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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 3

The World of Numbers

Complete Exercise Solutions

Exercise Sets 3.1 – 3.5

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IT

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 3

In-Text Questions

Think and Reflect · questions inside the running text

Directions: These are the questions that appear inside the running text of the chapter (the *Think and Reflect* boxes and the other "Can you …?" prompts). Each is answered below in the order in which it appears; the page number of the chapter is given with every question.

IN-TEXT 1 · P. 46

Think and Reflect. Why does a negative times a negative equal a positive? Think of it in terms of action and debt. If a negative number represents a debt, then multiplying by a negative number represents the removal of that debt. (**Hint:** If someone takes away (–) four of your debts that are each worth ₹3 (that is, –3), you are effectively ₹12 richer! Therefore, (–3) × (–4) = +12.)

ANSWER

Debt idea. A debt of ₹3 is –3. "Four debts of ₹3" is 4 × (–3) = –12. Multiplying by –4 means *removing* four such debts, which makes you ₹12 richer: (–3) × (–4) = +12.

Algebraic reason. We know (–3) × 0 = 0 and 0 = 4 + (–4). By the distributive law:

$$(-3) \times [4 + (-4)] = (-3) \times 4 + (-3) \times (-4) = 0$$
$$-12 + (-3) \times (-4) = 0$$
so $(-3) \times (-4) = 12$

Therefore, the product of two negative numbers is positive, because (–3) × (–4) must be the number that cancels –12, which is +12.

IN-TEXT 2 · P. 47

Think and Reflect. Can you explain why we need $q \neq 0$ in the definition of a rational number?

ANSWER

A rational number is p/q with integers p and q, q ≠ 0. The fraction p/q means "the number which, when multiplied by q, gives p".

If q = 0 and p ≠ 0, no number x satisfies $0 \times x = p$, because $0 \times x$ is always 0. If q = 0 and p = 0, every number x satisfies $0 \times x = 0$, so the value would not be unique.

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Therefore, division by zero has no meaning, and q must not be 0.

IN-TEXT 3 · P. 49

Think and Reflect. 1. While adding or subtracting two rational numbers having different denominators, how will you make the denominators equal?

ANSWER

Find the LCM of the denominators and rewrite each fraction with this LCM as denominator, multiplying the numerator and denominator by the same number (this does not change the value).

2/3 + 3/4: LCM of 3 and 4 is 12
2/3 = 8/12 and 3/4 = 9/12
2/3 + 3/4 = 8/12 + 9/12 = 17/12

(Multiplying the denominators, 3 × 4 = 12, also works and always gives a common denominator.)

Therefore, write both numbers with a common multiple of the denominators (preferably the LCM), then add or subtract the numerators.

IN-TEXT 4 · P. 49

Think and Reflect. 2. Verify the distributive law for rational numbers.

ANSWER

The law says: for rational numbers p, q, r, $p(q + r) = pq + pr$.

Example. Take p = 1/2, q = 1/3, r = 1/4.

p(q + r) = 1/2 × (1/3 + 1/4) = 1/2 × 7/12 = 7/24
pq + pr = 1/6 + 1/8 = 4/24 + 3/24 = 7/24

General proof. Let p = a/b, q = c/d, r = e/f. Then q + r = (cf + de)/(df), so p(q + r) = a(cf + de)/(bdf) = (acf + ade)/(bdf). Also pq + pr = ac/(bd) + ae/(bf) = (acf + ade)/(bdf). The two are equal.

Therefore, the distributive law holds for rational numbers.

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IN-TEXT 5 · P. 51

Think and Reflect. Try and represent 8/5 and –7/4 on a number line.

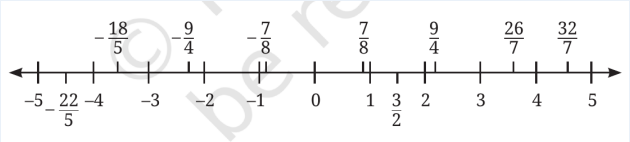


Fig. 3.7

ANSWER

8/5 = 1 3/5. It lies between 1 and 2. Divide the interval from 1 to 2 into 5 equal parts and move 3 parts to the right of 1.

