2$

So the length and breadth are both 5a – 3b units; the rectangle is a square.

(ii) **$36s^2 - 49t^2$**

$(6s)^2 - (7t)^2 = (6s - 7t)(6s + 7t)$

So the length is 6s + 7t units and the breadth is 6s – 7t units.

(i) 5a – 3b and 5a – 3b (ii) 6s + 7t and 6s – 7t (possible answers)

QUESTION 6

Find possible expressions for the length, breadth, and heights of each of the following cuboids whose volumes are given by the following expressions in cubic units.

(i) $6a^2 - 24b^2$ (ii) $3ps^2 - 15ps + 12p$

ANSWER

(i) **$6a^2 - 24b^2$**

$6a^2 - 24b^2 = 6(a^2 - 4b^2) = 6(a - 2b)(a + 2b)$

So possible dimensions are 6, (a – 2b) and (a + 2b) units.

(ii) **$3ps^2 - 15ps + 12p$**

$3ps^2 - 15ps + 12p = 3p(s^2 - 5s + 4) = 3p(s - 1)(s - 4)$

[$s^2 - 5s + 4$: $a + b = -5$, $ab = 4$: $a = -1$, $b = -4$]

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So possible dimensions are 3p, (s – 1) and (s – 4) units.

(i) 6, a – 2b, a + 2b (ii) 3p, s – 1, s – 4 (other groupings of the factors are also possible)

QUESTION 7

The village playground is shaped as a square of side 40 metres. A path of width *s* metres is created around the playground for people to walk. Find an expression for the area of the path in terms of *s*.

ANSWER

The path is built around (outside) the playground, so the playground and the path together form a larger square. The playground has side 40 m; the path adds s m on each side, so the larger square has side 40 + 2s.

Area of the larger square = $(40 + 2s)^2 = 1600 + 160s + 4s^2$

Area of the playground = $40^2 = 1600$

Area of the path = $1600 + 160s + 4s^2 - 1600 = 160s + 4s^2 = 4s(s + 40)$ square metres

The area of the path is $4s^2 + 160s = 4s(s + 40)$ square metres. (If the path were inside the playground, it would be $160s - 4s^2$.)

QUESTION 8

If a number plus its reciprocal equals 10/3, find the number.

ANSWER

Let the number be x ($x \neq 0$). Then $x + 1/x = 10/3$.

$3x^2 + 3 = 10x \Rightarrow 3x^2 - 10x + 3 = 0$

Split $-10x$ as $-9x - x$ (product $3 \times 3 = 9$): $3x^2 - 9x - x + 3 = 3x(x - 3) - 1(x - 3) = (3x - 1)(x - 3) = 0$

$x = 1/3$ or $x = 3$

Check: $3 + 1/3 = 10/3$ ✓ and $1/3 + 3 = 10/3$ ✓.

The number is 3 or 1/3.

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QUESTION 9

A rectangular pool has area $2x^2 + 7x + 3$ square *hastas*. If its width is $2x + 1$ *hastas*, find its length. *Hasta* was a unit used to measure length.

ANSWER

Area = length × width, so factorise $2x^2 + 7x + 3$. Split $7x$ as $6x + x$ (product $2 \times 3 = 6$):

$$2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1)$$

Since the width is $2x + 1$, the length is the other factor.

The length of the pool is $x + 3$ hastas.

QUESTION 10 *

If both $x - 2$ and $x - 1/2$ are factors of $px^2 + 5x + r$, show that $p = r$.

ANSWER

If $x - 2$ is a factor, then $x = 2$ makes the expression zero; if $x - 1/2$ is a factor, then $x = 1/2$ does.

$x = 2$: $p(2)^2 + 5(2) + r = 0 \Rightarrow 4p + r + 10 = 0$ ...(1)

$x = 1/2$: $p(1/2)^2 + 5(1/2) + r = 0 \Rightarrow p/4 + r + 5/2 = 0 \Rightarrow p + 4r + 10 = 0$ ...(2)

Subtract (2) from (1): $3p - 3r = 0 \Rightarrow p = r$

(Then $5p + 10 = 0$ gives $p = r = -2$; indeed $-2x^2 + 5x - 2 = -(2x - 1)(x - 2) = -2(x - 1/2)(x - 2)$.)

$p = r$.

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QUESTION 11 *

If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

ANSWER

Use $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$, and $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$.

