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SOLUTIONS BOOK

CLASS 8

Mathematics · Ganita Prakash

CHAPTER 5

Number Play

Complete In-text Questions and Figure it Out Solutions

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5.2 Checking Divisibility Quickly

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CLASS 8 MATHEMATICS · CHAPTER 5

5.1 Is This a Multiple Of?

Solutions · 40 questions

Directions (Questions 1–40): These are the questions, examples, activities and Figure it Out exercises that appear in Section 5.1 (pages 112–123) of the chapter “Number Play”, numbered in the order in which they appear. Where a question is open-ended, a model answer is given and you may add examples of your own.

QUESTION 1 *Sum of Consecutive Numbers · p. 112*

Anshu is exploring sums of consecutive numbers (7 = 3 + 4, 10 = 1 + 2 + 3 + 4, 12 = 3 + 4 + 5, 15 = 7 + 8 = 4 + 5 + 6 = 1 + 2 + 3 + 4 + 5). Now, he is wondering— “Can I write every natural number as a sum of consecutive numbers?” “Which numbers can I write as the sum of consecutive numbers in more than one way?” “Ohh, I know all odd numbers can be written as a sum of two consecutive numbers. Can we write all even numbers as a sum of consecutive numbers?” “Can I write 0 as a sum of consecutive numbers? Maybe I should use negative numbers.” Explore these questions and any others that may occur to you. Discuss them with the class.

ANSWER

Here every number from 1 to 32 is written as a sum of two or more consecutive natural numbers, in every possible way (found by trying all lengths).

Number	Ways to write it as a sum of consecutive natural numbers (2 or more terms)
1	— (none)
2	— (none)
3	1 + 2
4	— (none)
5	2 + 3
6	1 + 2 + 3

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Number	Ways to write it as a sum of consecutive natural numbers (2 or more terms)
7	3 + 4
8	— (none)
9	4 + 5 2 + 3 + 4
10	1 + 2 + 3 + 4
11	5 + 6
12	3 + 4 + 5
13	6 + 7
14	2 + 3 + 4 + 5
15	7 + 8 4 + 5 + 6 1 + 2 + 3 + 4 + 5
16	— (none)
17	8 + 9
18	5 + 6 + 7 3 + 4 + 5 + 6
19	9 + 10
20	2 + 3 + 4 + 5 + 6
21	10 + 11 6 + 7 + 8 1 + 2 + 3 + 4 + 5 + 6
22	4 + 5 + 6 + 7
23	11 + 12
24	7 + 8 + 9
25	12 + 13 3 + 4 + 5 + 6 + 7
26	5 + 6 + 7 + 8
27	13 + 14 8 + 9 + 10 2 + 3 + 4 + 5 + 6 + 7
28	1 + 2 + 3 + 4 + 5 + 6 + 7
29	14 + 15

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Number	Ways to write it as a sum of consecutive natural numbers (2 or more terms)
30	9 + 10 + 11 6 + 7 + 8 + 9 4 + 5 + 6 + 7 + 8
31	15 + 16
32	— (none)

a Can every natural number be written like this?

No. In the table, 1, 2, 4, 8, 16 and 32 have no way at all — these are the powers of 2, and the pattern continues (checked up to 200). All other numbers up to 32 can be written.

b Numbers that can be written in more than one way

For example 9 (4 + 5 and 2 + 3 + 4), 15 (three ways), 21, 25, 27, 30 … A number has exactly **one** way if it is an odd prime (3, 5, 7, 11, 13, …); the number of ways equals the number of odd factors of the number other than 1 (e.g. 15 has the odd factors 3, 5, 15 → 3 ways).

c Odd numbers and even numbers

Every odd number $2n + 1 = n + (n + 1)$ is a sum of two consecutive numbers. An even number can be written if it is not a power of 2: $6 = 1 + 2 + 3$, $10 = 1 + 2 + 3 + 4$, $12 = 3 + 4 + 5$, $14 = 2 + 3 + 4 + 5$, $18 = 5 + 6 + 7$ … but 2, 4, 8, 16 cannot (using natural numbers).

d Using negative numbers and 0

Yes, $0 = (–1) + 0 + 1$ and $0 = (–2) + (–1) + 0 + 1 + 2$. With negative numbers, **every** number can be written: the numbers cancel in pairs, e.g. $2 = (–1) + 0 + 1 + 2$ and $4 = (–3) + (–2) + (–1) + 0 + 1 + 2 + 3 + 4$. In general $n = (–(n – 1)) + … + 0 + … + (n – 1) + n$.

Therefore, only powers of 2 cannot be written as sums of consecutive natural numbers; odd primes have one way, many numbers have several ways, and using negative numbers every number (and 0) can be written.

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QUESTION 2 *Math Talk · p. 112*

Take any 4 consecutive numbers. For example, 3, 4, 5, and 6. Place '+' and '–' signs in between the numbers. How many different possibilities exist? Write all of them.

ANSWER

Between 3 and 4, between 4 and 5 and between 5 and 6 we can put either '+' or '–': 2 × 2 × 2 = 8 possibilities.

Sign pattern (between 3-4, 4-5, 5-6)	Expression
+ + +	3 + 4 + 5 + 6
+ + –	3 + 4 + 5 – 6
+ – +	3 + 4 – 5 + 6
+ – –	3 + 4 – 5 – 6
– + +	3 – 4 + 5 + 6
– + –	3 – 4 + 5 – 6
– – +	3 – 4 – 5 + 6
– – –	3 – 4 – 5 – 6

Therefore, 8 different expressions are possible (the tree diagram in the book lists them in this order).

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QUESTION 3 *Math Talk · p. 113*

Evaluate each expression and write the result next to it. Do you notice anything interesting? (Check if anyone in your class got an odd number.)

ANSWER

Expression	Result
3 + 4 + 5 + 6	18
3 + 4 + 5 – 6	6
3 + 4 – 5 + 6	8
3 + 4 – 5 – 6	-4
3 – 4 + 5 + 6	10
3 – 4 + 5 – 6	-2
3 – 4 – 5 + 6	0
3 – 4 – 5 – 6	-12

Observation: all eight results — 18, 6, 8, –4, 10, –2, 0, –12 — are **even**. Negative even numbers such as –4 and –12 are also even (they have the factor 2). No one will get an odd number.

Therefore, every one of the eight expressions gives an even number.

