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SOLUTIONS BOOK

CLASS 9

Mathematics · Ganita Manjari

CHAPTER 1

Orienting Yourself: The Use of Coordinates

Complete Exercise Solutions

Exercise Set 1.1 | Exercise Set 1.2 | End-of-Chapter Exercises | In-Text Questions

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1.1

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 1

Exercise Set 1.1

Solutions · 1 question

QUESTION 1

Fig. 1.3 shows Reiaan's room with points OABC marking its corners. The x- and y-axes are marked in the figure. Point O is the origin.

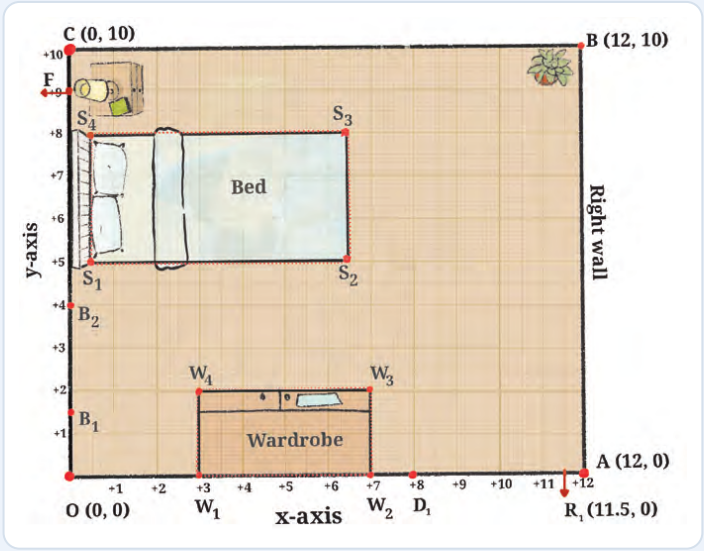


Fig. 1.3

Referring to Fig. 1.3, answer the following questions:

(i) If D_1R_1 represents the door to Reiaan's room, how far is the door from the left wall (the y-axis) of the room? How far is the door from the x-axis?

(ii) What are the coordinates of D_1?

(iii) If R_1 is the point (11.5, 0), how wide is the door? Do you think this is a comfortable width for the room door? If a person in a wheelchair wants to enter the room, will he/she be able to do so easily?

(iv) If B_1 (0, 1.5) and B_2 (0, 4) represent the ends of the bathroom door, is the bathroom door narrower or wider than the room door?

ANSWER

(i) **Distance of the door from the walls**

In Fig. 1.3 the door D_1R_1 lies along the bottom wall of the room, which is the x-axis. The point D_1 is marked at +8 on the x-axis, so it is 8 units, i.e. 8 ft (the scale is 1 unit = 1 ft), to the right of the y-axis.

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Distance from the left wall (y-axis) = 8 ft
Distance from the x-axis = 0 ft (the door is on the x-axis itself)

(ii) Coordinates of D_1

D_1 is on the x-axis, 8 units from O, so its y-coordinate is 0.

D_1 = (8, 0)

(iii) Width of the door

D_1 = (8, 0) and R_1 = (11.5, 0)
Width of the door = 11.5 – 8 = 3.5 ft

A width of 3.5 ft (about 1.07 m) is comfortable: ordinary room doors are about 2.5–3 ft wide. A wheelchair needs a clear opening of roughly 2.5–3 ft (about 80–90 cm). So a person in a wheelchair can enter easily through this door.

The door is 3.5 ft wide, a comfortable width, and a wheelchair can pass through it easily.

(iv) Bathroom door and room door

Width of the bathroom door = 4 – 1.5 = 2.5 ft
Width of the room door = 3.5 ft

The bathroom door (2.5 ft) is narrower than the room door (3.5 ft) by 1 ft.

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1.2

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Exercise Set 1.2

Solutions · 5 questions

GRAPH SHEET SETUP

On a graph sheet, mark the x-axis and y-axis and the origin O. Mark points from (– 7, 0) to (13, 0) on the x-axis and from (0, – 15) to (0, 12) on the y-axis. (Use the scale 1 cm = 1 unit.) Using Fig. 1.5, answer the given questions.

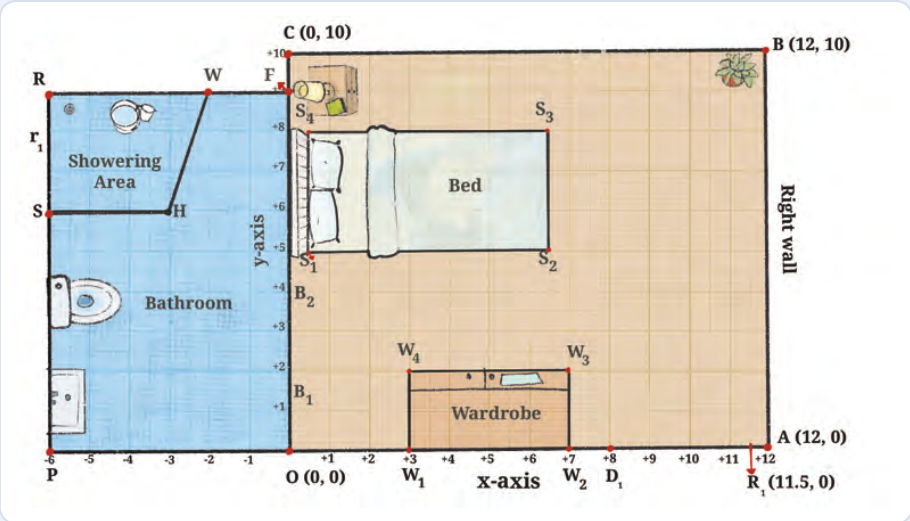


Fig. 1.5

ANSWER

Scale: 1 cm = 1 unit, so every 1 cm on the sheet is one unit on an axis.

x-axis: from (–7, 0) to (13, 0) is 7 + 13 = 20 units, so draw a horizontal line 20 cm long.

y-axis: from (0, –15) to (0, 12) is 15 + 12 = 27 units, so draw a vertical line 27 cm long.

Steps: (1) Draw the horizontal line and the vertical line so that they cross at right angles; the crossing point is the origin O (0, 0), and it is marked 7 cm from the left end of the x-axis and 15 cm from the lower end of the y-axis. (2) Mark points at every 1 cm along the x-axis: –7, –6, ..., –1, 0, 1, ..., 13, and along the y-axis: –15, –14, ..., –1, 0, 1, ..., 12. (3) Label the axes x and y and the origin O, and put arrowheads at the ends.

A graph sheet of at least 20 cm × 27 cm (for example, a full A4 sheet used in portrait) is needed, with O placed 7 cm from the left end and 15 cm from the bottom end of the axes.

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QUESTION 2

Place Reiaan's rectangular study table with three of its feet at the points (8, 9), (11, 9) and (11, 7).

(i) Where will the fourth foot of the table be?

(ii) Is this a good spot for the table?

(iii) What is the width of the table? The length? Can you make out the height of the table?

ANSWER

(i) The fourth foot

The feet of a rectangular table are the four corners of a rectangle. (8, 9) and (11, 9) have the same y-coordinate, so they are joined by a horizontal side. (11, 9) and (11, 7) have the same x-coordinate, so they are joined by a vertical side. The fourth corner has the x-coordinate of (8, 9) and the y-coordinate of (11, 7).

The fourth foot is at (8, 7).

(ii) Is it a good spot?