$25 = (a + b + c)^2 = a^2 + b^2 + c^2 + 2(10) \Rightarrow a^2 + b^2 + c^2 = 5$

$a^2 + b^2 + c^2 - ab - bc - ca = 5 - 10 = -5$

$a^3 + b^3 + c^3 - 3abc = 5 \times (-5) = -25$

Hence $a^3 + b^3 + c^3 - 3abc = -25$.

QUESTION 12 *

By factoring the expression, check that $n^3 - n$ is always divisible by 6 for all natural numbers n. Give reasons.

ANSWER

$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1) = (n - 1)\, n\, (n + 1)$

So $n^3 - n$ is the product of three consecutive natural numbers $(n - 1, n, n + 1)$. Among any three consecutive integers, at least one is even, and exactly one is a multiple of 3. So the product is divisible by 2 and by 3. Since 2 and 3 have no common factor, the product is divisible by $2 \times 3 = 6$.

Examples: $n = 2$: $8 - 2 = 6$; $n = 3$: $27 - 3 = 24 = 6 \times 4$; $n = 4$: $64 - 4 = 60 = 6 \times 10$.

$n^3 - n = (n - 1)n(n + 1)$ is a product of three consecutive numbers, so it is always divisible by 6.

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QUESTION 13 *

Find the value of

(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$

(ii) $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$

ANSWER

(i) $x^3 + y^3 - 12xy + 64$ when $x + y = -4$

Take $a = x$, $b = y$, $c = 4$. Then $3abc = 12xy$, so the expression is $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$.

$a + b + c = x + y + 4 = -4 + 4 = 0$

So the expression $= 0 \times (\ldots) = 0$

(ii) $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$

Take $a = x$, $b = -2y$, $c = -6$. Then $a^3 + b^3 + c^3 = x^3 - 8y^3 - 216$ and $3abc = 3(x)(-2y)(-6) = 36xy$, so the expression is $a^3 + b^3 + c^3 - 3abc$.

$a + b + c = x - 2y - 6 = (2y + 6) - 2y - 6 = 0$

So the expression $= 0$

(i) 0 (ii) 0

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CLASS 9 MATHS · GANITA MANJARI · CHAPTER 4

In-Text Questions

Solutions · 15 questions

EXAMPLE 1 · PATTERNS

Now try the same process with another set of three consecutive square numbers. Say 9, 16, 25.

Think and Reflect: Try and find other patterns like this one. For example, you could consider 4 consecutive squares and see if you can find a pattern.

ANSWER

A **The set 9, 16, 25**

$$(9 + 25) - (2 \times 16) = 34 - 32 = 2$$

B **A pattern for 4 consecutive squares**

Add the smallest and the largest of four consecutive squares and subtract the sum of the two middle squares. For 1, 4, 9, 16:

$$(1 + 16) - (4 + 9) = 17 - 13 = 4$$

For 4, 9, 16, 25: $(4 + 25) - (9 + 16) = 29 - 25 = 4$

It is always 4. Proof: for four consecutive numbers n, n + 1, n + 2, n + 3:

$$n^2 + (n + 3)^2 - (n + 1)^2 - (n + 2)^2 = (2n^2 + 6n + 9) - (2n^2 + 6n + 5) = 4$$

The result is always 4 for four consecutive squares (just as it is always 2 for three).

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THINK AND REFLECT · $(a + b)^2$ vs $a^2 + b^2$

1. What can you say about *a* and *b* if $(a + b)^2 < a^2 + b^2$?

2. What can you say about *a* and *b* if $(a + b)^2 > a^2 + b^2$?

3. When will $(a + b)^2$ be equal to $a^2 + b^2$?

Did you observe that $(a + b)^2$ and $a^2 + b^2$ are both positive? What term will decide which is larger? Use the expansion of $(a + b)^2$ to decide.

ANSWER

$(a + b)^2 = a^2 + b^2 + 2ab$, so $(a + b)^2 - (a^2 + b^2) = 2ab$. The term $2ab$ decides which of the two is larger.

1. $(a + b)^2 < a^2 + b^2$

This happens when $2ab < 0$, i.e. $ab < 0$: one of the numbers is positive and the other negative. Example: $a = 3$, $b = -2$: $(a + b)^2 = 1$ and $a^2 + b^2 = 13$.

2. $(a + b)^2 > a^2 + b^2$

This happens when $2ab > 0$, i.e. $ab > 0$: a and b are both positive or both negative. Example: $a = 10$, $b = 2$: $144 > 104$.

3. $(a + b)^2 = a^2 + b^2$

This happens when $2ab = 0$, i.e. $ab = 0$: at least one of the numbers is 0.