–7/4 = –1 3/4. It lies between –2 and –1. Divide the interval from –2 to –1 into 4 equal parts; the point is 3 parts to the left of –1 (equivalently 1 part to the right of –2).



8/5 and –7/4 on the number line

Therefore, 8/5 is at 1.6 and –7/4 is at –1.75 on the number line (compare Fig. 3.7).

IN-TEXT 6 · P. 52

No matter how close two rational numbers are on the number line, you can always find another rational number between them by taking their average. For example, a rational number between 1 and 3/2 can be found by taking their average: (1 + 3/2)/2 = 5/4. Try to explain why the average of two rational numbers *a* and *b*, which equals (*a* + *b*)/2, is always a rational number between *a* and *b*.

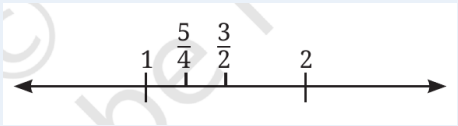


Fig. 3.9

ANSWER

It is rational: the sum *a* + *b* of two rational numbers is rational, and dividing a rational number by 2 gives a rational number.

It is between them. Let *a* < *b*.

$$(a + b)/2 - a = (b - a)/2 > 0 \Rightarrow (a + b)/2 > a$$
$$b - (a + b)/2 = (b - a)/2 > 0 \Rightarrow (a + b)/2 < b$$

So (*a* + *b*)/2 is exactly halfway between *a* and *b*. Repeating the process again and again gives infinitely many rational numbers between any two rational numbers.

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Therefore, the average of two rational numbers is a rational number lying between them.

IN-TEXT 7 · P. 53

Think and Reflect. Can √2 be written as a rational number *p*/*q*?

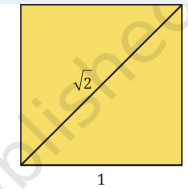


Fig. 3.10

ANSWER

No. Suppose √2 = p/q, where p and q have no common factor. Then $p^2 = 2q^2$, so p^2 is even and hence p is even, p = 2k. Then $4k^2 = 2q^2$, $q^2 = 2k^2$, so q is even too. This contradicts that p and q have no common factor (the proof in Section 3.5.1).

The length of the diagonal of a unit square is √2 (Fig. 3.10), yet it cannot be written as a ratio of integers.

Therefore, √2 is not a rational number; it is irrational.

IN-TEXT 8 · P. 55

Think and Reflect. Try to prove the irrationality of √3 using the approach of proof by contradiction. Will the same approach work for √5, √7, or √10?

ANSWER

Proof for √3. Suppose √3 = p/q with p, q integers having no common factor, q ≠ 0. Squaring, $3q^2 = p^2$. So 3 divides p^2. Since 3 is prime, 3 divides p; let p = 3k.

$$3q^2 = (3k)^2 = 9k^2 \Rightarrow q^2 = 3k^2$$

So 3 divides q^2, hence 3 divides q. Now 3 divides both p and q, which contradicts that they have no common factor. So √3 is irrational.

a **Does it work for √5, √7?** YES

Yes: 5 and 7 are prime numbers. From $5q^2 = p^2$ we get $5 \mid p^2 \Rightarrow 5 \mid p \Rightarrow q^2 = 5k^2 \Rightarrow 5 \mid q$, a contradiction; the same for 7.

b **Does it work for √10?** YES

Yes. $10q^2 = p^2$ gives $2 \mid p^2$ and $5 \mid p^2$, so $2 \mid p$ and $5 \mid p$ (2 and 5 are primes), hence $10 \mid p$; write p = 10k: $10q^2 = 100k^2$, $q^2 = 10k^2$, so 2 and 5 divide q. Again p and q have a common factor 2, a contradiction.

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Note. The method needs a step such as “prime divides $p^2 \Rightarrow$ it divides p”. It does not work for perfect squares like $\sqrt{4}$, since $\sqrt{4} = 2$ is rational.

Therefore, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ and $\sqrt{10}$ are all irrational.

IN-TEXT 9 · P. 55

Think and Reflect. We have seen how to obtain a line whose length is a rational number. How do we obtain lines whose lengths are irrational?