The table covers the region from x = 8 to x = 11 and y = 7 to y = 9. In Fig. 1.5 the bed ends at about x = 6.5, so there is a clear gap of about 1.5 ft between the bed and the table. The door D_1R_1 is on the bottom wall (y = 0), far from the table, and the wardrobe is at the bottom (y ≤ 2), so neither is blocked. The table also stays clear of the plant in the corner near B (12, 10).

Yes, it is a good spot: it is in free floor space and blocks neither the door, the bed nor the wardrobe.

(iii) Width, length and height

Horizontal side: from (8, 9) to (11, 9) = 11 – 8 = 3 ft
Vertical side: from (11, 9) to (11, 7) = 9 – 7 = 2 ft

So the length of the table is 3 ft (the longer side) and its width is 2 ft. The height cannot be found: the map shows only the floor, so it gives length and breadth but no information about heights (we would need a third coordinate for that).

Length = 3 ft, width = 2 ft; the height cannot be made out from this floor map.

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QUESTION 3

If the bathroom door has a hinge at B_1 and opens into the bedroom, will it hit the wardrobe? Are there any changes you would suggest if the door is made wider?

ANSWER

The bathroom door is B_1B_2, with $B_1 = (0, 1.5)$ and $B_2 = (0, 4)$, so the door is $4 - 1.5 = 2.5$ ft wide. The wardrobe is the rectangle $W_1(3, 0)$, $W_2(7, 0)$, $W_3(7, 2)$, $W_4(3, 2)$, so its left face is the line $x = 3$ and it spans $y = 0$ to $y = 2$.

With the hinge at B_1, the door swings into the bedroom about the point (0, 1.5). Every point of the door stays within 2.5 ft of the hinge, so the farthest point it can reach to the right is $x = 2.5$ (when the door is opened 90°, pointing along the line $y = 1.5$).

Farthest reach of the door = 2.5 ft from the y-axis
Left face of the wardrobe is at $x = 3$
Clearance = $3 - 2.5 = 0.5$ ft

The door will not hit the wardrobe; it clears it by 0.5 ft.

If the door is made wider than 2.5 ft, say w ft, it reaches $x = w$ when opened to 90° at the height $y = 1.5$, which lies on the face of the wardrobe (y from 0 to 2). So for $w = 3$ ft or more the door would hit the wardrobe. Possible changes: shift the wardrobe to the right so that its left face is at $x = w + 0.5$ or more (for example, W_1 at (4, 0) for a 3 ft door), reduce the width of the wardrobe accordingly, or fix the hinge at the other end B_2 or use a sliding door.

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QUESTION 4

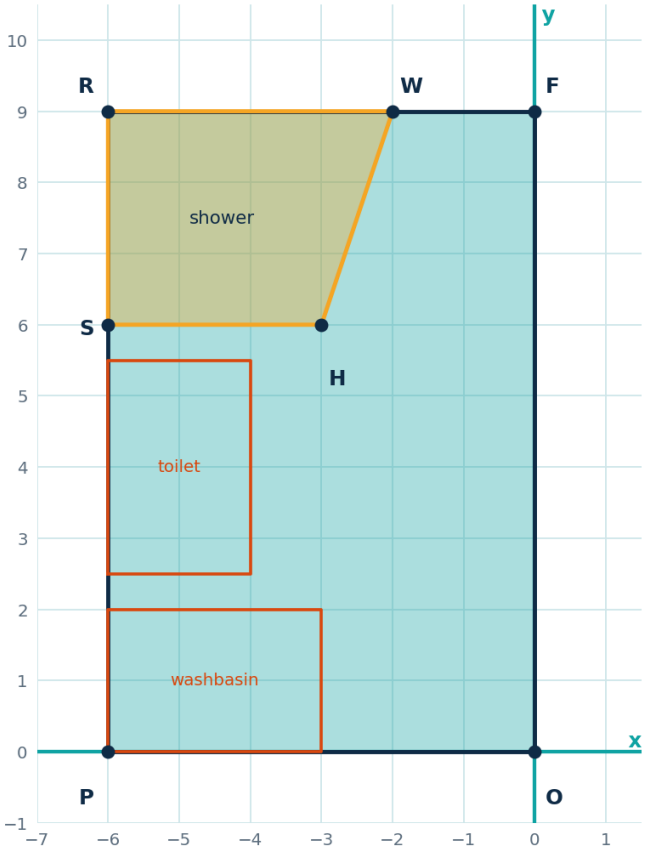
Look at Reiaan’s bathroom.

(i) What are the coordinates of the four corners O, F, R, and P of the bathroom?

(ii) What is the shape of the showering area SHWR in Reiaan’s bathroom? Write the coordinates of the four corners.

(iii) Mark off a 3 ft × 2 ft space for the washbasin and a 2 ft × 3 ft space for the toilet. Write the coordinates of the corners of these spaces.

ANSWER



Reiaan’s bathroom, with one possible placement of the washbasin and toilet

(i) Corners of the bathroom

From Fig. 1.5, the bathroom is 6 ft wide and 9 ft long. It lies to the left of the y-axis: its right wall is the part of the y-axis from O to F, and its left wall is 6 units to the left of the y-axis.

$$\text{O} = (0, 0),\ \text{F} = (0, 9),\ \text{R} = (-6, 9),\ \text{P} = (-6, 0)$$

Check: OP = 6 ft and OF = 9 ft, which agree with the size 6 ft × 9 ft of the bathroom.

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(ii) The showering area SHWR

S = (–6, 6) is on the left wall, H = (–3, 6), W = (–2, 9) is on the top wall and R = (–6, 9). In the quadrilateral SHWR the sides SH and RW are both horizontal, so they are parallel (SH = 3 ft, RW = 4 ft), while SR is vertical and so perpendicular to both. HW slants (it moves 1 unit to the right and 3 units up).

S = (–6, 6), H = (–3, 6), W = (–2, 9), R = (–6, 9)

SHWR is a trapezium (with two right angles, at S and R), having SH ∥ RW.

(iii) Washbasin and toilet

The positions can be chosen in different ways; here is one choice that leaves free space around the bathroom door B_1B_2 and the shower.

Washbasin (3 ft × 2 ft), in the corner at P: (–6, 0), (–3, 0), (–3, 2), (–6, 2)
Toilet (2 ft × 3 ft), against the left wall: (–6, 2.5), (–4, 2.5), (–4, 5.5), (–6, 5.5)

The washbasin occupies 3 × 2 = 6 sq ft and the toilet 2 × 3 = 6 sq ft; both lie inside the bathroom and do not overlap each other or the showering area (which begins at y = 6). Other correct placements are possible.

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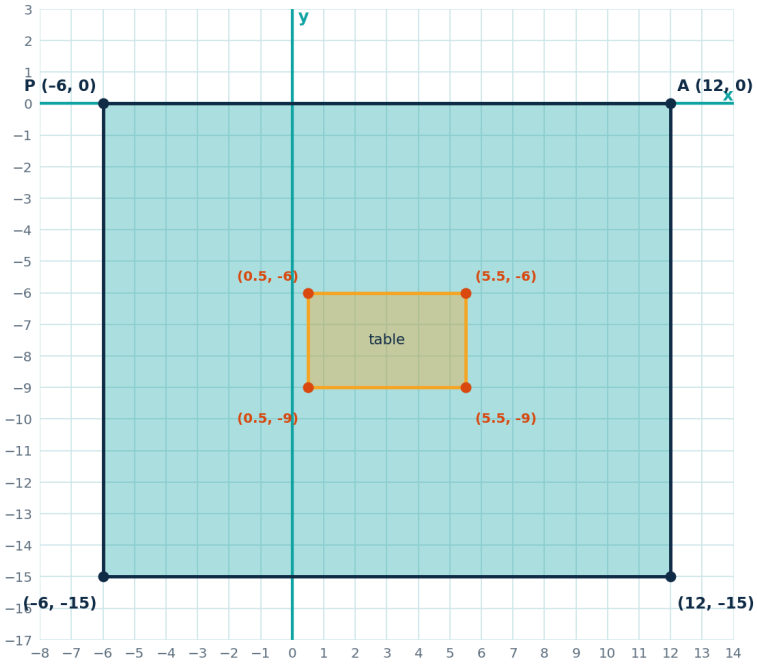
QUESTION 5

Other rooms in the house:

(i) Reiaan's room door leads from the dining room which has the length 18 ft and width 15 ft. The length of the dining room extends from point P to point A. Sketch the dining room and mark the coordinates of its corners.