$(a + b)^2 - (a^2 + b^2) = 2ab$: it is negative when a, b have opposite signs, positive when they have the same sign, and zero when $a = 0$ or $b = 0$.

THINK AND REFLECT · $b \to -b$

What if we replace *b* by $-b$ in $(a + b)^2 = a^2 + 2ab + b^2$?

ANSWER

$$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

We get the identity $(a - b)^2 = a^2 - 2ab + b^2$, which holds for all a and b, because the identity $(a + b)^2 = a^2 + 2ab + b^2$ holds for all numbers (including $-b$).

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THINK AND REFLECT · FIG. 4.4

Label the squares and rectangles in Fig. 4.4 so that it represents the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

ANSWER

The big square has side a + b + c, split into columns of widths a, b, c and rows of heights a, b, c. Each piece has area (row height) × (column width).

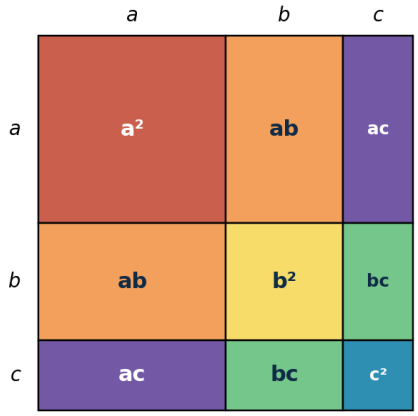


Fig. 4.4 with each piece labelled

Squares on the diagonal: $a \times a = a^2$, $b \times b = b^2$, $c \times c = c^2$

Rectangles: $a \times b = ab$ (two of them), $b \times c = bc$ (two of them), $c \times a = ca$ (two of them)

Total area $= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$.

FIG. 4.5 · $a^2 = (a + b)(a - b) + b^2$

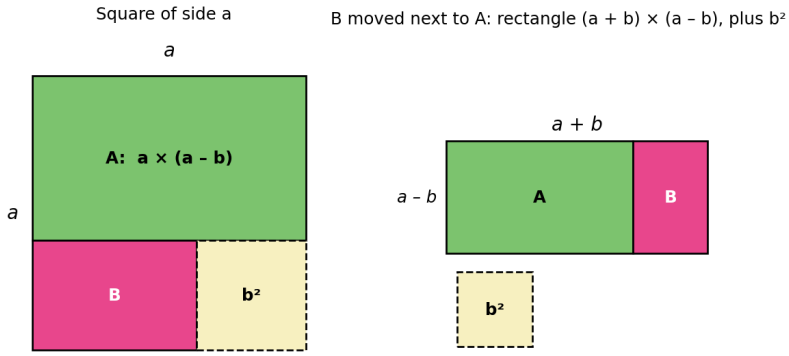
Look at the figure. Justify the identity $a^2 = (a + b)(a - b) + b^2$ for yourself.

(Fig. 4.5: a square of side a with a strip of width b removed and moved.)

ANSWER

Start with a square of side a (area a^2). Cut off a b × b square at a corner. The remaining L-shape splits into rectangle A ($a \times (a - b)$) and rectangle B ($(a - b) \times b$). Move B next to A: together they form a rectangle of sides (a + b) and (a – b).

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A square of side a rearranged

a^2 = [rectangle $(a + b)(a – b)$] + [square b^2]

$a^2 = (a + b)(a – b) + b^2$

Algebraic check: $(a + b)(a – b) + b^2 = a^2 – b^2 + b^2 = a^2$. Example (Śrīdharāchārya): $55^2 = (55 + 5)(55 – 5) + 5^2 = 3000 + 25 = 3025$.

The identity $a^2 = (a + b)(a – b) + b^2$ is justified geometrically and algebraically.

THINK AND REFLECT · SQUARES ENDING IN 5

1. Try to evaluate the following using a suitable identity:

(i) 35^2 (ii) 65^2 (iii) 85^2 (iv) 105^2

Do you observe any interesting pattern?

ANSWER

Use $a^2 = (a + 5)(a – 5) + 5^2$.

$35^2 = 40 \times 30 + 25 = 1200 + 25 = 1225$

$65^2 = 70 \times 60 + 25 = 4200 + 25 = 4225$

$85^2 = 90 \times 80 + 25 = 7200 + 25 = 7225$

$105^2 = 110 \times 100 + 25 = 11000 + 25 = 11025$

Pattern: the square of a number ending in 5 ends in 25, and the digits before 25 are n × (n + 1), where n is the number formed by the other digits: 35: 3 × 4 = 12 → 1225; 65: 6 × 7 = 42 → 4225; 85: 8 × 9 = 72 → 7225; 105: 10 × 11 = 110 → 11025.