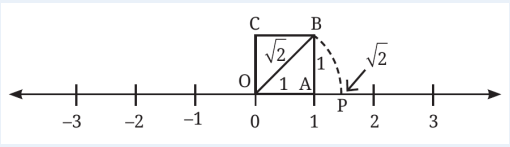


Fig. 3.11

ANSWER

By using the Baudhāyana–Pythagoras theorem. A right triangle with legs of rational lengths has hypotenuse $\sqrt{(a^2 + b^2)}$, which is irrational whenever $a^2 + b^2$ is not the square of a rational number.

For example, a right triangle with both legs 1 unit has hypotenuse $OB = \sqrt{(1^2 + 1^2)} = \sqrt{2}$ (Fig. 3.11). With a compass, draw an arc with centre O and radius OB; it cuts the number line at P, so $OP = \sqrt{2}$ and P represents $\sqrt{2}$.

Therefore, irrational lengths are obtained as hypotenuses of right triangles and transferred to the number line with a compass.

IN-TEXT 10 · P. 56

Think and Reflect. Try to extend this method for constructing line segments of lengths $\sqrt{3}$ and $\sqrt{5}$ using a ruler and a compass. Generalise this method to construct a line segment of any length of the form $\sqrt{n}$, where n is a positive integer.

ANSWER

a **$\sqrt{3}$**

Using $OP = \sqrt{2}$ on the number line (Fig. 3.11), draw a perpendicular to the number line at P and mark Q on it with PQ = 1 unit. Then $OQ^2 = (\sqrt{2})^2 + 1^2 = 3$, so $OQ = \sqrt{3}$. With centre O and radius OQ draw an arc cutting the number line at a point which represents $\sqrt{3}$.

b **$\sqrt{5}$**

Mark A at 2 on the number line (OA = 2). Draw a perpendicular at A and mark B with AB = 1. Then $OB^2 = 2^2 + 1^2 = 5$, so $OB = \sqrt{5}$. An arc with centre O and radius OB cuts the number line at the point representing $\sqrt{5}$.

c **$\sqrt{n}$**

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If √(n – 1) is already marked at a point X (OX = √(n – 1)), draw a perpendicular at X of length 1, giving a point Y. Then $OY^2 = (n - 1) + 1 = n$, so OY = √n; transfer this length to the number line with a compass. Starting from OA = 1, repeating this gives √2, √3, √4, √5, … one after another (the “square-root spiral”). Alternatively, write $n = a^2 + b^2$ and use legs a and b.

Therefore, every √n can be constructed using a ruler, a compass and the Baudhāyana–Pythagoras theorem.

IN-TEXT 11 · P. 57

Example 2: … = 0.375. (Can you tell for which rational numbers the decimal will be terminating?)

ANSWER

A rational number p/q in lowest terms has a terminating decimal exactly when the prime factors of q are only 2, only 5, or both 2 and 5.

Examples: 3/8 ($8 = 2^3$) = 0.375, 7/20 ($20 = 2^2 \times 5$) = 0.35 and 13/250 ($250 = 2 \times 5^3$) = 0.052 terminate. 1/3, 5/6 and 4/15 do not terminate, as 3 divides the denominator.

Therefore, p/q (lowest terms) terminates exactly when $q = 2^m \times 5^n$ for whole numbers m, n.

IN-TEXT 12 · P. 57

Think and Reflect. Try to find the decimal expansions of 10/3 and 11/12. What do you observe about the repetition of the digits after the decimal point?

ANSWER

10/3 = 3.3333… (the block 3 repeats)
11/12 = 0.91666… (the block 6 repeats)

In 10/3 the digit 3 repeats right after the decimal point. In 11/12 the digits 9 and 1 appear once, and then the digit 6 repeats; so the repetition may begin after a few non-repeating digits ($12 = 2^2 \times 3$ has the factor 2^2 which gives two non-repeating digits).

Therefore, 10/3 = 3.333… and 11/12 = 0.91666…; the repeating part may start immediately or after some digits.

