NCERT Archive

SOLUTIONS BOOK

CLASS 8

Maths · Ganita Prakash

CHAPTER 6

We Distribute, Yet Things Multiply

Complete Questions, Activities and Figure it Out Solutions

6.1 Multiplication
6.2 Special Cases
6.3 Mind the Mistake
6.4 This Way or That Way
6.5 Coin Conjoin

Contents

Class 8 Maths · Ganita Prakash · Chapter 6 — We Distribute, Yet Things Multiply

6.1 Some Properties of Multiplication — 3
22 questions · Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9 Q10 Q11 Q12 Q13 Q14 Q15 Q16 Q17 Q18 Q19 Q20 Q21 Q22

6.2 Special Cases of the Distributive Property — 19
21 questions · Q23 Q24 Q25 Q26 Q27 Q28 Q29 Q30 Q31 Q32 Q33 Q34 Q35 Q36 Q37 Q38 Q39 Q40 Q41 Q42 Q43

6.3 Mind the Mistake, Mend the Mistake — 33
1 question · Q44

6.4 This Way or That Way, All Ways Lead to the Bay — 35
20 questions · Q45 Q46 Q47 Q48 Q49 Q50 Q51 Q52 Q53 Q54 Q55 Q56 Q57 Q58 Q59 Q60 Q61 Q62 Q63 Q64

6.5 Coin Conjoin — 54
1 question · Q65

Every question is followed by its answer. Question numbers above are clickable and the PDF bookmarks panel lets you jump to any exercise set or question. Figures are reproduced from the NCERT textbook (© NCERT) for educational use.

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6.1

CLASS 8 MATHS · CHAPTER 6

Some Properties of Multiplication

Solutions · 22 questions

Directions (Questions 1–22): These are the questions, examples and activities that appear inside the chapter (Section 6.1, pp. 136–144), including the Figure it Out exercises, Math Talk and other in-text questions, numbered in the order in which they appear. Original question numbers and parts (a, b, c, …) are retained. Open-ended questions have model answers based on the chapter.

QUESTION 1 *Increments in Products · p. 136*

Consider the multiplication of two numbers, say, 23 × 27. 1. By how much does the product increase if the first number (23) is increased by 1? 2. What if the second number (27) is increased by 1? 3. How about when both numbers are increased by 1? Do you see a pattern that could help generalise our observations to the product of any two numbers?

ANSWER

First find the original product: 23 × 27 = 621. Then compute each new product and subtract.

a The three cases

Change	New product	Increase	With a = 23, b = 27
1. 23 → 24	24 × 27 = 648	648 − 621 = **27**	b
2. 27 → 28	23 × 28 = 644	644 − 621 = **23**	a
3. both increased by 1	24 × 28 = 672	672 − 621 = **51**	a + b + 1 = 23 + 27 + 1

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b **Pattern**

For two numbers a and b: increasing a by 1 increases ab by b; increasing b by 1 increases ab by a; increasing both by 1 increases ab by a + b + 1.

Therefore, the product increases by 27, 23 and 51; in general by b, a and a + b + 1 respectively.

QUESTION 2 *Try it · p. 138*

What would we get if we had expanded (a + 1) (b + 1) by first taking (b + 1) as a single term? Try it?

ANSWER

Treat (b + 1) as one term and use the distributive property (a + 1) c = ac + 1 × c with c = b + 1.

a **Expansion**

$(a + 1)(b + 1) = a(b + 1) + 1(b + 1)$
$= ab + a + b + 1$

Therefore, we again get ab + a + b + 1; the product increases by a + b + 1, the same as before.

QUESTION 3 *In-text question · p. 138*

Will the product always increase? Find 3 examples where the product decreases. (a is increased by 1 and b is decreased by 1, so the new product is (a + 1)(b – 1).)

ANSWER

We have $(a + 1)(b - 1) = ab + b - a - 1$, so the change in the product is $b - a - 1$.

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a When does the product decrease?

The change $b - a - 1$ is negative when $b - a - 1 < 0$, i.e. when b is not larger than a ($b \le a$). So the product does **not** always increase.

b Three examples

a, b	ab	(a + 1)(b − 1)	Change
a = 10, b = 3	30	11 × 2 = 22	−8 = 3 − 10 − 1
a = 7, b = 5	35	8 × 4 = 32	−3 = 5 − 7 − 1
a = 23, b = 20	460	24 × 19 = 456	−4 = 20 − 23 − 1

Therefore, the product does not always increase; for example 10 × 3, 7 × 5 and 23 × 20 all decrease when the first number is raised by 1 and the second lowered by 1.

QUESTION 4 *In-text question · p. 138*

What happens when a and b are negative integers? Check by substituting different values for a and b in each of the above cases. For example, a = –5, b = 8; a = –4, b = –5; etc.

ANSWER

Substitute the values in the three results: $a(b + 1) = ab + a$; $(a + 1)(b + 1) = ab + a + b + 1$; $(a + 1)(b - 1) = ab + b - a - 1$.

a Checks

Case	a = −5, b = 8 (ab = −40)	a = −4, b = −5 (ab = 20)
a(b + 1)	−5 × 9 = −45 = −40 + (−5) ✓	−4 × (−4) = 16 = 20 + (−4) ✓
(a + 1)(b + 1)	(−4)(9) = −36 = −40 − 5 + 8 + 1 ✓	(−3)(−4) = 12 = 20 − 4 − 5 + 1 ✓
(a + 1)(b − 1)	(−4)(7) = −28 = −40 + 8 + 5 − 1 ✓	(−3)(−6) = 18 = 20 − 5 + 4 − 1 ✓

Therefore, the same expressions for the increase hold when a and b are negative integers, because integers also satisfy the distributive property.

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QUESTION 5 *In-text question · p. 140*

Can you see how Identity 1, (a + m)(b + n) = ab + mb + an + mn, can be used when one or both numbers are decreased?

ANSWER

A decrease is an increase by a negative number, so m or n can be negative.

a Idea

- Decreasing a by 2 means a + m with m = −2; decreasing b by 4 means b + n with n = −4.
- Substitute the negative values in Identity 1 and use the rules of integer multiplication for mb, an and mn.

Example (a decreased by 2, b unchanged, n = 0): (a − 2)b = ab + (−2)b = ab − 2b

Therefore, Identity 1 works for decreases by taking m and/or n to be negative numbers.

QUESTION 6 *In-text question · p. 140*

Use Identity 1 to find how the product changes when (i) one number is decreased by 2 and the other increased by 3; (ii) both numbers are decreased, one by 3 and the other by 4. Verify the answers by finding the products without converting the subtractions to additions.

ANSWER

In Identity 1 the increase in the product is an + bm + mn.

a Using Identity 1

(i) m = −2, n = 3: increase = a(3) + b(−2) + (−2)(3) = 3a − 2b − 6
(ii) m = −3, n = −4: increase = a(−4) + b(−3) + (−3)(−4) = −4a − 3b + 12

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b Verification by direct expansion

(i) $(a - 2)(b + 3) = ab + 3a - 2b - 6$
(ii) $(a - 3)(b - 4) = ab - 4a - 3b + 12$

Numerical check with a = 23, b = 27 (ab = 621): (i) 21 × 30 = 630, increase 9 = 69 − 54 − 6; (ii) 20 × 23 = 460, change −161 = −92 − 81 + 12.

Therefore, the product changes by 3a − 2b − 6 in (i) and by −4a − 3b + 12 in (ii).

QUESTION 7 *In-text question · p. 140*

Check that (a + u)(b – v) = ab + ub – av – uv is the same as taking m = u and n = –v in Identity 1.

ANSWER

Put m = u and n = −v in $(a + m)(b + n) = ab + mb + an + mn$.

a Substitution

$ab + (u)b + a(-v) + (u)(-v) = ab + ub - av - uv$
which equals $(a + u)(b - v) = (a + u)b - (a + u)v = ab + ub - av - uv$

Therefore, both methods give ab + ub − av − uv, so they agree.

QUESTION 8 *In-text question · p. 140*

Expand (i) (a – u)(b + v), (ii) (a – u)(b – v).

ANSWER

Take m = −u in Identity 1 (with n = v and n = −v) or use the distributive property.

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a Expansions

(i) (a − u)(b + v) = (a − u)b + (a − u)v = ab − ub + av − uv
(ii) (a − u)(b − v) = (a − u)b − (a − u)v = ab − ub − av + uv

Therefore, (a − u)(b + v) = ab − ub + av − uv and (a − u)(b − v) = ab − ub − av + uv.

QUESTION 9 *Example 1 · p. 140*

Expand (3a/2)(a – b + 1/5).

ANSWER

The distributive property works for more than two terms in a bracket: multiply each term of the bracket by 3a/2.

a Multiplying term by term

(3a/2) × a = (3/2)(a × a) = (3/2) a^2
(3a/2) × b = (3/2) ab
(3a/2) × (1/5) = (3/2 × 1/5) a = (3/10) a

So (3a/2)(a − b + 1/5) = (3/2)a^2 − (3/2)ab + (3/10)a.

b Can any two terms be added?

No. (3/2)a^2, (3/2)ab and (3/10)a have different letter-numbers, so they are not like terms and cannot be combined.

Therefore, (3a/2)(a − b + 1/5) = (3/2)a^2 − (3/2)ab + (3/10)a, which cannot be simplified further.

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QUESTION 10 *Example 2 · p. 141*

Expand (a + b)(a + b).

ANSWER

Apply the distributive property, taking (a + b) as a single term.

a Expansion

$(a + b)(a + b) = (a + b)a + (a + b)b$
$= a^2 + ba + ab + b^2$
$ba = ab$, so $ba + ab = 2ab$ (like terms)

Therefore, $(a + b)(a + b) = a^2 + 2ab + b^2$.