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Reason: $(10n + 5)^2 = 100n^2 + 100n + 25 = 100\ n(n + 1) + 25$.

$35^2 = 1225$, $65^2 = 4225$, $85^2 = 7225$, $105^2 = 11025$: n5 squared is n(n + 1) followed by 25.

THINK AND REFLECT · FIG. 4.6

2. Observe the two rows of figures below. They represent an algebraic identity. Try to identify it.

(Fig. 4.6: top row: squares of sides a + b + c, a + b – c, a – b + c, a – b – c; bottom row: squares of sides 2a, 2b, 2c.)

ANSWER

The top row has squares of side (a + b + c), (a + b – c), (a – b + c) and (a – b – c); the bottom row has squares of side 2a, 2b and 2c. The two rows have the same total area, so:

$$(a + b + c)^2 + (a + b - c)^2 + (a - b + c)^2 + (a - b - c)^2 = (2a)^2 + (2b)^2 + (2c)^2$$

Check: the squares expand as $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$, $a^2 + b^2 + c^2 + 2ab - 2ac - 2bc$, $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$ and $a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$. Adding, all the cross-terms cancel and we get $4a^2 + 4b^2 + 4c^2$.

The identity is $(a + b + c)^2 + (a + b - c)^2 + (a - b + c)^2 + (a - b - c)^2 = 4a^2 + 4b^2 + 4c^2$.

THINK AND REFLECT · 7x = 2x + 5x

Suppose $7x$ is split as $2x + 5x$; can a similar rectangular arrangement be formed? Consider other possibilities and check.

ANSWER

With $x^2 + 2x + 5x + 12$ we place the x^2-tile, 2 x-tiles on its right and 5 x-tiles below it. The corner then needs a 2 by 5 array of unit tiles, i.e. 10 unit tiles, but we have 12. So the pieces do not fill a rectangle.

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In general, if 7x is split as ax + bx, the corner array is a by b, which needs ab unit tiles; here we need ab = 12 and a + b = 7.

1x + 6x: needs 1 × 6 = 6 unit tiles (not 12)
2x + 5x: needs 2 × 5 = 10 unit tiles (not 12)
3x + 4x: needs 3 × 4 = 12 unit tiles □
(4x + 3x gives the same rectangle turned round)

Only the split 3x + 4x (or 4x + 3x) works, giving the rectangle (x + 3) by (x + 4).

THINK AND REFLECT · ALGEBRA TILES

Algebra tiles can be used to represent products and find factors.

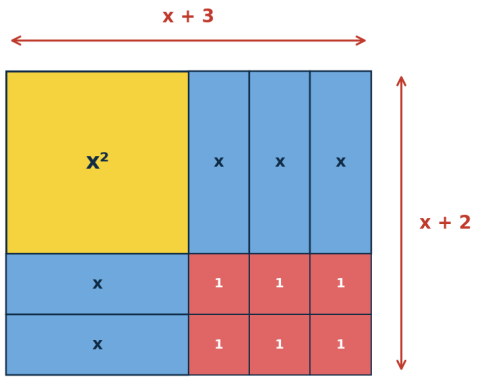
1. Figure out the product of x + 2 and x + 3 using algebra tiles.

2. Lay out algebra tiles for $x^2 + 11x + 30$ in such a way that you will see its factors.

ANSWER

1. Product of x + 2 and x + 3

Make a rectangle with one side an x-tile and 2 unit tiles (x + 2) and the other an x-tile and 3 unit tiles (x + 3). It contains one x^2-tile, 3 + 2 = 5 x-tiles and a 2 × 3 array of 6 unit tiles.