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IN-TEXT 13 · P. 58

Why do some rational numbers have repeating decimal representations? Imagine calculating 1/7 using long division. You are dividing by 7. What are the possible remainders at each step? They can only be 1, 2, 3, 4, 5, or 6 (why not 0?).

ANSWER

If a remainder were 0, the division would end and the decimal would terminate. For 1/7 this never happens: the remainders are 1, 3, 2, 6, 4, 5, 1, ... and 7 does not divide 10^n, so a remainder 0 would require $7 \mid 10^n \times 1$, which is impossible (7 is not a factor of 10^n).

As there are only 6 non-zero possible remainders, one of them must repeat within 7 steps, and then the digits repeat in a cycle.

Therefore, the remainder is never 0 for 1/7, so the remainders repeat and the decimal repeats.

IN-TEXT 14 · P. 58

Think and Reflect. The decimal expansion of p/q will be terminating precisely when the prime factors of q are only 2, only 5 or both 2 and 5. Can you explain why?

ANSWER

If $q = 2^m5^n$: multiply the numerator and denominator by $2^{(n-m)}$ if $n > m$, or by $5^{(m-n)}$ if $m > n$, to make the denominator 10^k where k is the larger of m, n. Then $p/q = a/10^k$, a terminating decimal.

Conversely: if p/q (lowest terms) is a terminating decimal, then $p/q = a/10^k = a/(2^k5^k)$. So q divides 2^k5^k (as p, q have no common factor), which means the only prime factors of q are 2 and 5.

Therefore, p/q terminates exactly when q has no prime factors other than 2 and 5.

IN-TEXT 15 · P. 64

Think and Reflect. Consider this puzzle: What is the square root of –1? We know that 1 × 1 = 1. We also know that (–1) × (–1) = 1. There is no Real Number that, when multiplied by itself, results in a negative number. Thus, √–1 cannot exist on number line.

ANSWER

The square of every real number is non-negative: positive × positive = positive, negative × negative = positive and 0 × 0 = 0. So no real number can satisfy $x^2 = -1$.

To solve this, mathematicians introduced a new number i with $i^2 = -1$ (an imaginary number). It lies off the real number line and is studied in later classes.

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Therefore, √–1 is not a real number; it is the imaginary number i.

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3.1

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 3

Exercise Set 3.1

Solutions · 4 questions

QUESTION 1

A merchant in the port city of Lothal is exchanging bags of spices for copper ingots. He receives 15 ingots for every 2 bags of spices. If he brings 12 bags of spices to the market, how many copper ingots will he leave with?

ANSWER

12 bags make 12 ÷ 2 = 6 groups of 2 bags. Each group gives 15 ingots.

$$6 \times 15 = 90$$

Therefore, he will leave with 90 copper ingots.

QUESTION 2

Look at the sequence of numbers on one column of the Ishango bone: 11, 13, 17, 19. What do these numbers have in common? List the next three numbers that fit this pattern.

ANSWER

11, 13, 17 and 19 are all **prime numbers** (each has exactly two factors, 1 and itself); they are the prime numbers between 10 and 20.

The next three prime numbers after 19 are 23, 29 and 31.

Therefore, they are prime numbers, and the next three numbers are 23, 29, 31.

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QUESTION 3

We know that Natural Numbers are closed under addition (the sum of any two natural numbers is always a natural number). Are they closed under subtraction? Provide a couple of examples to justify your answer.

ANSWER

No. The difference of two natural numbers need not be a natural number.

$3 - 5 = -2$ (not a natural number)
$4 - 4 = 0$ (not a natural number)
$2 - 7 = -5$ (not a natural number)

Therefore, natural numbers are not closed under subtraction.

QUESTION 4 *

Ancient Indians used the joints of their fingers to count, a practice still seen today. Each finger has 3 joints, and the thumb is used to count them. How many can you count on one hand? How does this relate to the ancient base-12 counting systems?

ANSWER

Each of the four fingers has 3 joints, and the thumb touches them one by one.

$4 \text{ fingers} \times 3 \text{ joints} = 12$

Relation to base 12. One hand counts up to 12, i.e. one complete “dozen” (a group of 12). Counting groups of 12 on the other hand’s 5 fingers can then reach $5 \times 12 = 60$ (which also relates to the base-60 system of the Babylonians). Counting in groups of twelve is exactly a base-12 system.