QUESTION 4 *Math Talk · p. 113*

Now, take four other consecutive numbers. Place the '+' and '–' signs as you have done before. Find out the results of each expression. What do you observe?

ANSWER

Take 5, 6, 7, 8 (the book shows 5 + 6 – 7 + 8 = 12 and 5 – 6 – 7 – 8 = –16).

Expression	Result
5 + 6 + 7 + 8	26
5 + 6 + 7 – 8	10

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Expression	Result
5 + 6 – 7 + 8	12
5 + 6 – 7 – 8	-4
5 – 6 + 7 + 8	14
5 – 6 + 7 – 8	-2
5 – 6 – 7 + 8	0
5 – 6 – 7 – 8	-16

Observation: again all results are even. Also, three results (–4, –2 and 0) are the same as for 3, 4, 5, 6.

Therefore, for 5, 6, 7, 8 also, all eight results are even.

QUESTION 5 *Math Talk · p. 113*

Repeat this for one more set of 4 consecutive numbers. Share your findings.

ANSWER

Take 9, 10, 11, 12 and compare it with the two earlier sets.

Pattern	3, 4, 5, 6	5, 6, 7, 8	9, 10, 11, 12
+ + +	18	26	42
+ + –	6	10	18
+ – +	8	12	20
+ – –	-4	-4	-4
– + +	10	14	22
– + –	-2	-2	-2
– – +	0	0	0
– – –	-12	-16	-24

Findings: (1) every result is even; (2) the results for the patterns "+ – –", "– + –" and "– – +" are –4, –2 and 0 in **all** three sets; (3) the other results change with the numbers chosen.

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Therefore, whichever four consecutive numbers we choose, all eight results are even, and –4, –2, 0 always appear.

QUESTION 6 *Math Talk · p. 113*

Do these patterns occur no matter which 4 consecutive numbers are chosen? Is there a way to find out through reasoning? Hint: Use algebra and describe the 8 expressions in a general form.

ANSWER

Let the four consecutive numbers be n, n + 1, n + 2, n + 3.

Pattern	Expression	Simplified
+ + +	n + (n+1) + (n+2) + (n+3)	4n + 6
+ + –	n + (n+1) + (n+2) – (n+3)	2n
+ – +	n + (n+1) – (n+2) + (n+3)	2n + 2
+ – –	n + (n+1) – (n+2) – (n+3)	–4
– + +	n – (n+1) + (n+2) + (n+3)	2n + 4
– + –	n – (n+1) + (n+2) – (n+3)	–2
– – +	n – (n+1) – (n+2) + (n+3)	0
– – –	n – (n+1) – (n+2) – (n+3)	–2n – 6

Each simplified value is either a multiple of 2 (4n + 6 = 2(2n + 3), 2n, 2n + 2 = 2(n + 1), 2n + 4, –2n – 6 …) or one of the fixed numbers –4, –2, 0. So all are even, and –4, –2, 0 occur for **every** choice of n.

Therefore, yes — the patterns hold for any 4 consecutive numbers, because the algebra works for every n.

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QUESTION 7 *Math Talk · p. 114*

Now take any 4 numbers, place '+' and '–' signs in the eight different ways, and evaluate the resulting expression. What do you observe about their parities? Repeat this with other sets of 4 numbers.

ANSWER

Pattern	2, 7, 9, 4	Parity	1, 6, 8, 12	Parity
+ + +	22	even	27	odd
+ + –	14	even	3	odd
+ – +	4	even	11	odd
+ – –	-4	even	-13	odd
– + +	8	even	15	odd
– + –	0	even	-9	odd
– – +	-10	even	-1	odd
– – –	-18	even	-25	odd

Therefore, for any four numbers all eight results have the same parity: all even for 2, 7, 9, 4 (sum 22) and all odd for 1, 6, 8, 12 (sum 27).

QUESTION 8 *Math Talk · p. 114*

Is there a way to explain why this happens? Hint: Think of the rules for parity of the sum or difference of two numbers.

ANSWER

a **Explanation 1: switching a sign changes the value by an even number**

Take a + b – c – d and switch +b to –b: the new expression is a – b – c – d. The change is (a + b – c – d) – (a – b – c – d) = a + b – c – d – a + b + c + d = **2b**, an even number. Two numbers whose difference is even have the same parity (both even or both odd).

b **Explanation 2: parity rules**

odd ± odd = even, even ± even = even, odd ± even = odd. So a + b and a – b have the same parity whatever the parities of a and b. In the same way a ± b + c and a ± b – c have the same parity, and finally all a ± b ± c ± d have the same parity.

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Therefore, all eight expressions made from a, b, c, d have the same parity, because changing one sign never changes the parity.

QUESTION 9 *Math Talk · p. 114*

Replace any negative sign in the expression a + b – c – d with a positive sign and find the difference between the two numbers.

ANSWER

Replace	New expression	New – old
–c by +c	a + b + c – d	(a + b + c – d) – (a + b – c – d) = 2c
–d by +d	a + b – c + d	(a + b – c + d) – (a + b – c – d) = 2d

Therefore, switching a negative sign to a positive sign changes the value by 2c or 2d — again an even number.

QUESTION 10 *Math Talk · p. 114*

What do you conclude from this observation? (Starting from any expression, we can get 7 expressions by switching one or more '+' and '–' signs.)

ANSWER

Switching one sign (from + to – or from – to +) changes the value by twice a number, which is even; so the parity does not change. Every one of the 8 expressions can be reached from any other by switching some signs, one at a time.

Therefore, all 8 expressions have the same parity.

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QUESTION 11 *Explanation 3 · p. 115*

Explanation 3: This can also be explained using the positive and negative token model you studied in the chapter on Integers. Try to think how.

ANSWER

Think of each number as that many tokens: a positive number as positive tokens (+1 each), a negative sign as negative tokens (–1 each).

- Whatever the signs are, the total number of tokens used is always the same: N = a + b + c + d (for natural numbers).
- If P of them are positive tokens and M are negative tokens, then P + M = N and the value of the expression is P – M = N – 2M.
- Since 2M is even, the value N – 2M has the same parity as N, whichever signs were chosen.

Therefore, all the expressions a ± b ± c ± d have the same parity as a + b + c + d.

QUESTION 12 *Math Talk · p. 115*

Is the phenomenon of all the expressions having the same parity limited to taking 4 numbers? What do you think?