QUESTION 11 *Example 3 · p. 141*

Expand $(a + b)(a^2 + 2ab + b^2)$.

ANSWER

Multiply (a + b) with each term of the second bracket, then collect like terms.

a Expansion

$(a + b)(a^2 + 2ab + b^2) = (a + b)a^2 + (a + b)(2ab) + (a + b)b^2$
$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$

b Collecting like terms

- $a \times a^2 = a \times (a \times a) = a^3$ (three factors of a); $b \times b^2 = b^3$.
- $a^2b + 2a^2b = 3a^2b$ and $ab^2 + 2ab^2 = 3ab^2$.

Therefore, $(a + b)(a^2 + 2ab + b^2) = a^3 + 3a^2b + 3ab^2 + b^3$.

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QUESTION 12 *Figure it Out · Q.1 · p. 142*

Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3 × 3 frame is given by the expression pq, as shown in the figure, write the expressions for the other numbers in the grid.

ANSWER

In the frame shown (3 × 5, 3 × 6, 3 × 7 / 4 × 5, 4 × 6, 4 × 7 / 5 × 5, 5 × 6, 5 × 7) the middle number is 4 × 6 = pq, so p = 4 and q = 6. The rows are p − 1, p, p + 1 and the columns are q − 1, q, q + 1.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

3 × 5	3 × 6	3 × 7
4 × 5	4 × 6	4 × 7
5 × 5	5 × 6	5 × 7

	pq	

Multiplication grid and 3 × 3 frame (NCERT figure, p. 142)

a Expressions in the frame

	column q − 1	column q	column q + 1
row p − 1	(p − 1)(q − 1)	(p − 1)q	(p − 1)(q + 1)
row p	p(q − 1)	**pq**	p(q + 1)
row p + 1	(p + 1)(q − 1)	(p + 1)q	(p + 1)(q + 1)

Therefore, the frame is (p−1)(q−1), (p−1)q, (p−1)(q+1) / p(q−1), pq, p(q+1) / (p+1)(q−1), (p+1)q, (p+1)(q+1).

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QUESTION 13 *Figure it Out · Q.2 · p. 143*

Expand the following products. (i) (3 + u)(v – 3) (ii) (2/3)(15 + 6a) (iii) (10a + b)(10c + d) (iv) (3 – x)(x – 6) (v) (–5a + b)(c + d) (vi) (5 + z)(y + 9)

ANSWER

Multiply every term of the first bracket by every term of the second bracket and collect like terms.

a Expansions

Product	Working	Result
(i) (3 + u)(v − 3)	3v − 9 + uv − 3u	**3v − 3u + uv − 9**
(ii) (2/3)(15 + 6a)	(2/3)(15) + (2/3)(6a) = 10 + 4a	**10 + 4a**
(iii) (10a + b)(10c + d)	100ac + 10ad + 10bc + bd	**100ac + 10ad + 10bc + bd**
(iv) (3 − x)(x − 6)	3x − 18 − x² + 6x	**−x² + 9x − 18**
(v) (−5a + b)(c + d)	−5ac − 5ad + bc + bd	**−5ac − 5ad + bc + bd**
(vi) (5 + z)(y + 9)	5y + 45 + yz + 9z	**5y + yz + 9z + 45**

Therefore, the products are as listed in the Result column.

QUESTION 14 *Figure it Out · Q.3 · p. 143*

Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.

ANSWER

Let the numbers be a and b. We need (a + 2)(b − 4) = ab.

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a Condition

$(a + 2)(b - 4) = ab - 4a + 2b - 8$

$ab - 4a + 2b - 8 = ab$ gives $2b = 4a + 8$, i.e. $b = 2a + 4$

b Three examples

a	b = 2a + 4	ab	(a + 2)(b − 4)
1	6	6	3 × 2 = 6
2	8	16	4 × 4 = 16
3	10	30	5 × 6 = 30

Therefore, the pairs (1, 6), (2, 8) and (3, 10) leave the product unchanged; any pair with b = 2a + 4 works.

QUESTION 15 *Figure it Out · Q.4 · p. 143*

Expand (i) $(a + ab - 3b^2)(4 + b)$, and (ii) $(4y + 7)(y + 11z - 3)$.

ANSWER

Multiply each term of the second bracket by each term of the first.

a (i)

$(a + ab - 3b^2)(4 + b) = 4(a + ab - 3b^2) + b(a + ab - 3b^2)$

$= 4a + 4ab - 12b^2 + ab + ab^2 - 3b^3$

$= 4a + 5ab + ab^2 - 12b^2 - 3b^3$

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b **(ii)**

$(4y + 7)(y + 11z - 3) = 4y(y + 11z - 3) + 7(y + 11z - 3)$
$= 4y^2 + 44yz - 12y + 7y + 77z - 21$
$= 4y^2 + 44yz - 5y + 77z - 21$

Therefore, (i) $4a + 5ab + ab^2 - 12b^2 - 3b^3$ and (ii) $4y^2 + 44yz - 5y + 77z - 21$.

QUESTION 16 *Figure it Out · Q.5 · p. 143*

Expand (i) $(a - b)(a + b)$, (ii) $(a - b)(a^2 + ab + b^2)$ and (iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$. Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?

ANSWER

In each product all the middle terms cancel in pairs.

a **Expansions**

(i) $(a - b)(a + b) = a^2 + ab - ab - b^2 = a^2 - b^2$
(ii) $(a - b)(a^2 + ab + b^2) = a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 = a^3 - b^3$
(iii) $(a - b)(a^3 + a^2b + ab^2 + b^3) = a^4 - b^4$

b **Pattern and next identity**

$(a - b)$ multiplied by a sum of all terms $a^{n-1}, a^{n-2}b, \ldots, b^{n-1}$ gives $a^n - b^n$. The next identity is $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - b^5$.

Check: $a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 - a^4b - a^3b^2 - a^2b^3 - ab^4 - b^5 = a^5 - b^5$

Therefore, the pattern is $(a - b)(a^{n-1} + a^{n-2}b + \ldots + b^{n-1}) = a^n - b^n$; the next identity is $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - b^5$.

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QUESTION 17 *In-text question · p. 143*

Use the following multiplications to find the product of a number with 11 in a single step. (a) 3874 × 11 (b) 5678 × 11

ANSWER

Since 11 = 10 + 1, a number × 11 = (number × 10) + number. Adding digit by digit, each digit of the answer is the sum of two neighbouring digits (with carries).

a (a) 3874 × 11

Units digit: 4
Tens: 7 + 4 = 11 → write 1, carry 1
Hundreds: 8 + 7 + 1 = 16 → write 6, carry 1
Thousands: 3 + 8 + 1 = 12 → write 2, carry 1
Ten-thousands: 3 + 1 = 4
So 3874 × 11 = 42614 (check: 38740 + 3874 = 42614)

b (b) 5678 × 11

Units digit: 8
Tens: 7 + 8 = 15 → 5, carry 1
Hundreds: 6 + 7 + 1 = 14 → 4, carry 1
Thousands: 5 + 6 + 1 = 12 → 2, carry 1
Ten-thousands: 5 + 1 = 6
So 5678 × 11 = 62458 (check: 56780 + 5678 = 62458)

Therefore, 3874 × 11 = 42614 and 5678 × 11 = 62458.

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QUESTION 18 *Math Talk · p. 144*

Describe a general rule to multiply a number (of any number of digits) by 11 and write the product in one line.

ANSWER

For dcba × 11 = dcba × 10 + dcba, add the digits of dcba0 and dcba column by column.

a The rule

- Write the last digit of the number as the last digit of the answer.
- Moving to the left, each digit of the answer is the sum of a digit of the number and the digit to its right (the digit just added), plus any carry.
- Finally write the first digit of the number (plus any carry) as the first digit of the answer.

dcba × 11 → d, (d + c), (c + b), (b + a), a (then carry over where a sum exceeds 9)

Therefore, to multiply by 11, keep the end digits and put the sum of every pair of neighbouring digits in between, carrying where needed.

QUESTION 19 *Math Talk · p. 144*

Evaluate (i) 94 × 11, (ii) 495 × 11, (iii) 3279 × 11, (iv) 4791256 × 11.

ANSWER

Apply the rule for 11 and check by number × 10 + number.

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a Products

Product	Digit-by-digit working (right to left)	Answer
(i) 94 × 11	4; 9 + 4 = 13 → 3 c1; 9 + 1 = 10	**1034**
(ii) 495 × 11	5; 9 + 5 = 14 → 4 c1; 4 + 9 + 1 = 14 → 4 c1; 4 + 1 = 5	**5445**
(iii) 3279 × 11	9; 7 + 9 = 16 → 6 c1; 2 + 7 + 1 = 10 → 0 c1; 3 + 2 + 1 = 6; 3	**36069**
(iv) 4791256 × 11	6; 11 → 1 c1; 8; 3; 10 → 0 c1; 17 → 7 c1; 12 → 2 c1; 4 + 1 = 5	**52703816**

Therefore, the products are 1034, 5445, 36069 and 52703816.

QUESTION 20 *Math Talk · p. 144*

Can we come up with a similar rule for multiplying a number by 101? Multiply 3874 by 101.

ANSWER

Since 101 = 100 + 1, 3874 × 101 = 3874 × 100 + 3874 = 387400 + 3874.

a Calculation

387400
+ 3874
= 391274

Digit-wise (as dcba × 101): d, c, (b + d), (a + c), b, a = 3, 8, (7 + 3), (4 + 8), 7, 4 = 3, 8, 10, 12, 7, 4. Carrying gives 391274.