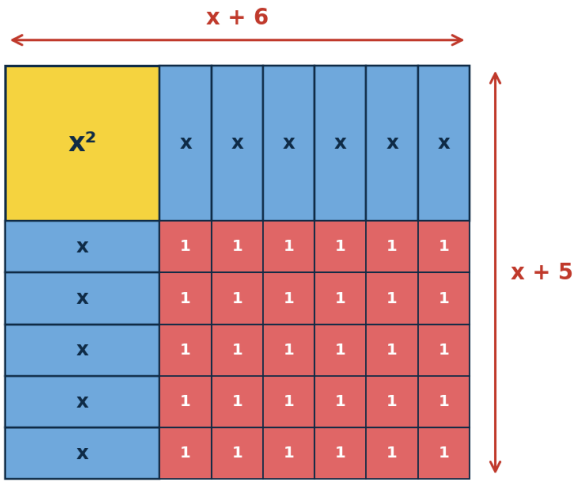
Tiles for (x + 3)(x + 2)

$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

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2. **Tiles for $x^2 + 11x + 30$**

Split 11x as 5x + 6x so that the unit tiles form a 5 by 6 array (5 × 6 = 30): put 5 x-tiles to the right of the x^2-tile and 6 x-tiles below it. The rectangle has sides x + 5 and x + 6.



Tiles for $x^2 + 11x + 30 = (x + 5)(x + 6)$

$(x + 2)(x + 3) = x^2 + 5x + 6$, and $x^2 + 11x + 30 = (x + 5)(x + 6)$.

THINK AND REFLECT · (x + a)(x + b)

We have seen that $(x + 3)(x + 4) = x^2 + 7x + 12$. Also $(x + 6)(x + 7) = x^2 + 13x + 42$. Generalise the pattern to get an expression for (x + a)(x + b).

Fill in the blanks with the appropriate expressions to make the equation true: $(px + a)(qx + b) = (____)x^2 + (____)x + ____$. Also, verify your answer using the distributive property.

ANSWER

A **(x + a)(x + b)**

In (x + 3)(x + 4) the coefficient of x is 7 = 3 + 4 and the constant is 12 = 3 × 4. In (x + 6)(x + 7): 13 = 6 + 7 and 42 = 6 × 7.

$(x + a)(x + b) = x^2 + (a + b)x + ab$

[Distributive property: $x^2 + bx + ax + ab$]

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B (px + a)(qx + b)

$$(px + a)(qx + b) = px \cdot qx + px \cdot b + a \cdot qx + ab$$
$$= pq\,x^2 + (pb + aq)\,x + ab$$

Check with (2x + 3)(3x + 1) (Fig. 4.8): p = 2, a = 3, q = 3, b = 1 gives $6x^2 + (2 \times 1 + 3 \times 3)x + 3 = 6x^2 + 11x + 3$, which agrees with the tiles: 6 x^2-tiles, 9 + 2 = 11 x-tiles and 3 unit tiles.

$(px + a)(qx + b) = (pq)x^2 + (pb + qa)x + ab$.

EXAMPLE 15 · NEW IDENTITIES

Verify $(x - y)(x^2 + xy + y^2) = x^3 - y^3$ for yourself by choosing different values of x and y. Predict what $(x + y)(x^2 - xy + y^2)$ will be.

ANSWER

A Verification of $(x - y)(x^2 + xy + y^2) = x^3 - y^3$

x = 3, y = 2: $(3 - 2)(9 + 6 + 4) = 19$ and $3^3 - 2^3 = 27 - 8 = 19$ ✓
x = 5, y = 1: $(4)(25 + 5 + 1) = 124$ and $125 - 1 = 124$ ✓

B $(x + y)(x^2 - xy + y^2)$

Replacing y by –y in the identity above gives $x^3 + y^3$. Direct multiplication:

$$(x + y)(x^2 - xy + y^2) = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3$$

Check: x = 3, y = 2: $5 \times (9 - 6 + 4) = 35 = 27 + 8$ ✓

$(x + y)(x^2 - xy + y^2) = x^3 + y^3$.

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THINK AND REFLECT · FACTORS OF $x^n - y^n$

Observe that $x - y$ is a common factor of $x^2 - y^2$ and $x^3 - y^3$. Do you think $x - y$ is also a factor of $x^4 - y^4$? Note that $x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 - y^2)(x^2 + y^2)$. Can you see how $x - y$ is a factor of $x^4 - y^4$? How about $x^5 - y^5$? Does this also have $x - y$ as a factor?

ANSWER

A **$x^4 - y^4$**

$$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) = (x - y)(x + y)(x^2 + y^2)$$

Since $x^2 - y^2 = (x - y)(x + y)$, the factor $x - y$ appears.

B **$x^5 - y^5$**

Yes. Multiplying $(x - y)$ by $x^4 + x^3y + x^2y^2 + xy^3 + y^4$ gives:

$$x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 - x^4y - x^3y^2 - x^2y^3 - xy^4 - y^5 = x^5 - y^5$$

so $x^5 - y^5 = (x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

$x - y$ is a factor of $x^4 - y^4$ and of $x^5 - y^5$; in fact it is a factor of $x^n - y^n$ for every natural number n.