ANSWER

No. The same reasoning works for any number of numbers: changing one sign changes the value by twice that number (even), so a ± b ± c ± … ± z all have the same parity as a + b + c + … + z. Example: 2 ± 3 ± 5 ± 6 ± 8 — 2 + 3 + 5 + 6 + 8 = 24 and 2 – 3 + 5 – 6 – 8 = –10, both even.

Therefore, the phenomenon is not limited to 4 numbers; it holds for any count of numbers.

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QUESTION 13 *Breaking Even · p. 115*

We know how to identify even numbers. Without computing them, find out which of the following arithmetic expressions are even: 43 + 37, 672 – 348, 4 × 347 × 3, 708 – 477, 809 + 214, 119 × 303, 543 – 479, 513^3.

ANSWER

Expression	Reason (parity rules)	Even?
43 + 37	odd + odd = even	**Yes**
672 – 348	even – even = even	**Yes**
4 × 347 × 3	has the factor 4 (so a factor 2)	**Yes**
708 – 477	even – odd = odd	No
809 + 214	odd + even = odd	No
119 × 303	odd × odd = odd	No
543 – 479	odd – odd = even	**Yes**
513^3	odd × odd × odd = odd	No

Therefore, the even expressions are 43 + 37, 672 – 348, 4 × 347 × 3 and 543 – 479.

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QUESTION 14 *Breaking Even · p. 115*

Using our understanding of how parity behaves under different operations, identify which of the following algebraic expressions give an even number for any integer values for the letter-numbers: 2a + 2b, 3g + 5h, 4m + 2n, 2u – 4v, 13k – 5k, 6m – 3n, $x^2 + 2$, $b^2 + 1$, 4k × 3j.

ANSWER

Expression	Always even?	Reason
2a + 2b	**Yes**	= 2(a + b), has the factor 2
3g + 5h	No (sometimes)	even only when g and h have the same parity (odd + odd or even + even)
4m + 2n	**Yes**	4m and 2n are both even; also = 2(2m + n)
2u – 4v	**Yes**	= 2(u – 2v), has the factor 2
13k – 5k	**Yes**	= 8k, has the factor 2
6m – 3n	No (sometimes)	= 3(2m – n): even only when n is even
$x^2 + 2$	No (sometimes)	x^2 has the parity of x, so the sum is even only when x is even
$b^2 + 1$	No (sometimes)	b^2 has the parity of b, so the sum is even only when b is odd
4k × 3j	**Yes**	= 12kj, has the factor 2

For 4m + 2q the book gives two justifications: 4m and 2q are even, so their sum is even; or 4m + 2q = 2(2m + q) has the factor 2.

Therefore, 2a + 2b, 4m + 2n, 2u – 4v, 13k – 5k and 4k × 3j always give even numbers; the other four do not always.

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QUESTION 15 *Breaking Even · p. 116*

Similarly, determine and explain which of the other expressions always give even numbers. Write a couple of examples and non-examples, as appropriate, for each expression.

ANSWER

Expression	Example (even result)	Non-example (odd result)
2a + 2b	a = 3, b = 5: 16	— (none)
3g + 5h	g = 2, h = 4: 26 (even)	g = 1, h = 2: 13 (odd)
4m + 2n	m = 1, n = 3: 10	— (none)
2u – 4v	u = 5, v = 2: 2	— (none)
13k – 5k	k = 3: 24	— (none)
6m – 3n	m = 2, n = 4: 0 (even)	m = 2, n = 3: 3 (odd)
$x^2 + 2$	x = 6: 38 (even)	x = 3: 11 (odd)
$b^2 + 1$	b = 3: 10 (even)	b = 4: 17 (odd)
4k × 3j	k = 1, j = 2: 24	— (none)

Expressions with no non-example are always even; those with a non-example are not always even.

Therefore, the expressions with at least one odd result (3g + 5h, 6m – 3n, $x^2 + 2$, $b^2 + 1$) do not always give an even number.

QUESTION 16 *Breaking Even · p. 116*

Write a few algebraic expressions which always give an even number.

ANSWER

Any expression that has 2 as a factor is always even.

Expression	Why it is always even
2n	has the factor 2
6a + 10b	= 2(3a + 5b)

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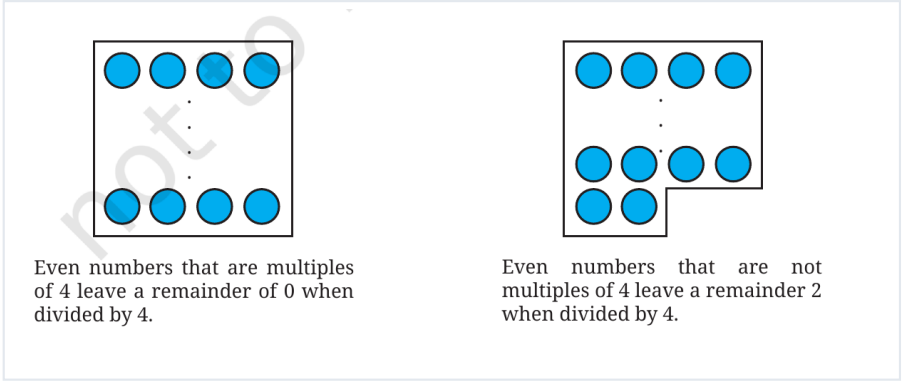
Expression	Why it is always even
8k – 2	= 2(4k – 1)
n(n + 1)	one of two consecutive numbers is even
$n^2 + n$	= n(n + 1)
(2n + 1) + (2m + 1)	odd + odd = even; = 2(n + m + 1)

Therefore, expressions such as 2n, 6a + 10b, 8k – 2 and n(n + 1) always give even numbers.

QUESTION 17 *Pairs to Make Fours · p. 116*

Take a pair of even numbers. Add them. Is the sum divisible by 4? Try this with different pairs of even numbers. When is the sum a multiple of 4, and when is it not? Is there a general rule or a pattern? (Even numbers leave remainder 0 or 2 when divided by 4.)