Therefore, 3874 × 101 = 391274.

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QUESTION 21 *Math Talk · p. 144*

What could be a general rule to multiply a number by 101 and write the product in one line? Extend this rule for multiplication by 1001, 10001, …

ANSWER

dcba × 101 = dcba00 + dcba; dcba × 1001 = dcba000 + dcba; dcba × 10001 = dcba0000 + dcba.

a The rules

- **101:** add the number to itself shifted two places. The last two digits stay as they are; every other digit is the sum of a digit and the digit two places to its right (with carries). For a 4-digit number: d, c, (b + d), (a + c), b, a.
- **1001:** shift three places: d, c, b, (a + d), c, b, a … with digits three places apart added (carry as needed); the last three digits stay.
- **10001** and 4-digit numbers: shifting four places does not overlap, so the number is simply written twice: dcba × 10001 = dcbadcba.
- In general, a number × $(10^k + 1)$ is the number added to itself shifted k places.

Therefore, multiplying by $10^k + 1$ means adding the number to a copy of itself shifted k places, adding digits that are k places apart.

QUESTION 22 *Math Talk · p. 144*

Use this to find (i) 89 × 101, (ii) 949 × 101, (iii) 265831 × 1001, (iv) 1111 × 1001, (v) 9734 × 99 and (vi) 23478 × 999.

ANSWER

Use ×(100 + 1), ×(1000 + 1), and ×99 = ×(100 − 1), ×999 = ×(1000 − 1).

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a Products

Product	Working	Answer
(i) 89 × 101	8900 + 89	**8989**
(ii) 949 × 101	94900 + 949	**95849**
(iii) 265831 × 1001	265831000 + 265831	**266096831**
(iv) 1111 × 1001	1111000 + 1111	**1112111**
(v) 9734 × 99	9734(100 − 1) = 973400 − 9734	**963666**
(vi) 23478 × 999	23478(1000 − 1) = 23478000 − 23478	**23454522**

Therefore, the products are 8989, 95849, 266096831, 1112111, 963666 and 23454522.

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6.2

CLASS 8 MATHS · CHAPTER 6

Special Cases of the Distributive Property

Solutions · 21 questions

Directions (Questions 23–43): These are the questions, examples and activities that appear inside the chapter (Section 6.2, pp. 145–149), including the Figure it Out exercises, Math Talk and other in-text questions, numbered in the order in which they appear. Original question numbers and parts (a, b, c, …) are retained. Open-ended questions have model answers based on the chapter.

QUESTION 23

In-text question · p. 145

The area of a square of sidelength 60 units is 3600 sq. units (60^2) and that of a square of sidelength 5 units is 25 sq. units (5^2). Can we use this to find the area of a square of sidelength 65 units? A square of sidelength 65 can be split into 4 regions: a square of sidelength 60, a square of sidelength 5, and two rectangles of sidelengths 60 and 5. Can you find the areas of the four parts?

ANSWER

The area of the big square is the sum of the areas of its four parts.

a Areas of the four parts

Part	Area
Square of side 60	$60^2 = 3600$
Square of side 5	$5^2 = 25$
Rectangle 60 × 5	300
Rectangle 5 × 60	300

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$65^2 = (60 + 5)^2 = 60^2 + 5^2 + 2 \times (60 \times 5) = 3600 + 25 + 600 = 4225$

Therefore, the four parts have areas 3600, 25, 300 and 300, and the square of side 65 has area 4225 sq. units.

QUESTION 24 *In-text question · p. 145*

What if we write 65^2 as $(30 + 35)^2$ or $(52 + 13)^2$? Draw the figures and check the area that you get.

ANSWER

Split a square of side 65 into a square of side 30 (or 52), a square of side 35 (or 13) and two equal rectangles.

a **$(30 + 35)^2$**

Squares: $30^2 = 900$ and $35^2 = 1225$; two rectangles $30 \times 35 = 1050$ each
Total = 900 + 1225 + 1050 + 1050 = 4225

b **$(52 + 13)^2$**

Squares: $52^2 = 2704$ and $13^2 = 169$; two rectangles $52 \times 13 = 676$ each
Total = 2704 + 169 + 676 + 676 = 4225

Therefore, both ways of splitting the square of side 65 give an area of 4225 sq. units.

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QUESTION 25 *Math Talk · p. 145*

If a and b are any two integers, is $(a + b)^2$ always greater than $a + b$? If not, when is it greater?

ANSWER

Let $s = a + b$, which is also an integer. We compare s^2 with s.

a **Comparison**

$s = a + b$	s^2	Compare
$s = 0$	0	$s^2 = s$
$s = 1$	1	$s^2 = s$
$s = 2, 3, 4, \ldots$	4, 9, 16, ...	$s^2 > s$
$s = -1, -2, \ldots$	1, 4, ...	$s^2 > s$ (positive vs negative)

So $(a + b)^2$ is not always greater: it is equal when $a + b = 0$ or $a + b = 1$, and greater for every other integer value of $a + b$. (It would be smaller only for fractions between 0 and 1, which are not integers.)

Therefore, $(a + b)^2 \geq a + b$ for integers; it is strictly greater unless $a + b$ is 0 or 1, when they are equal.

QUESTION 26 *In-text question · p. 145*

Use Identity 1A to find the values of 104^2, 37^2. (Hint: Decompose 104 and 37 into sums or differences of numbers whose squares are easy to compute.)

ANSWER

Identity 1A: $(a + b)^2 = a^2 + 2ab + b^2$.

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a Calculations

$104^2 = (100 + 4)^2 = 10000 + 2 \times 100 \times 4 + 16 = 10000 + 800 + 16 = 10816$

$37^2 = (30 + 7)^2 = 900 + 2 \times 30 \times 7 + 49 = 900 + 420 + 49 = 1369$

Therefore, $104^2 = 10816$ and $37^2 = 1369$.

QUESTION 27 *In-text question · p. 146*

Use Identity 1A to write the expressions for the following. (i) $(m + 3)^2$ (ii) $(6 + p)^2$

ANSWER

Use $(a + b)^2 = a^2 + 2ab + b^2$.

a Expressions

(i) $(m + 3)^2 = m^2 + 2 \times m \times 3 + 3^2 = m^2 + 6m + 9$

(ii) $(6 + p)^2 = 6^2 + 2 \times 6 \times p + p^2 = 36 + 12p + p^2$

Therefore, $(m + 3)^2 = m^2 + 6m + 9$ and $(6 + p)^2 = 36 + 12p + p^2$.

QUESTION 28 *Example · p. 146*

Expand $(6x + 5)^2$.

ANSWER

We can use Identity 1A or apply the distributive property directly; both give the same result.

NCERT ARCHIVE

a **Using the identity**

$(6x + 5)^2 = (6x)^2 + 2(6x \times 5) + 5^2 = 36x^2 + 60x + 25$

b **Using the distributive property**

$(6x + 5)(6x + 5) = 6x \times 6x + 5 \times 6x + 6x \times 5 + 5 \times 5 = 36x^2 + 25 + 60x = 36x^2 + 60x + 25$

Therefore, $(6x + 5)^2 = 36x^2 + 60x + 25$.

QUESTION 29 *In-text question · p. 146*

Expand $(3j + 2k)^2$ using both the identity and by applying the distributive property.

ANSWER

Take a = 3j and b = 2k in Identity 1A, and verify by multiplying term by term.

a **Using Identity 1A**

$(3j + 2k)^2 = (3j)^2 + 2(3j)(2k) + (2k)^2 = 9j^2 + 12jk + 4k^2$

b **Using the distributive property**

$(3j + 2k)(3j + 2k) = 9j^2 + 6jk + 6jk + 4k^2 = 9j^2 + 12jk + 4k^2$

Therefore, $(3j + 2k)^2 = 9j^2 + 12jk + 4k^2$.

NCERT ARCHIVE

QUESTION 30 *In-text question · p. 146*

Can we use 60^2 (= 3600) and 5^2 (= 25) to find the value of $(60 - 5)^2$ or 55^2? (The area of the square of sidelength 60 with the two rectangles of sidelengths 60 and 5 removed counts the small square of side 5 twice. What can we do with this expression to get the actual area?)

ANSWER

Draw a square of side 55 inside a square of side 60.

a Reasoning

- Start with 60^2 and remove the two 60×5 rectangles: $60^2 - (60 \times 5) - (5 \times 60)$.
- The small 5×5 square lies in both rectangles, so it has been removed twice; add it back once: $+ 5^2$.

$$(60 - 5)^2 = 60^2 - (60 \times 5) - (5 \times 60) + 5^2$$
$$= 3600 - 300 - 300 + 25 = 3025$$

Therefore, add back the area of the small square: $55^2 = 3025$ sq. units.

QUESTION 31 *In-text question · p. 147*

What is the expansion of $(a - b)^2$? (Think how the expansion of $(a + b)^2$ can be used. Hint: $(a - b)^2 = (a + (-b))^2$.)

ANSWER

We can multiply directly, or use Identity 1A with b replaced by $-b$.

a Distributive property

$$(a - b)(a - b) = a^2 - ba - ab + b^2 = a^2 - 2ab + b^2$$

NCERT ARCHIVE

b Using (a + (−b))²

$(a + (-b))^2 = a^2 + (-b)^2 + 2 \times a \times (-b) = a^2 + b^2 - 2ab$

Therefore, Identity 1B: $(a - b)^2 = a^2 - 2ab + b^2$.