Therefore, 12 can be counted on one hand, which matches the ancient base-12 (duodecimal) counting systems.

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3.2

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 3

Exercise Set 3.2

Solutions · 4 questions

QUESTION 1

The temperature in the high-altitude desert of Ladakh is recorded as 4 °C at noon. By midnight, it drops by 15 °C. What is the midnight temperature?

ANSWER

$4 - 15 = -11$

Therefore, the midnight temperature is –11 °C.

QUESTION 2

A spice trader takes a loan (debt) of ₹850. The next day, he makes a profit (fortune) of ₹1,200. The following week, he incurs a loss of ₹450. Write this sequence as an equation using integers and calculate his final financial standing.

ANSWER

Debt is negative, profit positive and loss negative: debt –850, profit +1200, loss –450.

$(-850) + 1200 + (-450) = 350 - 450 = -100$

Therefore, the equation is (–850) + 1200 + (–450) = –100; he is left with a debt of ₹100.

QUESTION 3

Calculate the following using Brahmagupta’s laws:

(i) (–12) × 5 (ii) (–8) × (–7) (iii) 0 – (–14) (iv) (–20) ÷ 4

ANSWER

i **(–12) × 5**

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A negative times a positive is negative: (–12) × 5 = –60

ii **(–8) × (–7)**

A negative times a negative is positive: (–8) × (–7) = 56

iii **0 – (–14)**

Subtracting a debt is the same as adding a fortune: 0 – (–14) = 0 + 14 = 14

iv **(–20) ÷ 4**

A negative divided by a positive is negative: (–20) ÷ 4 = –5

Therefore, the values are –60, 56, 14 and –5.

QUESTION 4

Explain, using a real-world example of debt, why subtracting a negative number is the same as adding a positive number (e.g., 10 – (–5) = 15).

ANSWER

Suppose you have ₹10, but you also owe a friend a debt of ₹5 (so your debt is –5). Taking away (subtracting) this debt means the friend cancels it.

Removing a debt of ₹5 makes you ₹5 better off, so your position becomes 10 + 5 = 15.

10 – (–5) = 10 + 5 = 15

Therefore, taking away a debt is the same as receiving that amount: subtracting –5 is the same as adding +5.

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3.3

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 3

Exercise Set 3.3

Solutions · 8 questions

QUESTION 1

Prove that the following rational numbers are equal:

(i) 2/3 and 4/6 (ii) 5/4 and 10/8

(iii) –3/5 and –6/10 (iv) 9/3 and 3

ANSWER

Two rational numbers a/b and c/d are equal exactly when ad = bc (cross-multiplication); equivalently, one is obtained from the other by multiplying numerator and denominator by the same number.

i **2/3 and 4/6**

$2 \times 6 = 12$ and $3 \times 4 = 12$, so $2/3 = 4/6$. (Also $2/3 = (2 \times 2)/(3 \times 2) = 4/6$.)

ii **5/4 and 10/8**

$5 \times 8 = 40$ and $4 \times 10 = 40$, so $5/4 = 10/8$.

iii **–3/5 and –6/10**

$(-3) \times 10 = -30$ and $5 \times (-6) = -30$, so $-3/5 = -6/10$.

iv **9/3 and 3**

$3 = 3/1$: $9 \times 1 = 9$ and $3 \times 3 = 9$, so $9/3 = 3$. (Also $9/3 = 9 \div 3 = 3$.)

Therefore, all four pairs of rational numbers are equal.

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QUESTION 2

Find the sum:

(i) 2/5 + 3/10 (ii) 7/12 + 5/8 (iii) –4/7 + 3/14

ANSWER

i **2/5 + 3/10**

LCM of 5 and 10 is 10: 4/10 + 3/10 = 7/10

ii **7/12 + 5/8**

LCM of 12 and 8 is 24: 14/24 + 15/24 = 29/24

iii **–4/7 + 3/14**

LCM of 7 and 14 is 14: –8/14 + 3/14 = –5/14

Therefore, the sums are 7/10, 29/24 and –5/14.