ANSWER



Even numbers leave a remainder of 0 or 2 on division by 4 (NCERT)

Pair	Remainders on ÷ 4	Sum	Multiple of 4?
4 + 8	0, 0	12	**Yes**
12 + 16	0, 0	28	**Yes**
2 + 6	2, 2	8	**Yes**
6 + 10	2, 2	16	**Yes**
4 + 2	0, 2	6	No
8 + 6	0, 2	14	No
12 + 10	0, 2	22	No

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Pair	Remainders on ÷ 4	Sum	Multiple of 4?
2 + 8	2, 0	10	No

a Rule

- Both multiples of 4 (remainders 0 and 0): the sum is a multiple of 4.
- Both leave remainder 2 (2 + 2 = 4): the sum is a multiple of 4.
- One multiple of 4 and one leaving remainder 2: the sum leaves remainder 2, so it is **not** a multiple of 4.

Therefore, two even numbers add up to a multiple of 4 exactly when their remainders (on division by 4) are the same — both 0 or both 2.

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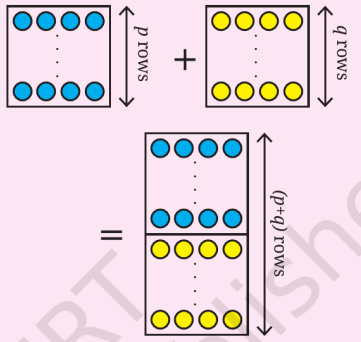
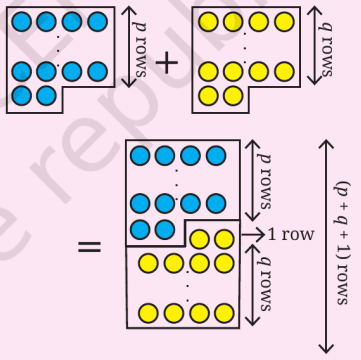
QUESTION 18 *Pairs to Make Fours · p. 117*

When will two even numbers add up to give a multiple of 4? This problem is similar to the question of identifying when adding two numbers will result in an even number. Can you see this? There are three cases to examine.

ANSWER

Explanation with Algebra and Visualisation			Examples
Adding two (even) numbers that are multiples of 4 will always give a multiple of 4.	$4p$ and $4q$. $4p + 4q$ $= 4\,(p + q)$.		4, 12, 16, 24, 36. 12 + 16 = 4 (3 + 4) = 28. 16 + 28 = 4 (4 + 7) = 44.
Adding two even numbers that are not multiples of 4 will always give a multiple of 4 because their remainders of 2 add up to 4.	$(4p + 2)$ and $(4q + 2)$. $(4p + 2) + (4q + 2)$ $= 4p + 4q + 4$ $= 4\,(p + q + 1)$.		2, 6, 10, 18, 22, 42. 2 + 6 = 8. 6 + 10 = 16. 22 + 6 = 28.

Cases 1 and 2 with algebra and visualisation (NCERT)

Case	Algebra	Result	Examples
1. Both multiples of 4	4p + 4q = 4(p + q)	multiple of 4	12 + 16 = 28; 16 + 28 = 44
2. Both leave remainder 2	(4p + 2) + (4q + 2) = 4(p + q + 1)	multiple of 4 (remainders 2 + 2 = 4)	2 + 6 = 8; 6 + 10 = 16; 22 + 6 = 28
3. One of each kind	4p + (4q + 2) = 4(p + q) + 2	**not** a multiple of 4 (remainder 2)	4 + 2 = 6; 8 + 6 = 14

This is similar to parity: even + even = even, odd + odd = even, even + odd = odd. Here 0 + 0 and 2 + 2 give a multiple of 4, while 0 + 2 does not.

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Therefore, the sum of two even numbers is a multiple of 4 in cases 1 and 2, and not in case 3.

QUESTION 19 *Pairs to Make Fours · p. 117*

What happens when we add a multiple of 4 to an even number that is not a multiple of 4? Is it similar to the case of the parity of the sum of an even and an odd number?

ANSWER

A multiple of 4 is 4p; an even number that is not a multiple of 4 is 4q + 2. Their sum is 4p + 4q + 2 = 4(p + q) + 2, which leaves a remainder of 2 when divided by 4.

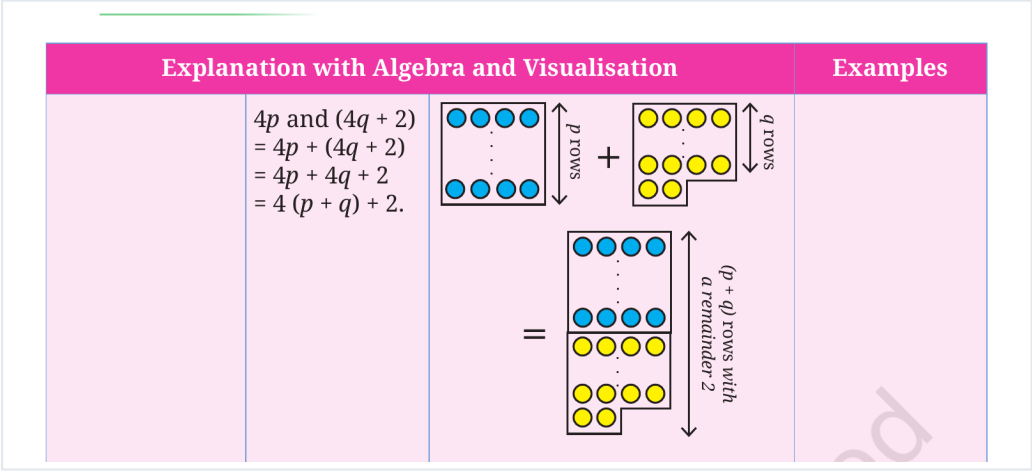
Yes, it is similar. Even + odd always gives an odd number (it is never even). In the same way, (multiple of 4) + (number leaving remainder 2) always leaves remainder 2 — it is never a multiple of 4.

Therefore, the sum is never a multiple of 4; it always leaves a remainder of 2.

QUESTION 20 *Pairs to Make Fours · p. 117*

Look at the following expressions and the visualisation (4p and (4q + 2) = 4p + (4q + 2) = 4p + 4q + 2 = 4(p + q) + 2). Write the corresponding explanation and examples.