QUESTION 32 *In-text question · p. 147*

Find the general expansion of $(a - b)^2$ using geometry, as we did for 55^2.

ANSWER

Draw a square of side $(a - b)$ inside a square of side a (with $a > b$).

a Geometric argument

- The big square of side a has area a^2.
- Remove the two rectangles of sides a and b along two adjacent sides: $a^2 - ab - ba$.
- The corner $b \times b$ square was removed twice, so add b^2 back once.

$(a - b)^2 = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$

Therefore, geometry also gives $(a - b)^2 = a^2 - 2ab + b^2$.

QUESTION 33 *In-text question · p. 147*

Use the identity $(a - b)^2$ to find the values of (a) 99^2 and (b) 58^2.

ANSWER

Write each number as a difference of a convenient number and a small number.

NCERT ARCHIVE

a Calculations

(a) $99^2 = (100 - 1)^2 = 10000 - 200 + 1 = 9801$
(b) $58^2 = (60 - 2)^2 = 3600 - 240 + 4 = 3364$

Therefore, $99^2 = 9801$ and $58^2 = 3364$.

QUESTION 34 *In-text question · p. 147*

Expand the following using both Identity 1B and by applying the distributive property. (i) (b – 6)² (ii) (–2a + 3)² (iii) (7y – 4z)²

ANSWER

Identity 1B: $(a - b)^2 = a^2 - 2ab + b^2$.

a Results

Expression	Using Identity 1B	Using the distributive property
(i) $(b - 6)^2$	$b^2 - 2(b)(6) + 36$	$(b - 6)(b - 6) = b^2 - 6b - 6b + 36$
(ii) $(-2a + 3)^2$	$(3 - 2a)^2 = 9 - 2(3)(2a) + 4a^2$	$(-2a + 3)(-2a + 3) = 4a^2 - 6a - 6a + 9$
(iii) $(7y - 4z)^2$	$(7y)^2 - 2(7y)(4z) + (4z)^2$	$49y^2 - 28yz - 28yz + 16z^2$

(i) $b^2 - 12b + 36$
(ii) $4a^2 - 12a + 9$
(iii) $49y^2 - 56yz + 16z^2$

Therefore, (i) $b^2 - 12b + 36$, (ii) $4a^2 - 12a + 9$, (iii) $49y^2 - 56yz + 16z^2$.

NCERT ARCHIVE

QUESTION 35 *Investigating Patterns · Pattern 1 · p. 148*

Look at the pattern $2(2^2 + 1^2) = 3^2 + 1^2$, $2(3^2 + 1^2) = 4^2 + 2^2$, $2(6^2 + 5^2) = 11^2 + 1^2$, $2(5^2 + 3^2) = 8^2 + 2^2$. Take a pair of natural numbers. Calculate the sum of their squares. Can you write twice this sum as a sum of two squares? Do the identities $(a + b)^2 = a^2 + 2ab + b^2$ and $(a – b)^2 = a^2 – 2ab + b^2$ help in explaining the observed pattern?

ANSWER

In every example twice the sum of the squares equals the square of the sum plus the square of the difference.

a Checking the examples

Pair	Twice the sum of squares	(sum)² + (difference)²
2, 1	$2(4 + 1) = 10$	$3^2 + 1^2 = 10$
3, 1	$2(9 + 1) = 20$	$4^2 + 2^2 = 20$
6, 5	$2(36 + 25) = 122$	$11^2 + 1^2 = 122$
5, 3	$2(25 + 9) = 68$	$8^2 + 2^2 = 68$

b Explanation using the identities

$(a + b)^2 + (a - b)^2 = (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2)$

$= 2a^2 + 2b^2$ (the $2ab$ and $-2ab$ cancel)

so $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$

Therefore, $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$ for any two numbers a and b.

NCERT ARCHIVE

QUESTION 36 *Investigating Patterns · Pattern 2 · p. 148*

Here is a related pattern. Try to describe the pattern using algebra to determine if the pattern always holds. 9 × 9 – 1 × 1 = 10 × 8; 8 × 8 – 6 × 6 = 14 × 2; 7 × 7 – 2 × 2 = 9 × 5; 10 × 10 – 4 × 4 = 14 × 6.

ANSWER

In each line the right side is (sum of the two numbers) × (difference of the two numbers), so the pattern appears to be $a^2 - b^2 = (a + b)(a - b)$.

a Checking the examples

Numbers	$a^2 - b^2$	$(a + b)(a - b)$
9, 1	$81 - 1 = 80$	$10 \times 8 = 80$
8, 6	$64 - 36 = 28$	$14 \times 2 = 28$
7, 2	$49 - 4 = 45$	$9 \times 5 = 45$
10, 4	$100 - 16 = 84$	$14 \times 6 = 84$

b Proof

$(a + b)(a - b) = a^2 - ab + ba - b^2 = a^2 - b^2$ (since $-ab + ba = 0$)

Therefore, Identity 1C: $(a + b)(a - b) = a^2 - b^2$ holds always.

QUESTION 37 *Try This · p. 148*

Use Identity 1C to calculate 98 × 102, and 45 × 55.

ANSWER

Write each product as $(a + b)(a - b)$ with a the number halfway between the two factors.

NCERT ARCHIVE

a Calculations

$98 \times 102 = (100 - 2)(100 + 2) = 100^2 - 2^2 = 10000 - 4 = 9996$

$45 \times 55 = (50 - 5)(50 + 5) = 50^2 - 5^2 = 2500 - 25 = 2475$

Therefore, 98 × 102 = 9996 and 45 × 55 = 2475.

QUESTION 38 *Try This · p. 148*

Show that (a + b) × (a – b) = a² – b² geometrically. (Hint: What do we get when this piece is moved?)

ANSWER

Take a square of side a (a > b) and cut out a smaller square of side b from one corner.

a Geometric argument

- The remaining L-shaped region has area $a^2 - b^2$.
- Cut the L along a line to get two rectangles: one $a \times (a - b)$ and one $b \times (a - b)$.
- Move the $b \times (a - b)$ piece to sit beside the other piece. Together they form one rectangle of length $(a + b)$ and width $(a - b)$.

Area of the rectangle = $(a + b)(a - b)$ = area of the L-shape = $a^2 - b^2$

Therefore, $(a + b)(a - b) = a^2 - b^2$ is shown by rearranging the L-shaped region into a rectangle.

NCERT ARCHIVE

QUESTION 39 *In-text question · p. 149*

Sridharacharya gave the modified form $a^2 = (a + b)(a - b) + b^2$ of Identity 1C. Why is this identity true?

ANSWER

Start from Identity 1C and move b^2 to the other side.

a Reason

$(a + b)(a - b) = a^2 - b^2$

adding b^2 to both sides: $(a + b)(a - b) + b^2 = a^2$

Example from the chapter: $31^2 = (31 + 1)(31 - 1) + 1^2 = 32 \times 30 + 1 = 961$.

Therefore, $a^2 = (a + b)(a - b) + b^2$ is just Identity 1C with b^2 added to both sides.

QUESTION 40 *Figure it Out · Q.1 · p. 149*

Which is greater: $(a - b)^2$ or $(b - a)^2$? Justify your answer.

ANSWER

Expand both expressions.

a Expansion

$(a - b)^2 = a^2 - 2ab + b^2$

$(b - a)^2 = b^2 - 2ab + a^2 = a^2 - 2ab + b^2$

Also, $b - a = -(a - b)$, and the square of a number equals the square of its negative.

Therefore, neither is greater; $(a - b)^2$ and $(b - a)^2$ are always equal.

NCERT ARCHIVE

QUESTION 41 *Figure it Out · Q.2 · p. 149*

Express 100 as the difference of two squares.

ANSWER

Use $a^2 - b^2 = (a + b)(a - b)$ and write 100 as a product of two factors.

a Method

$100 = 50 \times 2 = (a + b)(a - b)$, so $a + b = 50$, $a - b = 2$

$a = 26$, $b = 24$

Check: $26^2 - 24^2 = 676 - 576 = 100$

Another way: $100 = 10 \times 10$ gives $a = 10$, $b = 0$, i.e. $10^2 - 0^2$.

Therefore, $100 = 26^2 - 24^2$ (= 676 − 576).

QUESTION 42 *Figure it Out · Q.3 · p. 149*

Find 406^2, 72^2, 145^2, 1097^2, and 124^2 using the identities you have learnt so far.

ANSWER

Split each number into a sum or difference of numbers whose squares are easy.

NCERT ARCHIVE

a Calculations

Square	Working	Value
406^2	$(400 + 6)^2 = 160000 + 4800 + 36$	**164836**
72^2	$(70 + 2)^2 = 4900 + 280 + 4$	**5184**
145^2	$(150 - 5)^2 = 22500 - 1500 + 25$	**21025**
1097^2	$(1100 - 3)^2 = 1210000 - 6600 + 9$	**1203409**
124^2	$(120 + 4)^2 = 14400 + 960 + 16$	**15376**

Therefore, $406^2 = 164836$, $72^2 = 5184$, $145^2 = 21025$, $1097^2 = 1203409$ and $124^2 = 15376$.

QUESTION 43 *Math Talk · Q.4 · p. 149*

Do Patterns 1 and 2 hold only for counting numbers? Do they hold for negative integers as well? What about fractions? Justify your answer.