(ii) Place a rectangular 5 ft × 3 ft dining table precisely in the centre of the dining room. Write down the coordinates of the feet of the table.

ANSWER



The dining room and the dining table

(i) The dining room

P = (–6, 0) and A = (12, 0), so PA = 12 – (–6) = 18 ft, which is the length of the dining room. Reiaan's room door is on the x-axis and leads out of the bedroom, so the dining room lies on the other side of the line PA, i.e. below the x-axis, and extends 15 ft downwards (the width).

Corners: P = (–6, 0), A = (12, 0), (12, –15), (–6, –15)

(ii) The dining table

The centre of the dining room is the midpoint of the corners (–6, 0) and (12, –15):

Centre = ((–6 + 12) ÷ 2, (0 + (–15)) ÷ 2) = (3, –7.5)

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Take the 5 ft side along the length of the room. The feet are 5 ÷ 2 = 2.5 ft to the left and right of the centre and 3 ÷ 2 = 1.5 ft above and below it:

x: 3 – 2.5 = 0.5 and 3 + 2.5 = 5.5
y: –7.5 – 1.5 = –9 and –7.5 + 1.5 = –6

The feet of the table are at (0.5, –9), (5.5, –9), (5.5, –6) and (0.5, –6). (If the 5 ft side is placed along the width, the feet are (1.5, –10), (4.5, –10), (4.5, –5) and (1.5, –5).)

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EOC

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 1

End-of-Chapter Exercises

Solutions · 16 questions

QUESTION 1

What are the x-coordinate and y-coordinate of the point of intersection of the two axes?

ANSWER

The point where the axes meet is the origin O. It is on the x-axis, so its y-coordinate is 0, and it is on the y-axis, so its x-coordinate is 0.

x-coordinate = 0 and y-coordinate = 0, i.e. the origin is (0, 0).

QUESTION 2

Point W has x-coordinate equal to – 5. Can you predict the coordinates of point H which is on the line through W parallel to the y-axis? Which quadrants can H lie in?

ANSWER

A line parallel to the y-axis is vertical, and all its points are at the same distance from the y-axis. Hence every point of the line through W has x-coordinate –5.

H = (–5, y) for some number y

We can predict the x-coordinate (–5) but not the y-coordinate, because H can be anywhere on the line. Since x = –5 is negative, H lies:

in Quadrant II if y > 0 (coordinates (–, +))
in Quadrant III if y < 0 (coordinates (–, –))
on the negative x-axis, not in any quadrant, if y = 0

H = (-5, y); H can lie in Quadrant II or Quadrant III (or on the x-axis if y = 0).

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QUESTION 3

Consider the points R (3, 0), A (0, – 2), M (– 5, – 2) and P (– 5, 2). If they are joined in the same order, predict:

(i) Two sides of RAMP that are perpendicular to each other.

(ii) One side of RAMP that is parallel to one of the axes.

(iii) Two points that are mirror images of each other in one axis. Which axis will this be?

Now plot the points and verify your predictions.

ANSWER

(i) Two perpendicular sides

A and M have the same y-coordinate (–2), so AM is horizontal. M and P have the same x-coordinate (–5), so MP is vertical. A horizontal and a vertical line are perpendicular.

AM ⊥ MP (they meet at M at a right angle).

(ii) A side parallel to an axis

AM is horizontal (y = –2 at both ends), so it is parallel to the x-axis. (MP is also parallel to the y-axis.)

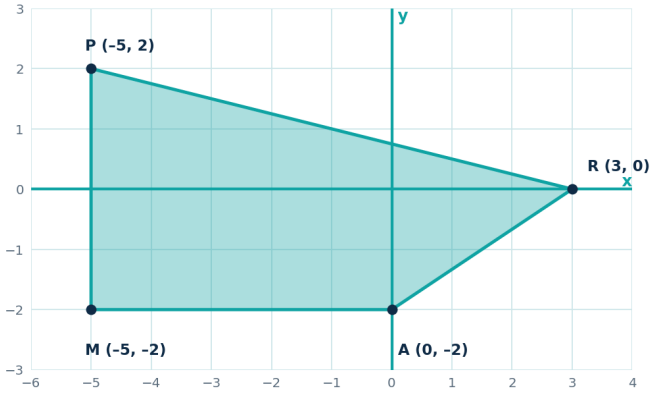
AM is parallel to the x-axis.

(iii) Mirror images in an axis

M = (–5, –2) and P = (–5, 2) have the same x-coordinate and opposite y-coordinates, so each is the reflection of the other in the x-axis.

M and P are mirror images of each other in the x-axis.

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Plot of RAMP: AM is horizontal, MP is vertical, and M, P are mirror images in the x-axis

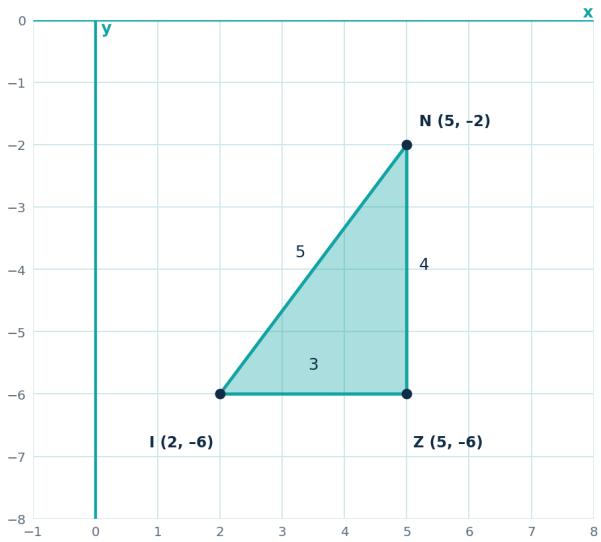
The plot confirms all three predictions. (RA and PR are not perpendicular to any other side, so AM ⊥ MP is the only perpendicular pair.)

QUESTION 4

Plot point Z (5, – 6) on the Cartesian plane. Construct a right-angled triangle IZN and find the lengths of the three sides.
(Comment: Answers may differ from person to person.**)**

ANSWER

Many triangles are possible. Take the right angle at Z. Choose I on the horizontal line through Z, say I = (2, –6), and N on the vertical line through Z, say N = (5, –2). Then ZI is parallel to the x-axis and ZN is parallel to the y-axis, so ∠IZN = 90°.



Right-angled triangle IZN

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ZI = 5 – 2 = 3 units
ZN = –2 – (–6) = 4 units
IN = $\sqrt{((5 - 2)^2 + (-2 - (-6))^2)} = \sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$ units

The sides are ZI = 3, ZN = 4 and IN = 5 units (this answer depends on the choice of I and N).

QUESTION 5

What would a system of coordinates be like if we did not have negative numbers? Would this system allow us to locate all the points on a 2-D plane?