ANSWER



Third case: a multiple of 4 plus a number leaving remainder 2 (NCERT)

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a Explanation

4p is shown as p rows of 4 counters and 4q + 2 as q rows of 4 counters with 2 extra. Putting them together gives (p + q) full rows of 4 and 2 counters left over — a remainder of 2.

b Examples

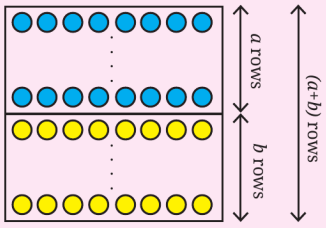
Multiple of 4	Number leaving remainder 2	Sum	Sum ÷ 4
4	2	6	1 remainder 2
8	6	14	3 remainder 2
12	10	22	5 remainder 2
16	14	30	7 remainder 2

Therefore, a multiple of 4 added to an even number that is not a multiple of 4 gives 4(p + q) + 2, which is not a multiple of 4.

QUESTION 21 *Always, Sometimes or Never · p. 118*

We examine different statements about factors and multiples and determine whether a statement is 'Always True', 'Sometimes True', or 'Never True'. We know that the sum of any two multiples of 2 is also a multiple of 2. 1. If 8 exactly divides two numbers separately, it must exactly divide their sum.

ANSWER

Explanation with Algebra and Visualisation			Examples
The two numbers have 8 as a factor; in other words, the two numbers are multiples of 8.	$8a$ and $8b$.		8 and 16. 16 and 56. 80 and 120.
As multiples of 8 are obtained by repeatedly adding 8, the sum of two multiples of 8 will also be a multiple of 8.	$8a + 8b$ $= 8\,(a + b)$.		$8 + 16 = 8(1 + 2)$ $= 24$. $16 + 56 = 72$. $80 + 120 = 200$.

Statement 1: explanation with algebra and visualisation (NCERT)

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a Explanation

Numbers that 8 divides are multiples of 8: 8a and 8b. Their sum is 8a + 8b = 8(a + b), a multiple of 8 (a rows of 8 and b rows of 8 make a + b rows of 8).

Numbers	Sum	Sum ÷ 8
8 and 16	24	3
16 and 56	72	9
80 and 120	200	25

Therefore, Statement 1 is always true.

QUESTION 22 *Always, Sometimes or Never · p. 118–119*

Statement 1 is always true. Determine if it is true with subtraction.

ANSWER

If 8 divides both numbers: 8a – 8b = 8(a – b), which is a multiple of 8. Examples: 56 – 16 = 40 = 8 × 5; 120 – 80 = 40 = 8 × 5.

In general, **if a divides M and a divides N, then a divides M + N and a divides M – N.**

Therefore, the statement is also always true with subtraction.

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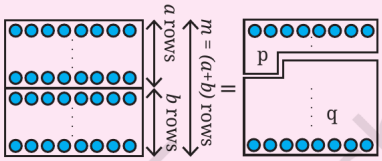
QUESTION 23 *Always, Sometimes or Never · p. 119*

2. If a number is divisible by 8, then 8 also divides any two numbers (separately) that add up to the number.

ANSWER

Explanation with Algebra and Visualisation			Examples
A number divisible by 8 is a multiple of 8.	$8m$		8, 16, 56, 72.
A number divisible by 8 can be expressed as a sum of two multiples of 8 or sum of two non-multiples of 8.	$8m = 8a + 8b$ $8m = p + q$ (p, q not multiples of 8)		$72 = 48 + 24$ $(8\times9 = 8\times6 + 8\times3)$. $72 = 50 + 22$

Statement 2 (NCERT)

a Examples and non-examples

Number	Split into	Does 8 divide both parts?
72	48 + 24	Yes (48 = 8 × 6, 24 = 8 × 3)
72	50 + 22	No (50 and 22 are not multiples of 8)

Algebra: 8m = 8a + 8b (both parts multiples of 8), but also 8m = p + q where p, q are not multiples of 8 (e.g. 72 = 50 + 22).

Therefore, Statement 2 is sometimes true.

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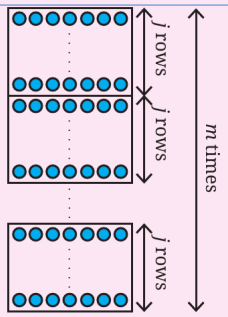
QUESTION 24 *Always, Sometimes or Never · p. 119*

3. If a number is divisible by 7, then all multiples of that number will be divisible by 7.

ANSWER

Explanation with Algebra and Visualisation			Examples
Numbers divisible by 7 will have 7 as a factor.	$7j$		$14 = 7 \times 2$ ($j = 2$). $42 = 7 \times 6$ ($j = 6$). $98 = 7 \times 14$ ($j = 14$).
This contains a total of mj rows. So this is also a multiple of 7.	$(7j) \times m$		Some multiples of 14: $28 = (7 \times 2) \times 2$. $70 = (7 \times 2) \times 5$. $154 = (7 \times 2) \times 11$

Statement 3 (NCERT)

A number divisible by 7 is 7j. Its multiples are (7j) × m = 7(jm), which has the factor 7. Example: 14 = 7 × 2 (j = 2); its multiples 28, 70, 154 are 7 × 4, 7 × 10, 7 × 22.

In general, **if A is divisible by k, then all multiples of A are divisible by k.**

Therefore, Statement 3 is always true.

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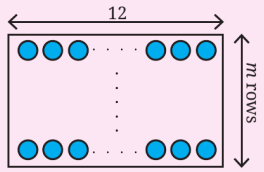
QUESTION 25 *Always, Sometimes or Never · p. 120*

4. If a number is divisible by 12, then the number is also divisible by all the factors of 12.

ANSWER

Explanation with Algebra and Visualisation			Examples
A number divisible by 12 is a multiple of 12.	$12m$		12, 24, 36, 48, 108, 132.
Factors of multiples of 12 will include factors of 12.	$12m$ $= 2 \times 6 \times m$ $= 3 \times 4 \times m$	 A factor of 12 covers a row fully. Hence, it covers all multiples of 12 fully.	Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24.

Statement 4 (NCERT)

A number divisible by 12 is 12m. If f is a factor of 12, say 12 = f × g, then 12m = f × (gm), so f divides 12m. Example: 24 is divisible by 12 and by every factor of 12 (1, 2, 3, 4, 6): 24 = 2 × 12 = 3 × 8 = 4 × 6 ...

In general, **if A is divisible by k, then A is divisible by all the factors of k.**

Therefore, Statement 4 is always true.