ANSWER

Both patterns were proved using only the distributive property and the rules of integer multiplication, so they are identities.

a Justification

- Pattern 1: $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$ and Pattern 2: $a^2 - b^2 = (a + b)(a - b)$ hold for any numbers a and b, because the algebra never used that a and b are counting numbers.
- Negative integers: $a = -3$, $b = 2$: $2(9 + 4) = 26$ and $(-1)^2 + (-5)^2 = 26$; $9 - 4 = 5$ and $(-1)(-5) = 5$.
- Fractions: $a = 1/2$, $b = 1/4$: $2(1/4 + 1/16) = 5/8$ and $(3/4)^2 + (1/4)^2 = 9/16 + 1/16 = 5/8$; $1/4 - 1/16 = 3/16$ and $(3/4)(1/4) = 3/16$.

Therefore, both patterns hold for all numbers: counting numbers, negative integers and fractions.

NCERT ARCHIVE

6.3

CLASS 8 MATHS · CHAPTER 6

Mind the Mistake, Mend the Mistake

Solutions · 1 question

Directions (Questions 44–44): These are the questions, examples and activities that appear inside the chapter (Section 6.3, p. 150), including the Figure it Out exercises, Math Talk and other in-text questions, numbered in the order in which they appear. Original question numbers and parts (a, b, c, …) are retained. Open-ended questions have model answers based on the chapter.

QUESTION 44 *Mind the Mistake, Mend the Mistake · p. 150*

We have expanded and simplified some algebraic expressions below to their simplest forms (cards 1 to 12). (i) Check each of the simplifications and see if there is a mistake. (ii) If there is a mistake, try to explain what could have gone wrong. (iii) Then write the correct expression.

ANSWER

Each card is checked by expanding it again correctly. Cards 5, 7, 10 and 12 are already correct.

NCERT ARCHIVE

a Check of the 12 cards

Card (as given)	Verdict	What went wrong	Correct expression
1. $-3p(-5p + 2q) = -3p + 5p - 2q = p - 2q$	Mistake	$-3p$ was not multiplied with each term; the signs were changed instead.	$15p^2 - 6pq$
2. $2(x - 1) + 3(x + 4) = 2x - 1 + 3x + 4 = 5x + 3$	Mistake	2 and 3 were multiplied with the first term only ($-1 \to -2$ and $4 \to 12$ were missed).	$5x + 10$
3. $y + 2(y + 2) = (y + 2)^2 = y^2 + 4y + 4$	Mistake	$y + 2(y + 2)$ is not $(y + 2)(y + 2)$; 2 multiplies only the bracket, and y is added.	$3y + 4$
4. $(5m + 6n)^2 = 25m^2 + 36n^2$	Mistake	The middle term $2ab = 2(5m)(6n) = 60mn$ was left out.	$25m^2 + 60mn + 36n^2$
5. $(-q + 2)^2 = q^2 - 4q + 4$	**Correct**	$(-q)^2 + 2(-q)(2) + 2^2 = q^2 - 4q + 4$.	**$q^2 - 4q + 4$** (no change)
6. $3a(2b \times 3c) = 6ab \times 9ac = 54a^2bc$	Mistake	3a was multiplied into each factor of the product $2b \times 3c$; distributivity applies to sums, not to products.	$3a \times 6bc = 18abc$
7. $(1/2)(10s - 6) + 3 = 5s - 3 + 3 = 5s$	**Correct**	$(1/2)(10s) = 5s$ and $(1/2)(6) = 3$, so $5s - 3 + 3 = 5s$.	**$5s$** (no change)
8. $5w^2 + 6w = 11w^2$	Mistake	$5w^2$ and $6w$ are unlike terms and cannot be added.	$5w^2 + 6w$ (cannot be simplified; $= w(5w + 6)$)
9. $2a^3 + 3a^3 + 6a^2b + 6ab^2 = 5a^3 + 12a^2b^2$	Mistake	Only $2a^3 + 3a^3$ are like terms; $6a^2b$ and $6ab^2$ are unlike and cannot be added or multiplied into a^2b^2.	$5a^3 + 6a^2b + 6ab^2$
10. $(x + 2)(x + 5) = (x + 2)x + (x + 2)5 = x^2 + 2x + 5x + 10 = x^2 + 7x + 10$	**Correct**	Every step follows the distributive property.	**$x^2 + 7x + 10$** (no change)
11. $(a + 2)(b + 4) = ab + 8$	Mistake	Only the first terms and the last terms were multiplied; the cross terms 4a and 2b were missed.	$ab + 4a + 2b + 8$
12. $ab^2 + a^2b + a^2b^2 = ab(a + b + ab)$	**Correct**	$ab \times b = ab^2$, $ab \times a = a^2b$, $ab \times ab = a^2b^2$ (taking out the common factor ab).	**$ab(a + b + ab)$** (no change)

Therefore, cards 1, 2, 3, 4, 6, 8, 9 and 11 contain mistakes (corrected above); cards 5, 7, 10 and 12 are correct.

NCERT ARCHIVE

6.4

CLASS 8 MATHS · CHAPTER 6

This Way or That Way, All Ways Lead to the Bay

Solutions · 20 questions

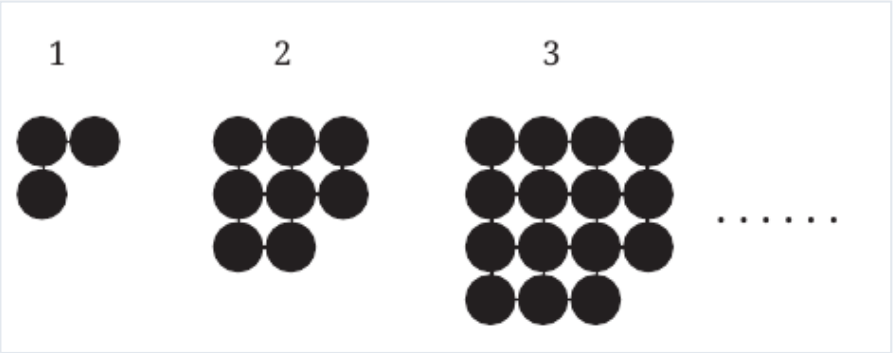
Directions (Questions 45–64): These are the questions, examples and activities that appear inside the chapter (Section 6.4, pp. 150–156), including the Figure it Out exercises, Math Talk and other in-text questions, numbered in the order in which they appear. Original question numbers and parts (a, b, c, …) are retained. Open-ended questions have model answers based on the chapter.

QUESTION 45 *Math Talk · p. 150*

Observe the pattern in the figure below. Draw the next figure in the sequence. How many circles does it have? How many total circles are there in Step 10? Write an expression for the number of circles in Step k.

ANSWER

Count the circles in the first steps: Step 1 has 3, Step 2 has 8 and Step 3 has 15 circles.



Circle pattern, Steps 1–3 (NCERT figure, p. 150)

NCERT ARCHIVE

a Counting and the formula

Step	Circles	As $(k + 1)^2 - 1$
1	3	$2^2 - 1$
2	8	$3^2 - 1$
3	15	$4^2 - 1$
4 (next figure)	**24**	$5^2 - 1$
10	**120**	$11^2 - 1$
k	**$k^2 + 2k$**	$(k + 1)^2 - 1$

Step 4 has 4 rows of 5 circles plus a last row of 4 circles: 5 + 5 + 5 + 5 + 4 = 24. Equivalent ways: $(k + 1)^2 - 1$, $k^2 + 2k$, $k(k + 1) + k$ and $k(k + 2)$; all simplify to $k^2 + 2k$.

Therefore, the next figure (Step 4) has 24 circles, Step 10 has 120 circles and Step k has $k^2 + 2k$ circles.

QUESTION 46 *In-text question · p. 152*

Use this formula ($k^2 + 2k$) to find the number of circles in Step 15.

ANSWER

Substitute $k = 15$.

a Calculation

$15^2 + 2 \times 15 = 225 + 30 = 255$

Check: $(15 + 1)^2 - 1 = 256 - 1 = 255$

Therefore, Step 15 has 255 circles.

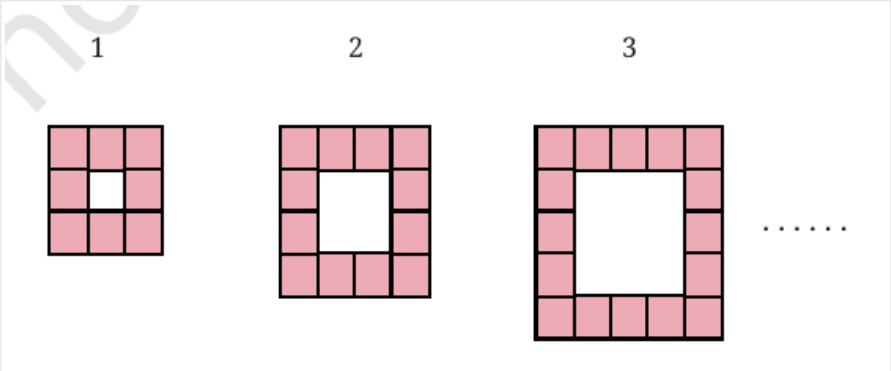
NCERT ARCHIVE

QUESTION 47 *In-text question · p. 152*

Consider the pattern made of square tiles in the picture below. How many square tiles are there in each figure?

ANSWER

In each step a square ring of tiles surrounds a square hole. Count tiles = outer square − hole.



Square tile pattern, Steps 1–3 (NCERT figure, p. 152)

a Tiles in each figure

Step	Outer square	Hole	Tiles
1	3 × 3 = 9	1 × 1 = 1	**8**
2	4 × 4 = 16	2 × 2 = 4	**12**
3	5 × 5 = 25	3 × 3 = 9	**16**

Therefore, the three figures have 8, 12 and 16 square tiles ($3^2 - 1^2$, $4^2 - 2^2$, $5^2 - 3^2$).