QUESTION 3

Find the difference:

(i) 5/6 – 1/4 (ii) 11/8 – 3/4 (iii) –7/9 – (–2/3)

ANSWER

i **5/6 – 1/4**

LCM 12: 10/12 – 3/12 = 7/12

ii **11/8 – 3/4**

LCM 8: 11/8 – 6/8 = 5/8

iii **–7/9 – (–2/3)**

–7/9 + 2/3 = –7/9 + 6/9 = –1/9

Therefore, the differences are 7/12, 5/8 and –1/9.

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QUESTION 4

Find the product:

(i) 2/3 × 3/10 (ii) 7/11 × 5/8 (iii) –4/7 × 5/14

ANSWER

i **2/3 × 3/10**

(2 × 3)/(3 × 10) = 6/30 = 1/5

ii **7/11 × 5/8**

(7 × 5)/(11 × 8) = 35/88

iii **–4/7 × 5/14**

(–4 × 5)/(7 × 14) = –20/98 = –10/49

Therefore, the products are 1/5, 35/88 and –10/49.

QUESTION 5

Find the quotient:

(i) 2/3 ÷ 3/10 (ii) 7/11 ÷ 5/8 (iii) –4/7 ÷ 5/14

ANSWER

To divide by a fraction, multiply by its reciprocal.

i **2/3 ÷ 3/10**

2/3 × 10/3 = 20/9

ii **7/11 ÷ 5/8**

7/11 × 8/5 = 56/55

iii **–4/7 ÷ 5/14**

–4/7 × 14/5 = –56/35 = –8/5

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Therefore, the quotients are 20/9, 56/55 and –8/5.

QUESTION 6

Show that: (1/2 + 3/4) × 8/3 = 1/2 × 8/3 + 3/4 × 8/3.

ANSWER

LHS = (1/2 + 3/4) × 8/3 = (2/4 + 3/4) × 8/3 = 5/4 × 8/3 = 40/12 = 10/3

RHS = 1/2 × 8/3 + 3/4 × 8/3 = 8/6 + 24/12 = 4/3 + 2 = 4/3 + 6/3 = 10/3

Therefore, LHS = RHS = 10/3; this is the distributive law.

QUESTION 7

Simplify the following using the distributive property: 7/9 (6/7 – 3/4).

ANSWER

7/9 (6/7 – 3/4) = 7/9 × 6/7 – 7/9 × 3/4
= 6/9 – 21/36 = 2/3 – 7/12
= 8/12 – 7/12 = 1/12

Check: 6/7 – 3/4 = 24/28 – 21/28 = 3/28 and 7/9 × 3/28 = 21/252 = 1/12 ✓.

Therefore, the value is 1/12.

QUESTION 8

Find the rational number *x* such that: 5/6 (*x* + 3/5) = 5/6 *x* + 1/2.

ANSWER

Expand the left side by the distributive law:

5/6 (x + 3/5) = 5/6 × x + 5/6 × 3/5 = 5/6 x + 15/30 = 5/6 x + 1/2

This is exactly the right-hand side, whatever *x* is. Subtracting 5/6 *x* from both sides leaves 1/2 = 1/2, which is always true.

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Therefore, the equation holds for every rational number x (it is an identity, so no particular value is forced).

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3.4

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 3

Exercise Set 3.4

Solutions · 6 questions

QUESTION 1

Represent the rational numbers 2/3, –5/4 and 1½ on a single number line.

ANSWER

2/3: divide the interval from 0 to 1 into 3 equal parts and take 2 parts to the right of 0.

–5/4 = –1¼: divide the interval from –2 to –1 into 4 equal parts; the point is 1 part to the left of –1.

1½ = 3/2: it is the midpoint of the interval from 1 to 2.



2/3, –5/4 and 1½ on one number line

Therefore, 2/3, –5/4 and 1½ are marked at 0.67, –1.25 and 1.5 on the number line.

QUESTION 2

Find three distinct rational numbers that lie strictly between –1/2 and 1/4.

ANSWER

Write both with denominator 8: –1/2 = –4/8 and 1/4 = 2/8. The fractions with denominator 8 between them are –3/8, –2/8, –1/8, 0, 1/8.