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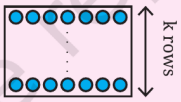
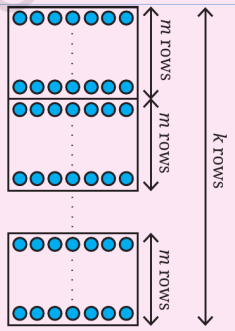
QUESTION 26 *Always, Sometimes or Never · p. 120*

5. If a number is divisible by 7, then it is also divisible by any multiple of 7.

ANSWER

Explanation with Algebra and Visualisation			Examples
Numbers divisible by 7 are multiples of 7.	$7k$		
Multiples of 7. $7k$ will be divisible by $7m$ if and only if m is a factor of k.	$7m$ *If* $k = ym$ then $7k \div 7m = 7ym \div 7m = y$		42 (7 × 6) is divisible by 7 but it is not divisible by 28 (7 × 4). 42 (7 × 6) is divisible by 7 and it is divisible by 14 (7 × 2).

Statement 5 (NCERT)

A number divisible by 7 is 7k. It is divisible by 7m (a multiple of 7) only if 7k ÷ 7m = k ÷ m is a whole number, i.e. only if m is a factor of k.

Number	Multiple of 7	Divisible?
42 = 7 × 6	14 = 7 × 2	**Yes** (m = 2 divides 6)
42 = 7 × 6	28 = 7 × 4	No (4 does not divide 6)

Therefore, Statement 5 is sometimes true.

QUESTION 27 *Math Talk · p. 121*

Examine each of the following statements, and determine whether it is 'Always true', 'Sometimes true', 'Never true'. 6. If a number is divisible by both 9 and 4, it must be divisible by 36.

ANSWER

9 = 3 × 3 and 4 = 2 × 2 have no common prime factor, so their LCM is 9 × 4 = 36.

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If A is divisible by 9 and by 4, then A is divisible by the LCM of 9 and 4 (its prime factorisation contains 2 × 2 × 3 × 3). Examples: 72 = 36 × 2, 108 = 36 × 3, 180 = 36 × 5.

Therefore, Statement 6 is always true.

QUESTION 28 *Math Talk · p. 121*

7. If a number is divisible by both 6 and 4, it must be divisible by 24.

ANSWER

In general, if A is divisible by k and also by m, then A is divisible by the LCM of k and m. Here LCM (6, 4) = 12, not 24.

Number	Divisible by 6 and 4?	Divisible by 24?
12	Yes	No
36	Yes	No
24	Yes	**Yes**
48	Yes	**Yes**

Numbers divisible by both 6 and 4 are the multiples of 12 (12k); such a number is divisible by 24 only when k is even.

Therefore, Statement 7 is sometimes true.

QUESTION 29 *Math Talk · p. 121*

8. When you add an odd number to an even number we get a multiple of 6. (Can I write an even and an odd number as 2n and 2n + 1 instead?)

ANSWER

a **Reasoning with parity**

All multiples of 6 are even, but odd + even is always odd. So the sum can never be a multiple of 6.

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b **Reasoning with algebra (yes, we can write an even number as 2n and an odd number**

Suppose (2n) + (2m + 1) = 6j. Then 2n + 2m = 6j – 1, i.e. 2(n + m) = 6j – 1. The left side is even and the right side is odd — impossible.

Therefore, Statement 8 is never true.

QUESTION 30 *What Remains? · p. 121*

Find a number that has a remainder of 3 when divided by 5. Write more such numbers.

ANSWER

These numbers are 3 more than a multiple of 5: 5 × 0 + 3, 5 × 1 + 3, 5 × 2 + 3, …

k	0	1	2	3	4	5	6
5k + 3	3	8	13	18	23	28	33

Therefore, 3, 8, 13, 18, 23, 28, 33, … all leave a remainder of 3 when divided by 5.

QUESTION 31 *What Remains? · p. 121*

Which algebraic expression(s) capture all such numbers? (i) 3k + 5 (ii) 3k – 5 (iii) 3k/5 (iv) 5k + 3 (v) 5k – 2 (vi) 5k – 3

ANSWER

Expression	Values (k = 1, 2, 3, 4, 5)	Captures all numbers leaving remainder 3 when divided by 5?
(i) 3k + 5	8, 11, 14, 17, 20	No (11 leaves remainder 1)
(ii) 3k – 5	–2, 1, 4, 7, 10	No (4 leaves remainder 4)
(iii) 3k/5	0.6, 1.2, 1.8, …	No (not even whole numbers)
(iv) 5k + 3	8, 13, 18, 23, 28	**Yes** (k = 0 gives 3)
(v) 5k – 2	3, 8, 13, 18, 23	**Yes** (k ≥ 1)
(vi) 5k – 3	2, 7, 12, 17, 22	No (remainder 2)

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Therefore, the expressions (iv) 5k + 3 and (v) 5k – 2 capture all such numbers.

QUESTION 32 *What Remains? · p. 122*

Let us consider another expression, 5k – 2, and see the values it takes for different values of k (k = 1, 2, 3, 4, 5 gives 3, 8, 13, 18, 23). Are there other expressions that generate numbers that are 3 more than a multiple of 5?

ANSWER

Adding or subtracting a multiple of 5 does not change the remainder, so any expression 5k + 3 ± 5t (t fixed) works.

Expression	Values	Condition
5k + 3	3, 8, 13, 18, …	k = 0, 1, 2, …
5k – 2	3, 8, 13, 18, …	k = 1, 2, 3, …
5k + 8	8, 13, 18, 23, …	k = 0, 1, 2, …
5k – 7	3, 8, 13, …	k = 2, 3, 4, …
10k + 3 and 10k + 8	3, 13, 23, … and 8, 18, 28, …	together they give all

Therefore, yes — for example 5k + 8, 5k – 7, or the pair 10k + 3 and 10k + 8 also generate all numbers that are 3 more than a multiple of 5.

QUESTION 33 *Figure it Out · Q.1 · p. 122*

The sum of four consecutive numbers is 34. What are these numbers?

ANSWER

Let the numbers be n, n + 1, n + 2, n + 3. Then n + (n + 1) + (n + 2) + (n + 3) = 4n + 6 = 34, so 4n = 28 and n = 7.

Check: 7 + 8 + 9 + 10 = 34.

Therefore, the numbers are 7, 8, 9 and 10.