QUESTION 48 *In-text question · p. 153*

How many are there in Step 4 of the sequence? What about Step 10?

ANSWER

In Step n the outer square has side n + 2 and the hole has side n.

NCERT ARCHIVE

a Calculations

Step 4: $6^2 - 4^2 = 36 - 16 = 20$

Step 10: $12^2 - 10^2 = 144 - 100 = 44$

Therefore, Step 4 has 20 tiles and Step 10 has 44 tiles.

QUESTION 49 *Math Talk · p. 153*

Write an algebraic expression for the number of tiles in Step n. Share your methods with the class. Can you find more than one method to arrive at the answer?

ANSWER

Several ways of counting give the same expression.

a Methods

Method	Expression
Outer square minus hole	$(n + 2)^2 - n^2 = n^2 + 4n + 4 - n^2 = 4n + 4$
Four sides, each with n + 1 tiles (going round the ring)	$4(n + 1) = 4n + 4$
Two full rows of n + 2 plus two side columns of n	$2(n + 2) + 2n = 4n + 4$

Check with n = 1, 2, 3: 8, 12, 16; with n = 4: 20; n = 10: 44.

Therefore, the number of tiles in Step n is $(n + 2)^2 - n^2 = 4n + 4$.

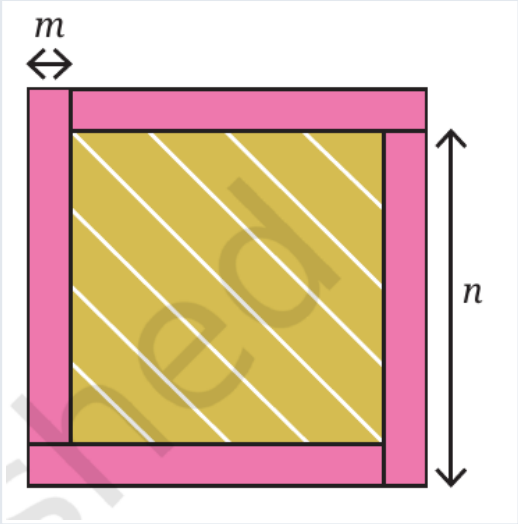
NCERT ARCHIVE

QUESTION 50 *In-text question · p. 153*

Find the area of the (interior) shaded region in the figure below. All four rectangles have the same dimensions. (Tadang's method: $(m + n)^2 - 4mn$. Yusuf's method: $(n - m)^2$.) By expanding both expressions, check that $(m + n)^2 - 4mn = (n - m)^2$.

ANSWER

Both methods describe the same region, so their expressions must be equal.



Four rectangles around a shaded square (NCERT figure, p. 153)

a Tadang's method

The whole figure is a square of side $(m + n)$, area $(m + n)^2$. Subtracting the four rectangles (each $m \times n$) leaves $(m + n)^2 - 4mn$.

b Yusuf's method

The shaded region is a square of side $(n - m)$, so its area is $(n - m)^2$.

c Checking by expansion

$$(m + n)^2 - 4mn = m^2 + 2mn + n^2 - 4mn = m^2 - 2mn + n^2$$
$$(n - m)^2 = n^2 - 2nm + m^2 = m^2 - 2mn + n^2$$

Therefore, the shaded area is $(m + n)^2 - 4mn = (n - m)^2 = m^2 - 2mn + n^2$.

NCERT ARCHIVE

QUESTION 51 *In-text question · p. 153*

Find out the area of the region with slanting lines in the figure (Fig. 1). All three rectangles have the same dimensions. (Methods: Anusha: $x^2 - xy$; Vaishnavi: $x(x + 2y) - 3xy$; Aditya: $x(x - y)$.)

ANSWER

The three equal rectangles (top, bottom and the vertical one) are each x long and y wide.

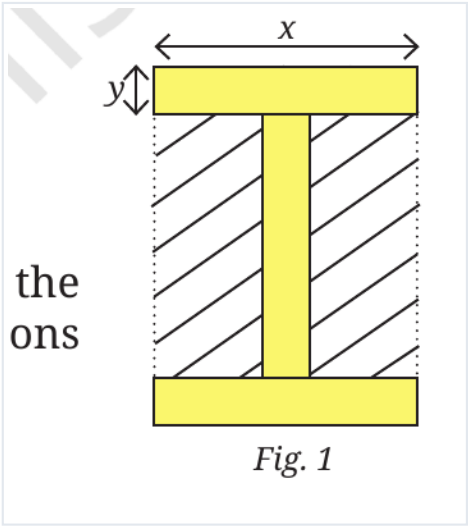
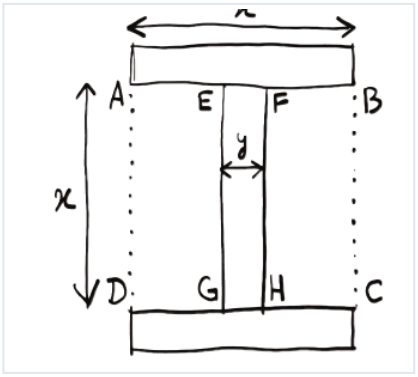


Fig. 1: three equal rectangles (NCERT figure, p. 153)

a Anusha's method

Area of ABCD (the square between the bars) = x^2; area of EFGH (the vertical rectangle) = xy. Required area = $x^2 - xy$.

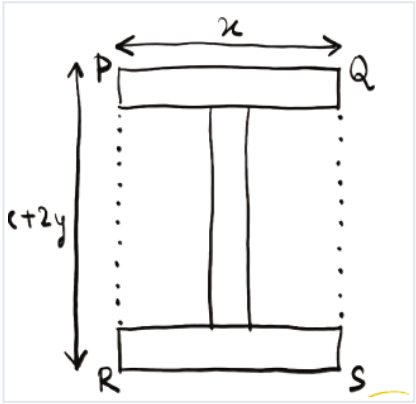


Anusha's figure (NCERT, p. 153)

NCERT ARCHIVE

b Vaishnavi's method

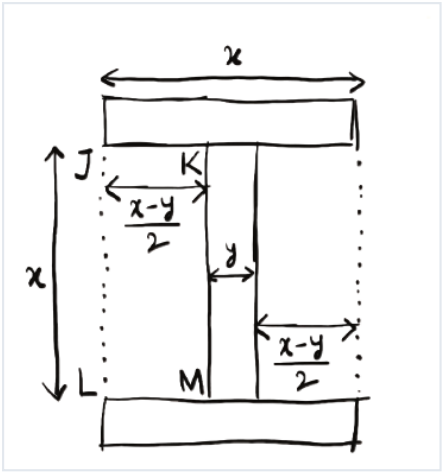
QS = y + x + y = x + 2y. Area of PQSR = x(x + 2y). Required area = x(x + 2y) − (area of the three rectangles) = x(x + 2y) − 3xy.



Vaishnavi's figure (NCERT, p. 153)

c Aditya's method

The required area is 2 times the area of JKLM, where JK = (x − y)/2 and KM = x. So the area = 2 × x(x − y)/2 = x(x − y).



Aditya's figure (NCERT, p. 154)

Therefore, the area of the region with slanting lines is $x^2 - xy$, which can also be written $x(x - y)$.

NCERT ARCHIVE

QUESTION 52 *In-text question · p. 154*

By expanding the expressions, verify that all three expressions are equivalent. If x = 8 and y = 3, find the area of the shaded region.

ANSWER

Expand Vaishnavi's and Aditya's expressions and compare with Anusha's $x^2 - xy$.

a Expansion

Anusha: $x^2 - xy$

Vaishnavi: $x(x + 2y) - 3xy = x^2 + 2xy - 3xy = x^2 - xy$

Aditya: $x(x - y) = x^2 - xy$

b Substituting x = 8, y = 3

$x^2 - xy = 64 - 24 = 40$

Check: $8(8 + 6) - 3 \times 24 = 112 - 72 = 40$ and $8(8 - 3) = 40$

Therefore, all three expressions equal $x^2 - xy$, and for x = 8, y = 3 the area is 40 square units.

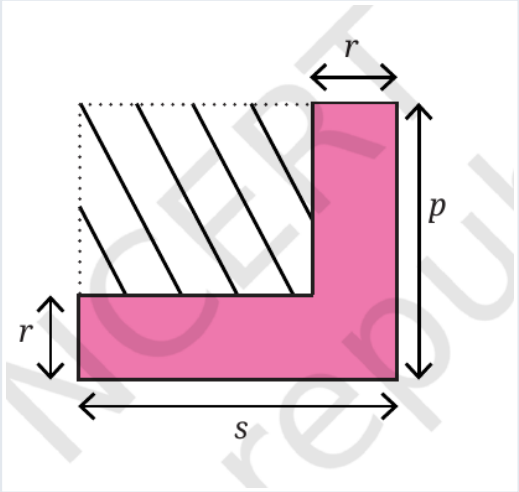
QUESTION 53 *Math Talk · p. 154*

Write an expression for the area of the dashed region in the figure below. Use more than one method to arrive at the answer. Substitute p = 6, r = 3.5, and s = 9, and calculate the area.

ANSWER

The figure is a rectangle of sides s (width) and p (height) from which an L-shaped strip of width r (along the bottom and the right side) has been removed; the dashed (slanted) region is what remains.