Taking three of them: –1/4 (= –2/8), 0 and 1/8. Check: –1/2 < –1/4 < 0 < 1/8 < 1/4 ✓. (The averages method also works: the average of –1/2 and 1/4 is –1/8.)

Therefore, –1/4, 0 and 1/8 are three rational numbers between –1/2 and 1/4 (other answers are possible).

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QUESTION 3

Simplify the expression: (–1/4) + (5/12).

ANSWER

LCM of 4 and 12 is 12: –3/12 + 5/12 = 2/12 = 1/6

Therefore, the value is 1/6.

QUESTION 4

A tailor has 15¾ metres of fine silk. If making one kurta requires 2¼ metres of silk, exactly how many kurtas can he make?

ANSWER

15¾ = 63/4 and 2¼ = 9/4
Number of kurtas = 63/4 ÷ 9/4 = 63/4 × 4/9 = 7

Therefore, the tailor can make exactly 7 kurtas.

QUESTION 5

Find three rational numbers between 3.1415 and 3.1416.

ANSWER

Write 3.1415 = 3.14150 and 3.1416 = 3.14160. The numbers with five decimal places in between are 3.14151, 3.14152, ..., 3.14159.

Therefore, 3.14152, 3.14155 and 3.14158 are three rational numbers between 3.1415 and 3.1416 (other answers are possible).

QUESTION 6 *

Can you think of other way(s) to find a rational number between any two rational numbers?

ANSWER

1. **Equal denominators**

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Write the two numbers with the same denominator and make the denominator larger by multiplying top and bottom by 10, 100, ... For example, 1/3 and 1/2 become 2/6 and 3/6, then 20/60 and 30/60, and 21/60, 22/60, ... lie in between.

2 Mediant method

For positive rationals $a/b < c/d$, the number $(a + c)/(b + d)$ lies between them. For example between 1/3 and 1/2: $(1 + 1)/(3 + 2) = 2/5$, and $1/3 < 2/5 < 1/2$ ✓.

3 Decimal method

Convert both numbers to decimals and write a decimal with more digits lying between them (as in Q5).

Therefore, besides the average, one can use equal denominators, the mediant $(a + c)/(b + d)$, or decimals; there are infinitely many rational numbers between any two.

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3.5

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 3

Exercise Set 3.5

Solutions · 5 questions

QUESTION 1

Without performing long division, determine which of the following rational numbers will have terminating decimals and which will be repeating: 7/20, 4/15 and 13/250. Then check your answers by explicitly performing the long divisions and expressing these rational numbers as decimals.

ANSWER

A rational number in lowest terms terminates exactly when the denominator has only the prime factors 2 and 5.

i **7/20** — TERMINATING

$20 = 2^2 \times 5$: only 2 and 5. Check: 7/20 = 35/100 = 0.35.

ii **4/15** — REPEATING

$15 = 3 \times 5$ has the prime factor 3. Long division gives 4/15 = 0.2666... (the digit 6 repeats).

iii **13/250** — TERMINATING

$250 = 2 \times 5^3$: only 2 and 5. Check: 13/250 = 52/1000 = 0.052.

Therefore, 7/20 = 0.35 and 13/250 = 0.052 terminate, while 4/15 = 0.2666... repeats.

QUESTION 2

Perform the long division for 1/13. Identify the repeating block of digits. Does it show cyclic properties if you evaluate 2/13? Now compute 3/13, 4/13, etc. What do you notice?

ANSWER

Long division of 1 by 13 has remainders 1, 10, 9, 12, 3, 4, 1, ..., so the digits repeat after 6 steps:

1/13 = 0.076923 076923... (repeating block 076923)

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Fraction	Decimal (repeating block)
1/13	0.076923 076923...
2/13	0.153846 153846...
3/13	0.230769 230769...
4/13	0.307692 307692...

Observation 1: 3/13 = 0.230769..., 4/13 = 0.307692... use the same six digits 0, 7, 6, 9, 2, 3 as 1/13, only started at a different place (cyclic shifts of 076923). The same happens for 9/13 = 0.692307..., 10/13 = 0.769230... and 12/13 = 0.923076....