THINK AND REFLECT · RATIONAL EXPRESSION

Try to simplify the following rational expression: $(36s^2 - 12st + t^2)/(t^2 + 2ts - 48s^2) = (6s - t)^2/((___ + ___)(___ + ___))$.

(Hint: Factor $t^2 + 2ts - 48s^2$ and simplify the rational expressions assuming that $t^2 + 2ts - 48s^2 \neq 0$.)

ANSWER

Numerator: $36s^2 - 12st + t^2 = (6s - t)^2$

Denominator: $t^2 + 2ts - 48s^2$: find a, b with $a + b = 2s$ and $ab = -48s^2$: $a = 8s$, $b = -6s$

$t^2 + 2ts - 48s^2 = (t + 8s)(t - 6s)$

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So the blanks are filled as (t + 8s)(t + (–6s)), i.e. (t + 8s)(t – 6s). Since (6s – t)² = (t – 6s)² and t – 6s ≠ 0 (the denominator is not zero), we can cancel:

$$(36s^2 - 12st + t^2)/(t^2 + 2ts - 48s^2) = (t - 6s)^2/((t + 8s)(t - 6s)) = (t - 6s)/(t + 8s)$$

The expression simplifies to (t – 6s)/(t + 8s).

THINK AND REFLECT · JAMES AND RESHMA

James and Reshma were talking about algebraic identities they learnt in school. James: (a – b)²(a + b) = (a² – 2ab + b²)(a + b). Reshma: I have a different idea. (a – b)²(a + b) = (a – b)[(a – b)(a + b)] = (a – b)(a² – b²). I will find this product to get the answer.

According to you, who is correct and why? Try to combine more such identities and find new results.

ANSWER

Both are correct: they use different (valid) identities for the same product.

James: $(a^2 - 2ab + b^2)(a + b) = a^3 + a^2b - 2a^2b - 2ab^2 + ab^2 + b^3 = a^3 - a^2b - ab^2 + b^3$

Reshma: $(a - b)(a^2 - b^2) = a^3 - ab^2 - a^2b + b^3 = a^3 - a^2b - ab^2 + b^3$

The two results are the same, because $(a - b)^2 = (a - b)(a - b)$ and $(a - b)(a + b) = a^2 - b^2$. Using different identities for the same expression must give the same answer.

More results by combining identities:

$(a + b)^2(a - b) = (a + b)(a^2 - b^2) = a^3 + a^2b - ab^2 - b^3$

$(a^2 - b^2)^2 = (a + b)^2(a - b)^2 = a^4 - 2a^2b^2 + b^4$

$(a + b)^2(a - b)^2 = [(a + b)(a - b)]^2 = (a^2 - b^2)^2$

Both James and Reshma are correct; the product is $a^3 - a^2b - ab^2 + b^3$.

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EXAMPLE 17 · SAIRA'S RECTANGLE

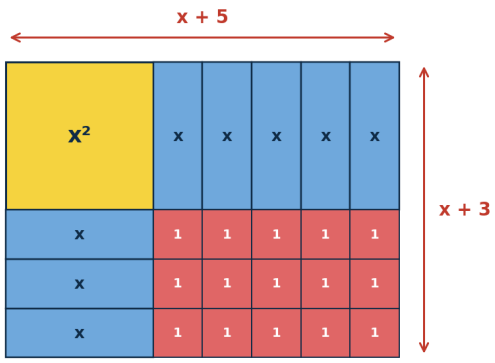
Saira has arranged a square of side *x* units, 8 rectangular strips of sides *x* units and width 1 unit, and 15 squares of side 1 unit to form a bigger rectangle. Find the length and breadth of the rectangle in terms of *x*. ... Now draw Saira's rectangle using these pieces.

ANSWER

Total area = $x^2 + 8x + 15$. Factorising: a + b = 8 and ab = 15 give a = 3, b = 5.

$$x^2 + 8x + 15 = x^2 + 3x + 5x + 15 = x(x + 3) + 5(x + 3) = (x + 3)(x + 5)$$

To draw the rectangle: place the x^2-tile, then 5 strips of the x-tile to its right and 3 strips below it, and fill the corner with a 5 by 3 array of unit squares (15 squares). The strips used are 5 + 3 = 8, as given.



Saira's rectangle: (x + 5) by (x + 3)

Length = x + 5 units and breadth = x + 3 units.

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