NCERT ARCHIVE



Dashed region and L-shaped strip (NCERT figure, p. 154)

a Method 1 – dimensions of the dashed rectangle

Width = s − r, height = p − r

Area = (p − r)(s − r) = ps − pr − sr + r²

b Method 2 – whole rectangle minus the L-shape

L-shape = pr + sr − r² (the corner r × r square is counted twice)

Area = ps − (pr + sr − r²) = ps − pr − sr + r²

c Substituting p = 6, r = 3.5, s = 9

(6 − 3.5)(9 − 3.5) = 2.5 × 5.5 = 13.75

Check: 54 − 21 − 31.5 + 12.25 = 13.75

Therefore, the dashed region has area ps − pr − sr + r² = (p − r)(s − r), which is 13.75 sq. units for the given values.

NCERT ARCHIVE

QUESTION 54 *Figure it Out · Q.1 · p. 154*

Compute these products using the suggested identity. (i) 46^2 using Identity 1A for $(a + b)^2$ (ii) 397×403 using Identity 1C for $(a + b)(a – b)$ (iii) 91^2 using Identity 1B for $(a – b)^2$ (iv) 43×45 using Identity 1C for $(a + b)(a – b)$

ANSWER

Write each number in the form required by the identity.

a Calculations

Product	Working	Value
(i) 46^2	$(40 + 6)^2 = 1600 + 480 + 36$	**2116**
(ii) 397×403	$(400 - 3)(400 + 3) = 160000 - 9$	**159991**
(iii) 91^2	$(100 - 9)^2 = 10000 - 1800 + 81$	**8281**
(iv) 43×45	$(44 - 1)(44 + 1) = 1936 - 1$	**1935**

Therefore, $46^2 = 2116$, $397 \times 403 = 159991$, $91^2 = 8281$ and $43 \times 45 = 1935$.

QUESTION 55 *Figure it Out · Q.2 · p. 154*

Use either a suitable identity or the distributive property to find each of the following products. (i) $(p – 1)(p + 11)$ (ii) $(3a – 9b)(3a + 9b)$ (iii) $–(2y + 5)(3y + 4)$ (iv) $(6x + 5y)^2$ (v) $(2x – 1/2)^2$ (vi) $(7p) \times (3r) \times (p + 2)$

ANSWER

Apply the distributive property or Identities 1A, 1B and 1C.

NCERT ARCHIVE

a Products

Product	Working	Result
(i) $(p - 1)(p + 11)$	$p^2 + 11p - p - 11$	**$p^2 + 10p - 11$**
(ii) $(3a - 9b)(3a + 9b)$	Identity 1C: $(3a)^2 - (9b)^2$	**$9a^2 - 81b^2$**
(iii) $-(2y + 5)(3y + 4)$	$-(6y^2 + 8y + 15y + 20)$	**$-6y^2 - 23y - 20$**
(iv) $(6x + 5y)^2$	$(6x)^2 + 2(6x)(5y) + (5y)^2$	**$36x^2 + 60xy + 25y^2$**
(v) $(2x - 1/2)^2$	$(2x)^2 - 2(2x)(1/2) + (1/2)^2$	**$4x^2 - 2x + 1/4$**
(vi) $(7p)(3r)(p + 2)$	$21pr(p + 2)$	**$21p^2r + 42pr$**

Therefore, the products are as listed in the Result column.

QUESTION 56 *Figure it Out · Q.3 · p. 155*

For each statement identify the appropriate algebraic expression(s). (i) Two more than a square number: $2 + s$, $(s + 2)^2$, $s^2 + 2$, $s^2 + 4$, $2s^2$, 2^2s. (ii) The sum of the squares of two consecutive numbers: $m^2 + n^2$, $(m + n)^2$, $m^2 + 1$, $m^2 + (m + 1)^2$, $m^2 + (m - 1)^2$, $(m + (m + 1))^2$, $(2m)^2 + (2m + 1)^2$

ANSWER

A square number is s^2 for some number s; consecutive numbers differ by 1.

a (i) Two more than a square number

A square number is s^2, so two more than it is **$s^2 + 2$**. The others do not match: $2 + s$ adds 2 to s, $(s + 2)^2$ squares $s + 2$, $s^2 + 4$ adds 4, $2s^2$ doubles s^2 and $2^2s = 4s$.

NCERT ARCHIVE

b **(ii) Sum of the squares of two consecutive numbers**

- **$m^2 + (m + 1)^2$**: squares of m and the next number m + 1.
- **$m^2 + (m - 1)^2$**: squares of m and the previous number m − 1.
- **$(2m)^2 + (2m + 1)^2$**: squares of the consecutive numbers 2m and 2m + 1.
- Not suitable: $m^2 + n^2$ (n is not tied to m), $(m + n)^2$ and $(m + (m + 1))^2$ (squares of sums), $m^2 + 1$ (second number is 1).

Therefore, (i) $s^2 + 2$; (ii) $m^2 + (m + 1)^2$, $m^2 + (m - 1)^2$ and $(2m)^2 + (2m + 1)^2$ each describe the sum of the squares of two consecutive numbers.

QUESTION 57 *Math Talk · Figure it Out · Q.4 · p. 155*

Consider any 2 by 2 square of numbers in a calendar, as shown in the figure. Find products of numbers lying along each diagonal – 4 × 12 = 48, 5 × 11 = 55. Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens. (Hint: Label the numbers in each 2 by 2 square as a, a + 1, a + 7, a + 8.)

ANSWER

In a calendar, the number below a date is 7 more than the date.

February

Su	M	Tu	W	Th	F	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

February calendar with a 2 × 2 square (NCERT figure, p. 155)

NCERT ARCHIVE

a Diagonal products of some 2 × 2 squares

2 × 2 square	Product along ↘	Product along ↙	Difference
4, 5 / 11, 12	4 × 12 = 48	5 × 11 = 55	7
2, 3 / 9, 10	2 × 10 = 20	3 × 9 = 27	7
16, 17 / 23, 24	16 × 24 = 384	17 × 23 = 391	7

b Why it happens

Numbers: a, a + 1, a + 7, a + 8

↙ product − ↘ product = (a + 1)(a + 7) − a(a + 8)

= a^2 + 8a + 7 − a^2 − 8a = 7

Therefore, in every 2 × 2 square the product along the diagonal from top-right to bottom-left exceeds the other diagonal product by exactly 7.

QUESTION 58 *Figure it Out · Q.5 · p. 155*

Verify which of the following statements are true. (i) (k + 1)(k + 2) – (k + 3) is always 2. (ii) (2q + 1)(2q – 3) is a multiple of 4. (iii) Squares of even numbers are multiples of 4, and squares of odd numbers are 1 more than multiples of 8. (iv) (6n + 2)² – (4n + 3)² is 5 less than a square number.

ANSWER

Expand each expression and test with examples where needed.

a (i) False

(k + 1)(k + 2) − (k + 3) = k^2 + 3k + 2 − k − 3 = k^2 + 2k − 1

k = 1 gives 2, but k = 2 gives 7, so it is not always 2.

NCERT ARCHIVE

b (ii) False

$(2q + 1)(2q - 3) = 4q^2 - 6q + 2q - 3 = 4q^2 - 4q - 3 = 4(q^2 - q) - 3$
This is 3 less than a multiple of 4 (an odd number), never a multiple of 4. E.g. $q = 2$: $5 \times 1 = 5$.

c (iii) True

Even: $(2n)^2 = 4n^2$, a multiple of 4.
Odd: $(2n + 1)^2 = 4n^2 + 4n + 1 = 4n(n + 1) + 1$; $n(n + 1)$ is a product of consecutive numbers, hence even, so $4n(n + 1)$ is a multiple of 8.

d (iv) False

$(6n + 2)^2 - (4n + 3)^2 = 36n^2 + 24n + 4 - 16n^2 - 24n - 9 = 20n^2 - 5$
Adding 5 gives $20n^2$, which is not a perfect square for any $n \geq 1$ (e.g. $n = 1$: 20). So the statement fails.

Therefore, (iii) is true; (i), (ii) and (iv) are false.

QUESTION 59 *Figure it Out · Q.6 · p. 155*

A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?

ANSWER

Write the numbers as $n_1 = 7a + 3$ and $n_2 = 7b + 5$.

a Sum, difference and product

Sum: $n_1 + n_2 = 7a + 7b + 8 = 7(a + b + 1) + 1 \rightarrow$ remainder 1
Difference: $n_2 - n_1 = 7(b - a) + 2 \rightarrow$ remainder 2 ($n_1 - n_2 = 7(a - b - 1) + 5 \rightarrow$ remainder 5)
Product: $n_1 n_2 = 49ab + 35a + 21b + 15 = 7(7ab + 5a + 3b + 2) + 1 \rightarrow$ remainder 1

NCERT ARCHIVE

Example: 10 and 12: sum 22 = 3 × 7 + 1; difference 2; product 120 = 17 × 7 + 1.

Therefore, the remainders are 1 for the sum, 2 for the difference (larger − smaller; 5 if taken the other way) and 1 for the product.

QUESTION 60 *Figure it Out · Q.7 · p. 155*

Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.

ANSWER

Try a few sets first: 3, 4, 5 → $4^2 - 3 \times 5 = 1$; 9, 10, 11 → $100 - 99 = 1$.

a Pattern

For 5, 6, 7: $6^2 - 5 \times 7 = 36 - 35 = 1$. The answer is always 1.

b Algebraic form

Numbers $n - 1$, n, $n + 1$: $n^2 - (n - 1)(n + 1) = 1$

Left side: $n^2 - (n^2 - n + n - 1) = n^2 - n^2 + 1 = 1$ = right side

Therefore, $n^2 - (n - 1)(n + 1) = 1$ is a true identity: the result is always 1.

NCERT ARCHIVE

QUESTION 61 *Figure it Out · Q.8 · p. 156*

What is the algebraic expression describing the following steps – add any two numbers. Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.