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QUESTION 34 *Figure it Out · Q.2 · p. 122*

Suppose p is the greatest of five consecutive numbers. Describe the other four numbers in terms of p.

ANSWER

Consecutive numbers differ by 1, so going down from p we subtract 1, 2, 3, 4.

Therefore, the five numbers are p – 4, p – 3, p – 2, p – 1 and p.

QUESTION 35 *Figure it Out · Q.3 · p. 122*

For each statement below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra. (i) The sum of two even numbers is a multiple of 3. (ii) If a number is not divisible by 18, then it is also not divisible by 9. (iii) If two numbers are not divisible by 6, then their sum is not divisible by 6. (iv) The sum of a multiple of 6 and a multiple of 9 is a multiple of 3. (v) The sum of a multiple of 6 and a multiple of 3 is a multiple of 9.

ANSWER

	Verdict	Examples / non-examples	Algebra
(i)	Sometimes	2 + 4 = 6 and 4 + 8 = 12 are multiples of 3; 2 + 6 = 8 and 6 + 8 = 14 are not	2m + 2n = 2(m + n): a multiple of 3 only when m + n is
(ii)	Sometimes	30 is not divisible by 18 and not by 9 (true); 27 is not divisible by 18 but is divisible by 9 (false)	Numbers not divisible by 18 include odd multiples of 9 (9, 27, 45, …) and non-multiples of 9
(iii)	Sometimes	9 + 11 = 20 (not a multiple of 6); 8 + 10 = 18 is a multiple of 6	Remainders on ÷ 6 may add up to 6 (e.g. 2 + 4)
(iv)	**Always**	12 + 18 = 30 = 3 × 10	6x + 9y = 3(2x + 3y)
(v)	Sometimes	18 + 9 = 27 is a multiple of 9; 12 + 9 = 21 is not	6x + 3y = 3(2x + y): a multiple of 9 only when 2x + y is a multiple of 3

Therefore, (i), (ii), (iii) and (v) are sometimes true, and (iv) is always true.

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QUESTION 36 *Figure it Out · Q.4 · p. 122*

Find a few numbers that leave a remainder of 2 when divided by 3 and a remainder of 2 when divided by 4. Write an algebraic expression to describe all such numbers.

ANSWER

If x leaves remainder 2 on division by 3 and by 4, then x – 2 is a multiple of both 3 and 4, i.e. a multiple of LCM (3, 4) = 12.

n	0	1	2	3	4
12n + 2	2	14	26	38	50

Therefore, such numbers are 2, 14, 26, 38, 50, … and all of them are given by 12n + 2.

QUESTION 37 *Figure it Out · Q.5 · p. 122*

“I hold some pebbles, not too many, When I group them in 3’s, one stays with me. Try pairing them up — it simply won’t do, A stubborn odd pebble remains in my view. Group them by 5, yet one’s still around, But grouping by seven, perfection is found. More than one hundred would be far too bold, Can you tell me the number of pebbles I hold?”

ANSWER

Clue	Meaning
Groups of 3: one left	remainder 1 on ÷ 3
Pairs: one odd pebble left	remainder 1 on ÷ 2
Groups of 5: one left	remainder 1 on ÷ 5
Groups of 7: perfect	divisible by 7
Not more than a hundred	less than 100

Remainder 1 on division by 3, 2 and 5 means (number – 1) is a multiple of LCM (3, 2, 5) = 30. So the number is 30k + 1: 31, 61, 91 (121 is more than 100).

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Candidate	31	61	91
Divisible by 7?	No	No	**Yes** (7 × 13)

Therefore, the number of pebbles is 91.

QUESTION 38 *Figure it Out · Q.6 · p. 122*

Tathagat has written several numbers that leave a remainder of 2 when divided by 6. He claims, "If you add any three such numbers, the sum will always be a multiple of 6." Is Tathagat's claim true?

ANSWER

Three such numbers are 6a + 2, 6b + 2 and 6c + 2.

Sum = (6a + 2) + (6b + 2) + (6c + 2) = 6a + 6b + 6c + 6 = 6(a + b + c + 1), a multiple of 6.
Example: 8 + 14 + 20 = 42 = 6 × 7.

Therefore, Tathagat's claim is true.

QUESTION 39 *Figure it Out · Q.7 · p. 123*

When divided by 7, the number 661 leaves a remainder of 3, and 4779 leaves a remainder of 5. Without calculating, can you say what remainders the following expressions will leave when divided by 7? Show the solution both algebraically and visually. (i) 4779 + 661 (ii) 4779 – 661

ANSWER

a Algebraically

Write 4779 = 7p + 5 and 661 = 7q + 3.

- (i) 4779 + 661 = 7p + 7q + 8 = 7(p + q + 1) + 1 → remainder **1**.
- (ii) 4779 – 661 = 7p + 5 – 7q – 3 = 7(p – q) + 2 → remainder **2**.

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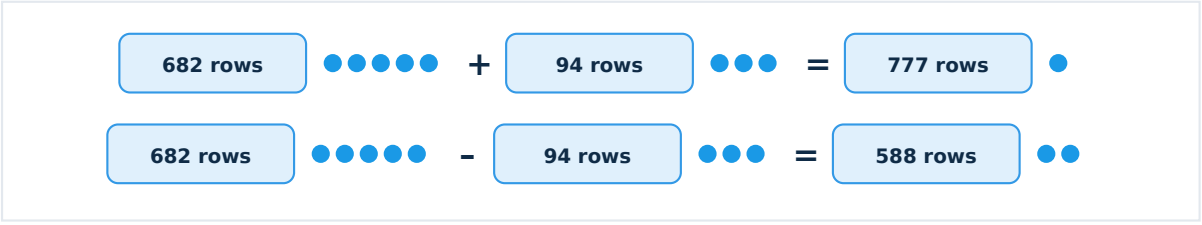
b **Visually (rows of 7 counters; leftover counters are the remainder)**



Each bar is a block of rows of 7 counters. Top: 5 + 3 = 8 leftovers = 1 more row + 1. Bottom: 5 – 3 = 2 leftovers.

Check: 4779 + 661 = 5440 = 7 × 777 + 1 and 4779 – 661 = 4118 = 7 × 588 + 2.

Therefore, the remainders are (i) 1 and (ii) 2.