ANSWER

Without negative numbers we could only use distances measured to the right of the origin along the x-axis and upwards along the y-axis. All the points we could name would have both coordinates 0 or positive, so we could describe only the points of the first quadrant and the positive parts of the axes.

Points to the left of the y-axis or below the x-axis (Quadrants II, III and IV) could not be given coordinates. We could, of course, choose the origin far to the bottom-left so that a limited region of interest lies in the first quadrant (as in the computer screen of Question 15), but then points beyond the origin would still be left out.

No. Without negative numbers only the points with x ≥ 0 and y ≥ 0 can be located; the rest of the plane cannot.

QUESTION 6 *

Are the points M (– 3, – 4), A (0, 0) and G (6, 8) on the same straight line? Suggest a method to check this without plotting and joining the points.

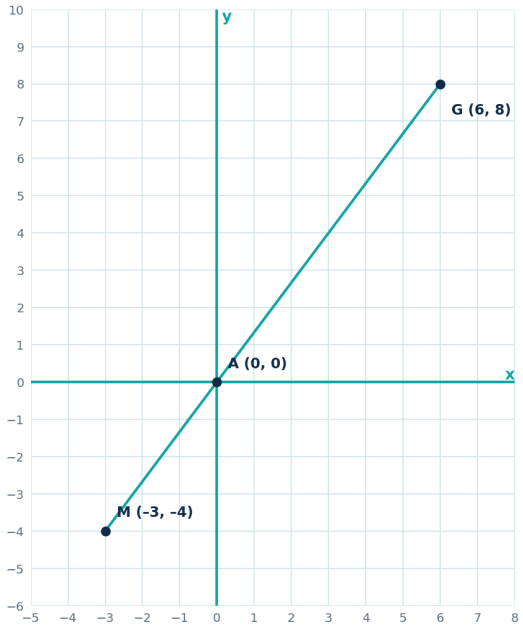
ANSWER

Method. Three points are on one straight line exactly when the distance between the two farthest points is the sum of the other two distances. Find the three distances using the distance formula $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$.

MA = $\sqrt{((0 - (-3))^2 + (0 - (-4))^2)} = \sqrt{(9 + 16)} = 5$
AG = $\sqrt{((6 - 0)^2 + (8 - 0)^2)} = \sqrt{(36 + 64)} = 10$
MG = $\sqrt{((6 - (-3))^2 + (8 - (-4))^2)} = \sqrt{(81 + 144)} = \sqrt{225} = 15$

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Here MA + AG = 5 + 10 = 15 = MG, so A lies on the segment MG. (Equivalently, going from A, the shifts to G are (6, 8) and to M are (–3, –4) = –½ × (6, 8): both are in the same ratio, so the points are along one line.)



M, A and G on one line

Yes, M, A and G are on the same straight line.

QUESTION 7 *

Use your method (from Problem 6) to check if the points R (– 5, – 1), B (– 2, – 5) and C (4, – 12) are on the same straight line. Now plot both sets of points and check your answers.

ANSWER

$RB = \sqrt{((-2-(-5))^2 + (-5-(-1))^2)} = \sqrt{(9+16)} = 5$

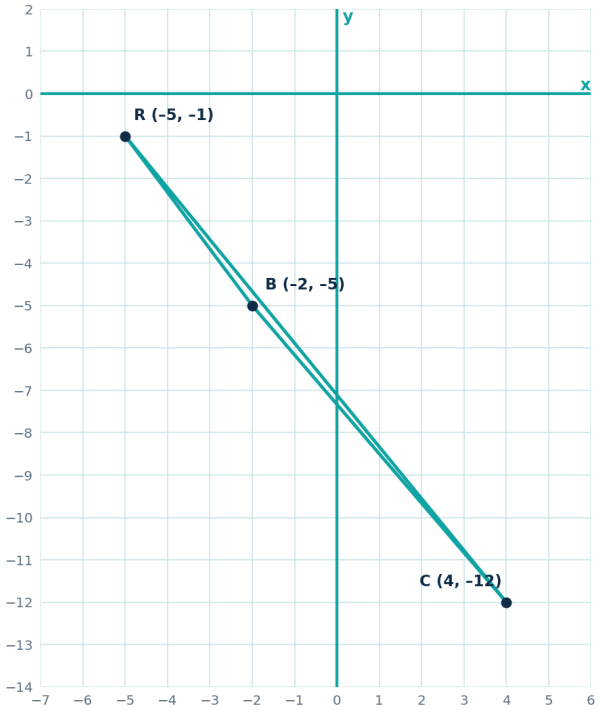
$BC = \sqrt{((4-(-2))^2 + (-12-(-5))^2)} = \sqrt{(36+49)} = \sqrt{85}$

$RC = \sqrt{((4-(-5))^2 + (-12-(-1))^2)} = \sqrt{(81+121)} = \sqrt{202}$

If the points were on a line we would have RB + BC = RC, i.e. $5 + \sqrt{85} = \sqrt{202}$. Squaring both sides: $(5 + \sqrt{85})^2 = 25 + 85 + 10\sqrt{85} = 110 + 10\sqrt{85}$, while $(\sqrt{202})^2 = 202$. Equality would need $10\sqrt{85} = 92$, i.e. $8500 = 8464$, which is false. (Numerically $5 + \sqrt{85} \approx 14.2195$ but $\sqrt{202} \approx 14.2127$; the points are very nearly, but not exactly, on a line.)

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The same conclusion follows from the shifts: from R to B the shifts are (3, –4), and from B to C they are (6, –7); these are not in the same ratio (3 : 6 = 1 : 2, but 4 : 7 ≠ 1 : 2).



R, B, C do not lie on one line (the dashed line is RC)

No, R, B and C are not on the same straight line. Plotting both sets agrees: M, A, G (Question 6) are collinear, while R, B, C form a (thin) triangle.

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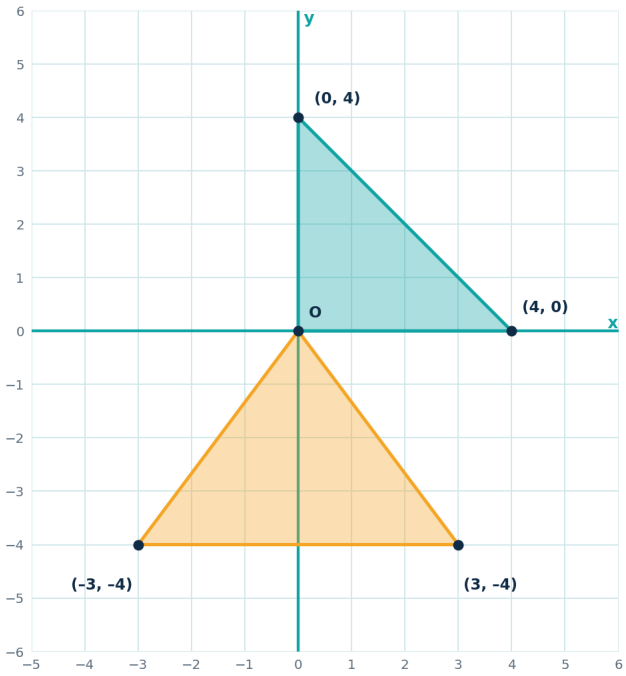
QUESTION 8 *

Using the origin as one vertex, plot the vertices of:

(i) A right-angled isosceles triangle.

(ii) An isosceles triangle with one vertex in Quadrant III and the other in Quadrant IV.

ANSWER



(i) in teal, (ii) in amber

(i) A right-angled isosceles triangle

Take the right angle at the origin, with the two equal sides along the axes: O (0, 0), A (4, 0), B (0, 4).

$$OA = 4,\ OB = 4,\ AB = \sqrt{(4^2 + 4^2)} = \sqrt{32}$$

OA = OB and ∠AOB = 90°, so OAB is a right-angled isosceles triangle.