ANSWER

Let the two numbers be a and b.

a **Expression**

Sum = a + b
Half of the sum = (a + b)/2
Product = (a + b) × (a + b)/2 = $(a + b)^2/2$

This is half of $(a + b)^2$, i.e. half of the square of the sum. For example a = 3, b = 5: 8 × 4 = 32 = 64/2.

Therefore, the result is (a + b) × (a + b)/2 = (1/2)$(a + b)^2$, half of the square of the sum.

QUESTION 62 *Figure it Out · Q.9 · p. 156*

Which is larger? Find out without fully computing the product. (i) 14 × 26 or 16 × 24 (ii) 25 × 75 or 26 × 74

ANSWER

Both pairs of factors have the same sum, so use the difference of squares.

a **(i) 14 × 26 or 16 × 24**

Both pairs add up to 40: 14 × 26 = (20 − 6)(20 + 6) = 400 − 36; 16 × 24 = (20 − 4)(20 + 4) = 400 − 16
Since 16 < 36, 400 − 16 is larger. (Difference: 20.)

NCERT ARCHIVE

b (ii) 25 × 75 or 26 × 74

Both pairs add up to 100: $25 \times 75 = 50^2 - 25^2$; $26 \times 74 = 50^2 - 24^2$
Since $24^2 < 25^2$, 26 × 74 is larger. (Difference: 625 − 576 = 49.)

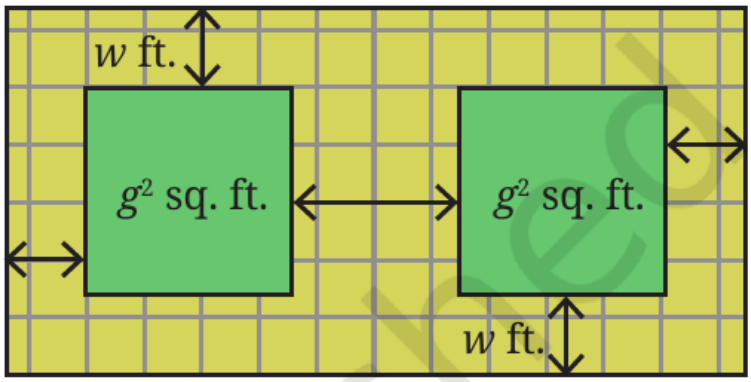
Therefore, 16 × 24 is larger than 14 × 26, and 26 × 74 is larger than 25 × 75.

QUESTION 63 *Figure it Out · Q.10 · p. 156*

A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g² sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to be tiled. Write an expression for the area that needs to be tiled.

ANSWER

In the plan the path runs along all four edges (width w) and also between the two squares (a gap of 2w, since w is left beside each square).



Plan of the park (NCERT figure, p. 156)

a Size of the whole park

Length = w + g + 2w + g + w = 2g + 4w
Breadth = w + g + w = g + 2w

NCERT ARCHIVE

b Area to be tiled

Area of park = $(2g + 4w)(g + 2w) = 2g^2 + 8gw + 8w^2$

Area of two squares = $2g^2$

Tiled area = $2g^2 + 8gw + 8w^2 - 2g^2 = 8gw + 8w^2 = 8w(w + g)$

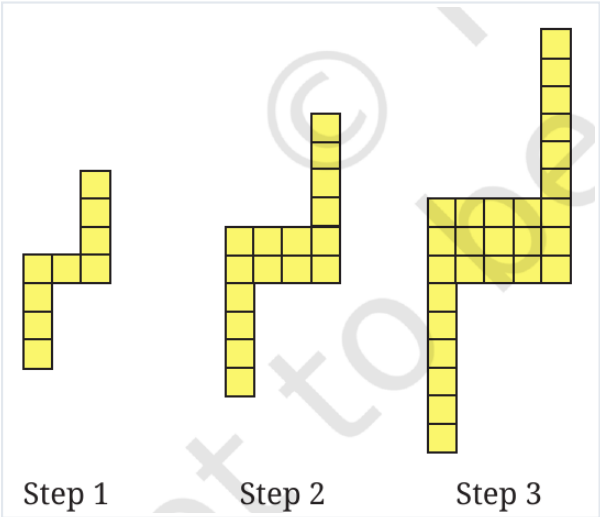
Therefore, the area to be tiled is $8w(w + g) = 8w^2 + 8gw$ square feet.

QUESTION 64 *Figure it Out · Q.11 · p. 156*

For each pattern shown below (the yellow pattern (a) and the blue pattern (b)), (i) Draw the next figure in the sequence. (ii) How many basic units are there in Step 10? (iii) Write an expression to describe the number of basic units in Step y.

ANSWER

Count the basic units in Steps 1, 2 and 3 and look for a square in each figure.

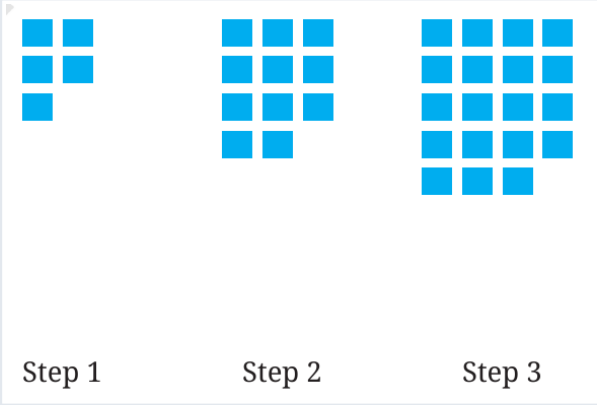


Pattern (a): yellow squares (NCERT figure, p. 156)

NCERT ARCHIVE

a Pattern (a)

- Step 1: a block 1 × 3 with arms of 3 squares: 3 + 3 + 3 = 9 = 3². Step 2: block 2 × 4 with arms of 4: 8 + 4 + 4 = 16 = 4².
- In Step y the block is y × (y + 2) and each arm has y + 2 squares: y(y + 2) + 2(y + 2) = (y + 2)². Step 3 therefore has 25 = 5² squares (the printed Step 3 shows 6 squares in each arm, which would make 27; the pattern of Steps 1 and 2 gives arms of 5).
- **Step 4** (next figure): block 4 × 6 = 24 with two arms of 6 squares: 36 = 6² squares.
- **Step 10:** (10 + 2)² = 144 squares. **Step y:** (y + 2)².



Pattern (b): blue squares (NCERT figure, p. 156)

b Pattern (b)

- Step 1: a 2 × 2 block and 1 extra: 2² + 1 = 5. Step 2: 3 × 3 and 2 extra: 3² + 2 = 11. Step 3: 4 × 4 and 3 extra: 4² + 3 = 19.
- **Step 4** (next figure): a 5 × 5 block with 4 extra squares: 5² + 4 = 29.
- **Step 10:** 11² + 10 = 131 squares. **Step y:** (y + 1)² + y.

Therefore, (a) Step 4 has 36, Step 10 has 144, Step y has (y + 2)² units; (b) Step 4 has 29, Step 10 has 131, Step y has (y + 1)² + y units.

NCERT ARCHIVE

6.5

CLASS 8 MATHS · CHAPTER 6

Coin Conjoin

Solutions · 1 question

Directions (Questions 65–65): These are the questions, examples and activities that appear inside the chapter (Coin Conjoin, p. 158), including the Figure it Out exercises, Math Talk and other in-text questions, numbered in the order in which they appear. Original question numbers and parts (a, b, c, …) are retained. Open-ended questions have model answers based on the chapter.

QUESTION 65 *Coin Conjoin · p. 158*

Arrange 10 coins in a triangle as shown in the figure. The task is to turn the triangle upside down by moving one coin at a time. How many moves are needed? What is the minimum number of moves? A triangle of 3 coins can be inverted with a single move, and a triangle of 6 coins by moving 2 coins. The 10-coin triangle can be flipped with just 3 moves; did you figure out how? Find out the minimum possible moves needed to flip the next bigger triangle having 15 coins. Try the same for bigger triangular numbers. Is there a simple way to calculate the minimum number of coin moves needed for any such triangular arrangement?

ANSWER

Place the coins on a triangular grid. Each coin that is already in a position needed by the upside-down triangle can stay; only the other coins have to move, one move each.

a The 10-coin triangle

- Rows of 1, 2, 3, 4 coins. The 6 coins lying on the sides between the corners, together with the centre coin (7 coins in all), are also part of the upside-down triangle (rows of 4, 3, 2, 1 coins).
- Only the 3 corner coins are in the wrong places. Moving each corner coin to one of the 3 corner positions of the upside-down triangle inverts it: **3 moves**, and no arrangement keeps more than 7 coins in place, so 3 is the minimum.

NCERT ARCHIVE

b 15 coins and bigger triangles

I checked every possible placement of the upside-down triangle on the triangular grid and counted the largest number of coins that can stay put. The minimum number of moves is the number of coins that cannot stay:

Rows	Coins (triangular number)	Coins that can stay	Minimum moves
2	3	2	1
3	6	4	2
4	10	7	3
5	**15**	10	**5**
6	21	14	7
7	28	19	9
8	36	24	12
9	45	30	15

c A simple rule

In every case checked the minimum number of moves equals one third of the number of coins, rounded down: for a triangle with n rows, $\lfloor n(n+1)/6 \rfloor$ (e.g. 15 coins: 30/6 = 5; 21 coins: 42/6 = 7).

Therefore, the 10-coin triangle needs 3 moves, the 15-coin triangle needs 5 moves, and (for the triangles checked, up to 45 coins) the minimum is the number of coins divided by 3, rounded down.

NCERT ARCHIVE