Observation 2: 2/13 = 0.153846... has a different block 153846 = 2 × 076923, which is also cyclic: 5/13 = 0.384615..., 6/13 = 0.461538..., 7/13 = 0.538461..., 8/13 = 0.615384... and 11/13 = 0.846153... are cyclic shifts of 153846.

Therefore, 1/13 has the repeating block 076923, and the multiples n/13 give only two cyclic families of digits: shifts of 076923 (n = 1, 3, 4, 9, 10, 12) and shifts of 153846 (n = 2, 5, 6, 7, 8, 11).

QUESTION 3

Classify the following numbers as rational or irrational:

(i) √81

(ii) √12

(iii) 0.33333 …

(iv) 0.123451234512345 …

(v) 1.01001000100001 … (Notice the pattern: Is it repeating a single block?)

(vi) 23.560185612239874790120

Find the explicit fractions in case they are rational.

ANSWER

(i) √81 — RATIONAL

√81 = 9 = 9/1

(ii) √12 — IRRATIONAL

√12 = 2√3. Since √3 is irrational, 2√3 is irrational (if 2√3 = p/q then √3 = p/(2q) would be rational, a contradiction).

(iii) 0.33333 … — RATIONAL

Let x = 0.333...; 10x = 3.333...; 10x – x = 3; 9x = 3; x = 1/3

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(iv) 0.123451234512345 … RATIONAL

The block 12345 repeats. Let x = 0.12345 12345…; multiply by 10^5:

$$100000x = 12345.12345\ldots$$
$$100000x - x = 12345 \Rightarrow 99999x = 12345$$
$$x = 12345/99999 = 4115/33333$$

(v) 1.01001000100001 … IRRATIONAL

The number of zeros between successive 1s keeps increasing (1, 2, 3, 4, … zeros), so no single block repeats and the decimal does not terminate. Hence it is irrational.

(vi) 23.5601856122398747901 20 RATIONAL

It is a terminating decimal (it stops after 21 decimal places), so it equals a fraction with denominator a power of 10:

$$23.560185612239874790120 = 2356018561223987479012 0/10^{21} = 589004640305996869753/25000000000000000000$$

Therefore, (i), (iii), (iv), (vi) are rational and (ii), (v) are irrational.

QUESTION 4

The number 0.9 with a bar over the 9 (which means 0.99999 …) is a rational number. Using algebra (let *x* = 0.99999 …), multiply by 10, and subtract), explain why 0.99999 … is exactly equal to 1.

ANSWER

$$\text{Let } x = 0.999\ldots$$
$$10x = 9.999\ldots$$
$$10x - x = 9.999\ldots - 0.999\ldots = 9$$
$$9x = 9$$
$$x = 1$$

Since the repeating 9s cancel exactly in the subtraction, the number 0.999… equals 1; it is just a second decimal form of 1 (like 1 = 1.000…).

Therefore, 0.999… = 1 exactly.

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QUESTION 5 *

We have seen that the repeating block of 1/7 is a cyclic number. Try to find more numbers (*n*) whose reciprocals (1/*n*) produce decimals with repeating blocks that are cyclic.

ANSWER

For 1/7 the block 142857 has 6 digits (one less than 7), and multiplying it by 1 to 6 only rotates the digits. Doing long division for other primes gives the same behaviour for:

n	Repeating block of 1/n	Length
7	142857	6
17	0588235294117647	16
19	052631578947368421	18
23	0434782608695652173913	22

In each case the block has n – 1 digits (the remainders 1, 2, ..., n – 1 all occur), and multiplying the block by 1, 2, ..., n – 1 only shifts its digits cyclically. For example 2/17 = 0.1176470588235294..., whose block 1176470588235294 is a cyclic shift of 0588235294117647. Such primes are called full-reptend primes: 7, 17, 19, 23, 29, 47, 59, 61, 97, ... Note 13 does not work (its block has only 6 digits, not 12).

Therefore, n = 17, 19, 23, 29, … (like 7) have reciprocals with cyclic repeating blocks of n – 1 digits.

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