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(ii) An isosceles triangle with vertices in Quadrants III and IV

Take O (0, 0), P (–3, –4) in Quadrant III and Q (3, –4) in Quadrant IV (P and Q are mirror images in the y-axis).

$OP = \sqrt{(3^2 + 4^2)} = 5$
$OQ = \sqrt{(3^2 + 4^2)} = 5$
$PQ = 3 - (-3) = 6$

OP = OQ = 5, so OPQ is an isosceles triangle. (Many other answers are possible.)

QUESTION 9 *

The following table shows the coordinates of points S, M and T. In each case, state whether M is the midpoint of segment ST. Justify your answer.

S	M	T	Is M the midpoint of ST? Yes or No	Reason for your answer
(–3, 0)	(0, 0)	(3, 0)		
(2, 3)	(3, 4)	(4, 5)		
(0, 0)	(0, 5)	(0, –10)		
(–8, 7)	(0, –2)	(6, –3)		

When M is the mid-point of ST, can you find any connection between the coordinates of M, S and T?

ANSWER

The midpoint of the segment joining S (x_1, y_1) and T (x_2, y_2) is found by taking the average of the coordinates, $((x_1 + x_2) \div 2, (y_1 + y_2) \div 2)$. Let us compare it with M in each row.

S	M	T	Is M the midpoint of ST?	Reason for your answer
(–3, 0)	(0, 0)	(3, 0)	Yes	Midpoint of ST = ((–3 + 3) ÷ 2, (0 + 0) ÷ 2) = (0, 0) = M
(2, 3)	(3, 4)	(4, 5)	Yes	Midpoint of ST = ((2 + 4) ÷ 2, (3 + 5) ÷ 2) = (3, 4) = M
(0, 0)	(0, 5)	(0, –10)	No	Midpoint of ST = ((0 + 0) ÷ 2, (0 – 10) ÷ 2) = (0, –5) ≠ M. (M is above S, but T is below S.)
(–8, 7)	(0, –2)	(6, –3)	No	Midpoint of ST = ((–8 + 6) ÷ 2, (7 – 3) ÷ 2) = (–1, 2) ≠ M

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Connection. When M is the midpoint of ST, the x-coordinate of M is the average of the x-coordinates of S and T, and the y-coordinate of M is the average of the y-coordinates of S and T:

x-coordinate of M = (x-coordinate of S + x-coordinate of T) ÷ 2
y-coordinate of M = (y-coordinate of S + y-coordinate of T) ÷ 2

M is the midpoint of ST in rows 1 and 2 only. In general, the midpoint of (x_1, y_1) and (x_2, y_2) is $((x_1 + x_2) \div 2, (y_1 + y_2) \div 2)$.

QUESTION 10 *

Use the connection you found to find the coordinates of B given that M (–7, 1) is the midpoint of A (3, – 4) and B (*x*, *y*).

ANSWER

Since M is the midpoint of AB, each coordinate of M is the average of the corresponding coordinates of A and B.

$(3 + x) \div 2 = -7 \Rightarrow 3 + x = -14 \Rightarrow x = -17$
$(-4 + y) \div 2 = 1 \Rightarrow -4 + y = 2 \Rightarrow y = 6$

Check: the midpoint of A (3, –4) and B (–17, 6) is ((3 – 17) ÷ 2, (–4 + 6) ÷ 2) = (–7, 1) = M.

B = (–17, 6).

QUESTION 11 *

Let P, Q be points of trisection of AB, with P closer to A, and Q closer to B. Using your knowledge of how to find the coordinates of the midpoint of a segment, how would you find the coordinates of P and Q? Do this for the case when the points are A (4, 7) and B (16, –2).

ANSWER

Trisection means that AB is divided into three equal parts: AP = PQ = QB. Then P is the midpoint of AQ and Q is the midpoint of PB. Let A = (x_1, y_1) and B = (x_2, y_2); write the first coordinates of P and Q as p and q.

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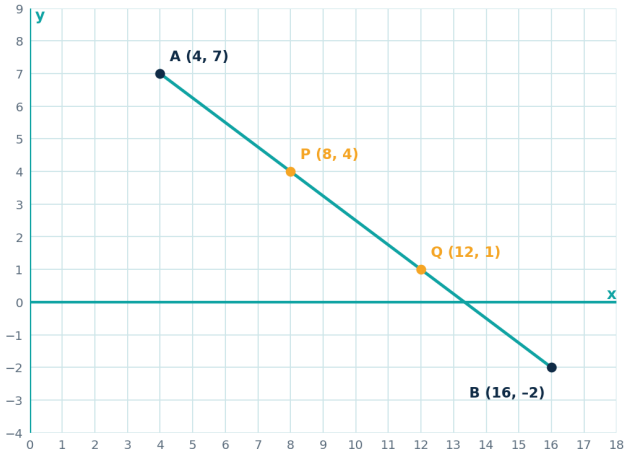
P is the midpoint of AQ: $p = (x_1 + q) \div 2$
Q is the midpoint of PB: $q = (p + x_2) \div 2$
Solving these two equations: $p = (2x_1 + x_2) \div 3$ and $q = (x_1 + 2x_2) \div 3$

The y-coordinates are found in the same way. So P is obtained by adding one-third of the shift from A to B to A, and Q by adding two-thirds of the shift.

For A (4, 7) and B (16, –2):

$P = ((2 \times 4 + 16) \div 3, (2 \times 7 + (–2)) \div 3) = (24 \div 3, 12 \div 3) = (8, 4)$
$Q = ((4 + 2 \times 16) \div 3, (7 + 2 \times (–2)) \div 3) = (36 \div 3, 3 \div 3) = (12, 1)$

Check: each step from A to P, P to Q and Q to B is a shift of (4, –3), so AP = PQ = QB = 5; also the midpoint of AQ is ((4 + 12) ÷ 2, (7 + 1) ÷ 2) = (8, 4) = P and the midpoint of PB is (12, 1) = Q.



Points of trisection P and Q of AB

P = (8, 4) and Q = (12, 1).

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QUESTION 12 *

(i) Given the points A (1, – 8), B (– 4, 7) and C (–7, – 4), show that they lie on a circle K whose center is the origin O (0, 0). What is the radius of circle K?

(ii) Given the points D (–5, 6) and E (0, 9), check whether D and E lie within the circle, on the circle, or outside the circle K.

ANSWER

(i) The points lie on a circle with centre O

Points lie on a circle with centre O if they are all at the same distance from O.

$$OA = \sqrt{(1^2 + (-8)^2)} = \sqrt{(1 + 64)} = \sqrt{65}$$
$$OB = \sqrt{((-4)^2 + 7^2)} = \sqrt{(16 + 49)} = \sqrt{65}$$
$$OC = \sqrt{((-7)^2 + (-4)^2)} = \sqrt{(49 + 16)} = \sqrt{65}$$

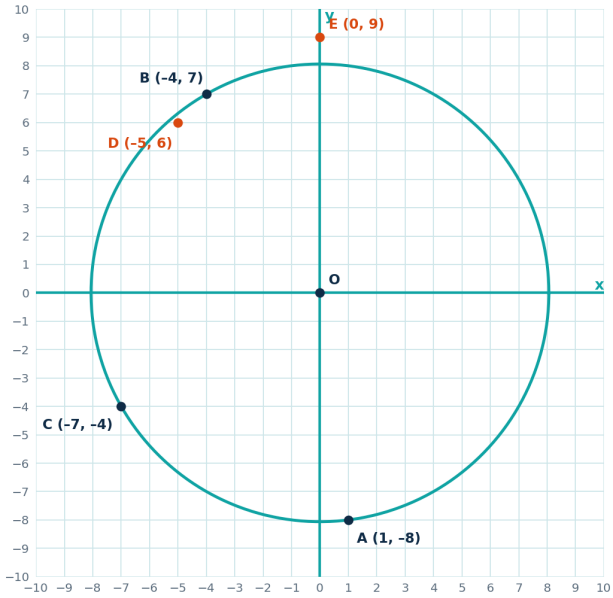
OA = OB = OC = √65, so A, B, C lie on the circle K with centre O and radius √65 (≈ 8.06 units).

(ii) Positions of D and E

A point is inside the circle if its distance from O is less than √65, on the circle if it is equal to √65, and outside if it is greater.

$$OD = \sqrt{((-5)^2 + 6^2)} = \sqrt{(25 + 36)} = \sqrt{61} < \sqrt{65}$$
$$OE = \sqrt{(0^2 + 9^2)} = \sqrt{81} = 9 > \sqrt{65}\ (\approx 8.06)$$

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Circle K with centre O and radius √65

D lies inside the circle K, and E lies outside the circle K.

QUESTION 13 *

The midpoints of the sides of triangle ABC are the points D, E, and F. Given that the coordinates of D, E, and F are (5, 1), (6, 5), and (0, 3), respectively, find the coordinates of A, B and C.

ANSWER

The question does not say which side each midpoint belongs to; we take D as the midpoint of BC, E as the midpoint of CA and F as the midpoint of AB. Let A = (a_1, a_2), B = (b_1, b_2), C = (c_1, c_2). Using the x-coordinates (the y-coordinates work in the same way):

$$(b_1 + c_1) \div 2 = 5 \Rightarrow b_1 + c_1 = 10$$
$$(c_1 + a_1) \div 2 = 6 \Rightarrow c_1 + a_1 = 12$$
$$(a_1 + b_1) \div 2 = 0 \Rightarrow a_1 + b_1 = 0$$

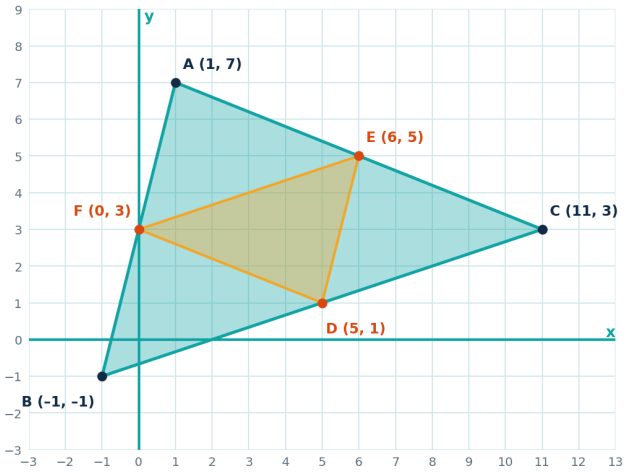
Adding all three: $2(a_1 + b_1 + c_1) = 22$, so $a_1 + b_1 + c_1 = 11$. Subtracting each equation in turn: $a_1 = 11 - 10 = 1$, $b_1 = 11 - 12 = -1$, $c_1 = 11 - 0 = 11$.

For the y-coordinates: $b_2 + c_2 = 2$, $c_2 + a_2 = 10$, $a_2 + b_2 = 6$. Their sum gives $a_2 + b_2 + c_2 = 9$, so $a_2 = 9 - 2 = 7$, $b_2 = 9 - 10 = -1$, $c_2 = 9 - 6 = 3$.

A = (1, 7), B = (–1, –1), C = (11, 3)

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Check: midpoint of BC = ((–1 + 11) ÷ 2, (–1 + 3) ÷ 2) = (5, 1) = D; midpoint of CA = ((11 + 1) ÷ 2, (3 + 7) ÷ 2) = (6, 5) = E; midpoint of AB = ((1 – 1) ÷ 2, (7 – 1) ÷ 2) = (0, 3) = F.



Triangle ABC and the triangle DEF of midpoints

A = (1, 7), B = (–1, –1), C = (11, 3). (If the midpoints are assigned to the sides differently, the same three points are obtained, only named in a different order.)

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QUESTION 14

A city has two main roads which cross each other at the centre of the city. These two roads are along the North–South (N–S) direction and East–West (E–W) direction. All the other streets of the city run parallel to these roads and are 200 m apart. There are 10 streets in each direction.

(i) Using 1 cm = 200 m, draw a model of the city in your notebook. Represent the roads/streets by single lines.

(ii) There are street intersections in the model. Each street intersection is formed by two streets — one running in the N–S direction and another in the E–W direction. Each street intersection is referred to in the following manner: If the second street running in the N–S direction and 5th street in the E–W direction meet at some crossing, then we call this street intersection (2, 5). Using this convention, find:

(a) how many street intersections can be referred to as (4, 3).

(b) how many street intersections can be referred to as (3, 4).

ANSWER

(i) Model of the city

Since the streets are 200 m apart and 1 cm = 200 m, neighbouring streets are 1 cm apart in the model. Draw 10 vertical lines (the N–S streets) and 10 horizontal lines (the E–W streets), each 1 cm from the next, so that the 10 lines in each direction span 9 cm. The two main roads are two of these lines and they cross at the centre of the model. Number the N–S streets 1, 2, …, 10 from the west and the E–W streets 1, 2, …, 10 from the south.

(ii) Street intersections

The intersection (a, b) is formed by exactly one N–S street (the a-th) and exactly one E–W street (the b-th), which cross at exactly one point.

(a) (4, 3): the 4th N–S street meets the 3rd E–W street at only one crossing → 1 intersection
(b) (3, 4): the 3rd N–S street meets the 4th E–W street at only one crossing → 1 intersection

Exactly one street intersection is referred to as (4, 3) and exactly one as (3, 4). These are different intersections, so the order of the numbers matters: (4, 3) ≠ (3, 4). In all there are 10 × 10 = 100 intersections.

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QUESTION 15

A computer graphics program displays images on a rectangular screen whose coordinate system has the origin at the bottom-left corner. The screen is 800 pixels wide and 600 pixels high. A circular icon of radius 80 pixels is drawn with its centre at the point A (100, 150). Another circular icon of radius 100 pixels is drawn with its centre at the point B (250, 230). Determine:

(i) whether any part of either circle lies outside the screen.

(ii) whether the two circles intersect each other.

ANSWER

(i) Is any part outside the screen?

The screen is the region $0 \le x \le 800$ and $0 \le y \le 600$. A circle with centre (h, k) and radius r extends from h – r to h + r in x and from k – r to k + r in y.

Circle A: x from 100 – 80 = 20 to 100 + 80 = 180; y from 150 – 80 = 70 to 150 + 80 = 230

Circle B: x from 250 – 100 = 150 to 250 + 100 = 350; y from 230 – 100 = 130 to 230 + 100 = 330

All these values lie within 0 to 800 (for x) and 0 to 600 (for y), so no part of either circle lies outside the screen.

(ii) Do the circles intersect?

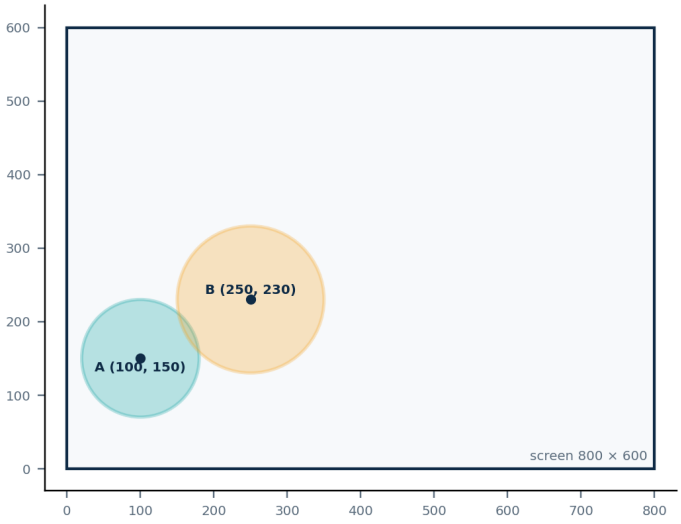
$AB = \sqrt{((250 - 100)^2 + (230 - 150)^2)} = \sqrt{(150^2 + 80^2)} = \sqrt{(22500 + 6400)} = \sqrt{28900} = 170$

Sum of radii = 80 + 100 = 180

Difference of radii = 100 – 80 = 20

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Two circles cut each other at two points when the distance between their centres is less than the sum of the radii and greater than the difference of the radii. Here 20 < 170 < 180.



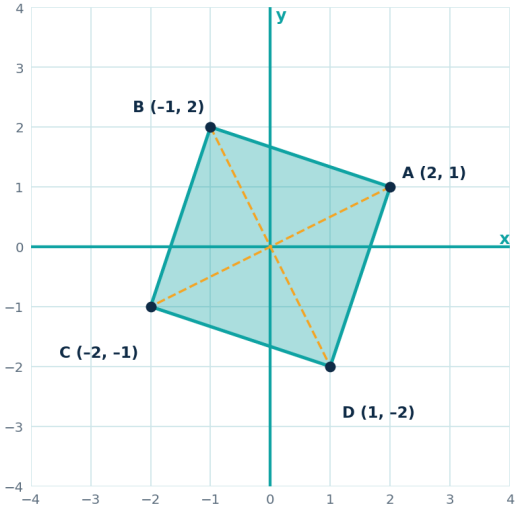
The screen and the two circular icons

Yes, the two circles intersect each other (at two points), because AB = 170 is less than 80 + 100 = 180.

QUESTION 16

Plot the points A (2, 1), B (–1, 2), C (–2, –1), and D (1, –2) in the coordinate plane. Is ABCD a square? Can you explain why? What is the area of this square?

ANSWER



Quadrilateral ABCD

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Find the lengths of the sides and the diagonals:

$AB = \sqrt{((-1-2)^2 + (2-1)^2)} = \sqrt{(9+1)} = \sqrt{10}$
$BC = \sqrt{((-2-(-1))^2 + (-1-2)^2)} = \sqrt{(1+9)} = \sqrt{10}$
$CD = \sqrt{((1-(-2))^2 + (-2-(-1))^2)} = \sqrt{(9+1)} = \sqrt{10}$
$DA = \sqrt{((2-1)^2 + (1-(-2))^2)} = \sqrt{(1+9)} = \sqrt{10}$
$AC = \sqrt{((-2-2)^2 + (-1-1)^2)} = \sqrt{(16+4)} = \sqrt{20}$
$BD = \sqrt{((1-(-1))^2 + (-2-2)^2)} = \sqrt{(4+16)} = \sqrt{20}$

All four sides are equal, so ABCD is a rhombus. The diagonals are equal as well ($AC = BD = \sqrt{20}$). Also, in triangle ABC, $AB^2 + BC^2 = 10 + 10 = 20 = AC^2$, so by the Baudhāyana–Pythagoras theorem $\angle B = 90°$. A rhombus with a right angle is a square.

$\text{Area} = \text{side}^2 = (\sqrt{10})^2 = 10$ square units

Check using the diagonals: area = ½ × AC × BD = ½ × √20 × √20 = 10 square units.

Yes, ABCD is a square (equal sides and equal diagonals), and its area is 10 square units.

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IT

CLASS 9 MATHS · GANITA MANJARI · CHAPTER 1

In-Text Questions

Solutions · 6 questions

FIG. 1.1 · WINDOWS

Let us examine Fig. 1.1 to understand the layout of the room. Notice that this only shows the map of the floor. Do you see why the position of the windows cannot be marked on this map?

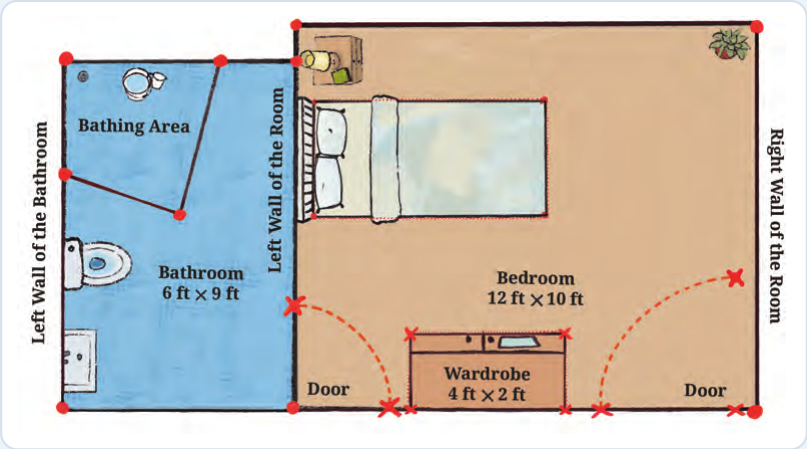


Fig. 1.1: Sketch of Reiaan's room

ANSWER

The sketch is a map of the floor, i.e. a view from above in which only two measurements are shown: how far a point is along the room and how far across it. Windows are fixed in the walls, and their position is mainly a matter of height above the floor (where the window starts and ends on the wall). The height is a third direction that a flat, 2-D floor map does not show.

At best, a window would appear as a gap in a wall line, and even then only one of its two dimensions (its length along the wall) would be seen, not its height. Thus the exact position of a window cannot be marked on this map.

> **A floor map shows only length and breadth (2-D). The position of a window needs its height above the floor as well, and a 2-D map cannot show height.**

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THINK AND REFLECT 1.1

1. What are the standard widths for a room door? Look around your home and in school.

2. Are the doors in your school suitable for people in wheelchairs?

ANSWER

1. Standard widths of doors

The answer depends on what you measure at home and in school; measure the clear opening of the doors with a tape. Typical values are: bathroom doors about 2.5 ft (75 cm), ordinary room doors about 2.5–3 ft (75–90 cm), and main doors about 3–3.5 ft (90–105 cm). In Fig. 1.3 the room door is 3.5 ft wide and the bathroom door is 2.5 ft.

2. Doors and wheelchairs

A wheelchair is about 24–26 inches (60–65 cm) wide and needs a clear opening of at least about 32 inches (about 80 cm; a margin up to 90 cm or 3 ft is better). So the doors in your school are suitable if their clear opening is at least this wide, there is no step or high threshold at the door, and there is free space in front of the door to turn the chair. Narrow bathroom-type doors (2.5 ft ≈ 75 cm) are generally too narrow.

Answers will differ from school to school: compare your measured widths with the width needed for a wheelchair (about 80–90 cm).

ACTIVITY · FIG. 1.4

Copy Fig. 1.4 and mark S and Q in your diagram. Mark any point P in Quadrant I and any point R in Quadrant III, and write down their coordinates.

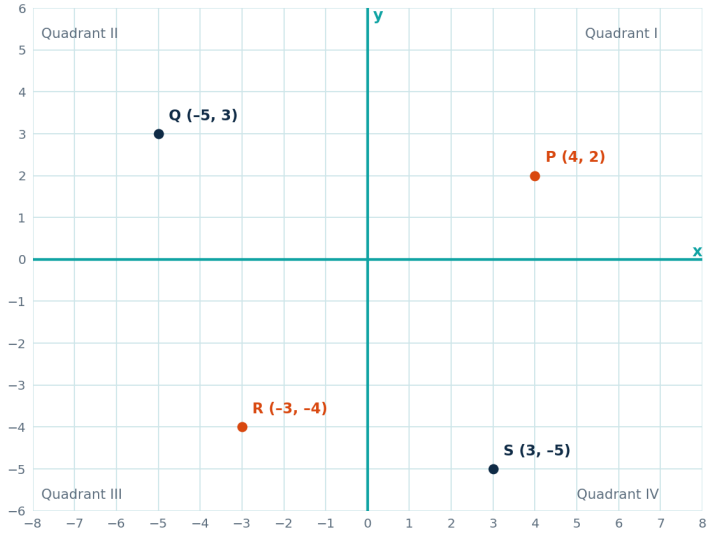
ANSWER

Draw the x-axis and the y-axis on a graph sheet. S (3, –5) is 3 units to the right of the y-axis and 5 units below the x-axis (Quadrant IV). Q (–5, 3) is 5 units to the left of the y-axis and 3 units above the x-axis (Quadrant II). For P take any point with both coordinates positive and for R any point with both coordinates negative, for example:

P = (4, 2) in Quadrant I (4 units to the right, 2 units up)

R = (–3, –4) in Quadrant III (3 units to the left, 4 units down)

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Points S, Q, P and R

P (4, 2) lies in Quadrant I and R (–3, –4) lies in Quadrant III. (Any point with x > 0, y > 0 will do for P, and any point with x < 0, y < 0 for R.)

THINK AND REFLECT 1.3

1. What is the x-coordinate of a point on the y-axis?

2. Is there a similar generalisation for a point on the x-axis?

3. Does point Q (*y*, *x*) ever coincide with point P (*x*, *y*)? Justify your answer.

4. If $x \neq y$, then $(x, y) \neq (y, x)$; and $(x, y) = (y, x)$ if and only if $x = y$. Is this claim true?

ANSWER

1. x-coordinate of a point on the y-axis

The x-coordinate of a point is its distance from the y-axis. A point on the y-axis is at distance 0 from it, so its x-coordinate is 0: the points are of the form (0, y).

2. A similar generalisation for the x-axis

Yes. The y-coordinate is the distance from the x-axis, which is 0 for a point on the x-axis. So every point on the x-axis has y-coordinate 0, i.e. it is of the form (x, 0).

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3. Can Q (y, x) coincide with P (x, y)?

Two points coincide only if their x-coordinates are equal and their y-coordinates are equal. For Q (y, x) and P (x, y) this means y = x and x = y, which both say x = y. So they coincide when x = y (for example, P (3, 3) and Q (3, 3)), and not otherwise. For x ≠ y, P and Q are different points (for example, P (2, 5) and Q (5, 2)).

Q coincides with P exactly when x = y, i.e. when the point lies on the line through the origin that bisects the first and third quadrants.

4. Is the claim true?

Yes. (x, y) = (y, x) means that the first coordinates are equal (x = y) and the second coordinates are equal (y = x); both conditions say x = y. So (x, y) = (y, x) if and only if x = y. Conversely, if x ≠ y, the first coordinates are different, so (x, y) ≠ (y, x). Thus the ordering of the coordinates matters.

The claim is true.

THINK AND REFLECT 1.4

1. In moving from A (3, 4) to D (7, 1), what distance has been covered along the x-axis? What about the distance along the y-axis?

2. Can these distances help you find the distance AD?

ANSWER

1. Distances along the axes

Along the x-axis: 7 – 3 = 4 units (to the right)
Along the y-axis: 4 – 1 = 3 units (downwards)

2. Finding the distance AD

Yes. Let C = (3, 1). Then AC is vertical (length 3) and CD is horizontal (length 4), so triangle ACD is right-angled at C with AD as its hypotenuse. By the Baudhāyana–Pythagoras theorem:

$AD^2 = CD^2 + AC^2 = 4^2 + 3^2 = 16 + 9 = 25$
$AD = 5$ units

The shifts 4 (along the x-axis) and 3 (along the y-axis) are the legs of a right triangle, so AD = √(4² + 3²) = 5 units.

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THINK AND REFLECT 1.4 · REFLECTION

1. What has remained the same and what has changed with this reflection?

2. Would these observations be the same if ΔADM is reflected in the x-axis (instead of the y-axis)?

ANSWER

1. Reflection in the y-axis

A (3, 4), M (9, 6), D (7, 1) are reflected to A′ (–3, 4), M′ (–9, 6), D′ (–7, 1).

Same: the y-coordinates of all points; the lengths of the sides (AD = 5, DM = √29, MA = √40 as before); hence the size and shape of the triangle; the distances of the points from the y-axis

Changed: the sign of every x-coordinate (3 → –3, 9 → –9, 7 → –7); the position of the triangle (Quadrant I → Quadrant II); its orientation (it becomes a mirror image, so the vertices A, M, D are now arranged in the opposite sense)

2. Reflection in the x-axis

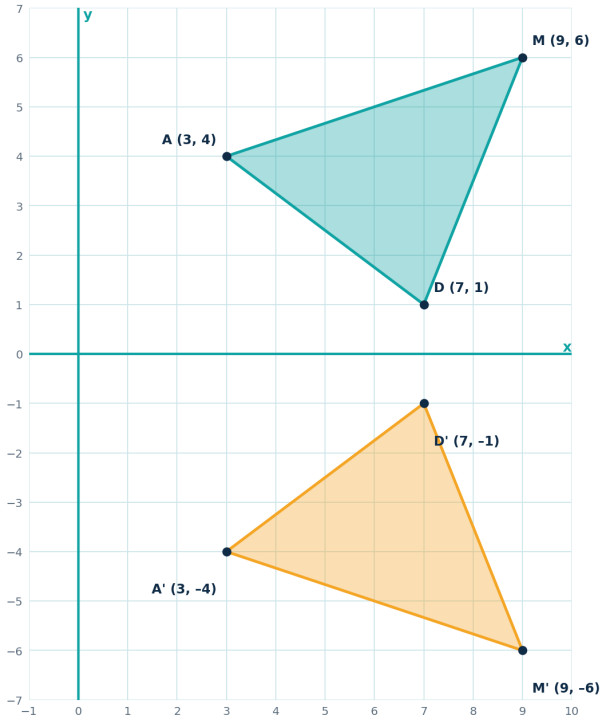
Now (x, y) goes to (x, –y): A′ (3, –4), D′ (7, –1), M′ (9, –6). The triangle moves to Quadrant IV. The lengths of the sides are again unchanged:

$A'D' = \sqrt{((7 - 3)^2 + (-1 - (-4))^2)} = \sqrt{(16 + 9)} = 5$

$D'M' = \sqrt{((9 - 7)^2 + (-6 - (-1))^2)} = \sqrt{(4 + 25)} = \sqrt{29}$

$M'A' = \sqrt{((3 - 9)^2 + (-4 - (-6))^2)} = \sqrt{(36 + 4)} = \sqrt{40}$

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ΔADM (teal) and its reflection in the x-axis (amber)

The observations are similar, except for the roles of the coordinates: now the x-coordinates stay the same and the y-coordinates change sign. The side lengths (and hence the shape and size) are preserved in both reflections.

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