QUESTION 40 *Figure it Out · Q.8 · p. 123*

Find a number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5. What is the smallest such number? Can you give a simple explanation of why it is the smallest?

ANSWER

Each remainder is 1 less than the divisor (2 = 3 – 1, 3 = 4 – 1, 4 = 5 – 1). So the number is 1 less than a multiple of 3, 4 and 5.

The common multiples of 3, 4 and 5 are the multiples of LCM (3, 4, 5) = 60. So the numbers are 60k – 1: 59, 119, 179, ...

Why smallest: the smallest positive common multiple is 60 (k = 1); k = 0 would give –1, which is not a natural number. So the smallest number is 60 – 1 = 59.

Check: 59 = 3 × 19 + 2 = 4 × 14 + 3 = 5 × 11 + 4.

Therefore, the smallest such number is 59.

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CLASS 8 MATHEMATICS · CHAPTER 5

5.2 Checking Divisibility Quickly

Solutions · 33 questions

Directions (Questions 41–73): These are the questions, examples, activities and Figure it Out exercises that appear in Section 5.2 (pages 123–131) of the chapter “Number Play”, numbered in the order in which they appear. Where a question is open-ended, a model answer is given and you may add examples of your own.

QUESTION 41 *Checking Divisibility Quickly · p. 123*

Similarly, explain using algebra why the divisibility shortcuts for 5, 2, 4, and 8 work.

ANSWER

A number …dcba = … + 1000d + 100c + 10b + a, where a, b, c, d are the units, tens, hundreds, thousands digits.

Divisor	Place values that are always multiples of the divisor	Only this part decides	Shortcut
5	10b, 100c, 1000d, … (multiples of 10, hence of 5)	a	units digit is 0 or 5
2	10b, 100c, … (multiples of 10, hence of 2)	a	units digit is 0, 2, 4, 6, 8
4	100c, 1000d, … (multiples of 100 = 4 × 25)	10b + a	last two digits divisible by 4
8	1000d, … (multiples of 1000 = 8 × 125)	100c + 10b + a	last three digits divisible by 8

Since the other terms are multiples of the divisor, the whole number is divisible by it if and only if the remaining part is.

Therefore, the shortcuts work because all higher place values are multiples of 10, 100 or 1000.

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QUESTION 42 *A Shortcut for Divisibility by 9 · p. 123*

Can you say, without actually calculating, which of these numbers are divisible by 9: 999, 909, 900, 90, 990?

ANSWER

Number	999	909	900	90	990
As 9 × …	9 × 111	9 × 101	9 × 100	9 × 10	9 × 110

Therefore, all of them are divisible by 9 (each is made only of 9s and 0s).

QUESTION 43 *A Shortcut for Divisibility by 9 · p. 124*

Can we say that any number made up of only the digits ‘0’ and ‘9’, in any order, will always be divisible by 9?

ANSWER

Yes. In the expanded form every term is 9 × (place value) or 0 × (place value), and each is a multiple of 9. Example: 99009 = 9 × 10000 + 9 × 1000 + 0 × 100 + 0 × 10 + 9 × 1.

But this shortcut cannot identify all multiples of 9: 99 and 109 both end in 9, yet 99 is divisible by 9 and 109 is not.

Therefore, any number made only of 0s and 9s is divisible by 9.

QUESTION 44 *A Shortcut for Divisibility by 9 · p. 124*

Is 10 divisible by 9? If not, what is the remainder? Check the divisibility of other multiples of 10 (10, 20, 30, …) by 9.

ANSWER

Number	10	20	30	40	50	60	70	80	90
Remainder ÷ 9	1	2	3	4	5	6	7	8	0

10 = 9 + 1, so 10t = 9t + t. The 9t part is a multiple of 9, so the remainder is the number of tens, t (and 0 if t = 9).

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Therefore, 10 is not divisible by 9 (remainder 1); for any multiple of 10 the remainder equals the number of tens.

QUESTION 45 *A Shortcut for Divisibility by 9 · p. 124*

Similarly, look at the remainder when the multiples of 100 (100, 200, 300, …) are divided by 9. What do you notice?

ANSWER

Number	100	200	300	400	500	600	700	800	900
Remainder ÷ 9	1	2	3	4	5	6	7	8	0

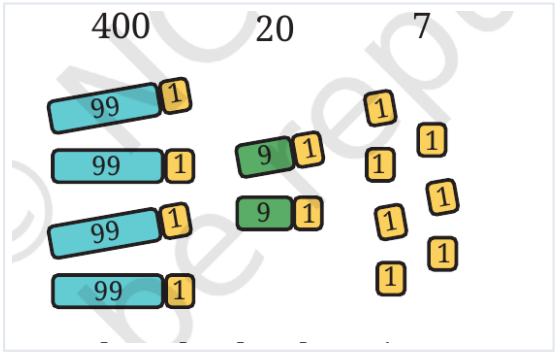
100 = 99 + 1, so 100h = 99h + h, and 99h is a multiple of 9.

Therefore, the remainder is the same as the number of hundreds, for any multiple of 100.

QUESTION 46 *A Shortcut for Divisibility by 9 · p. 124*

Using this observation, find the remainder when 427 is divided by 9.

ANSWER



The number 427 as 400 + 20 + 7 in counters (NCERT)

- 4 hundreds → remainder 4
- 2 tens → remainder 2
- 7 units → remainder 7

Adding the remainders: 4 + 2 + 7 = 13. One more group of 9 can be made from 13, leaving 13 – 9 = 4.

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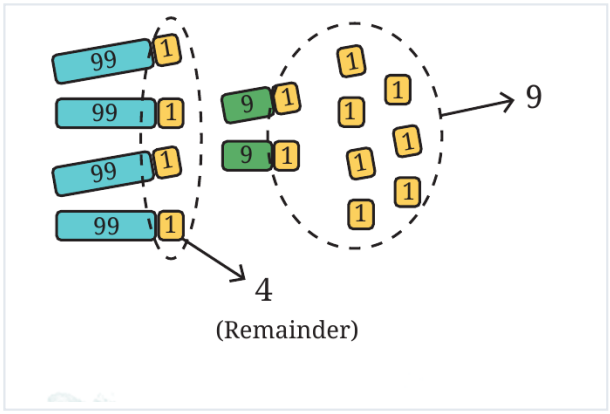


Grouping the leftovers: one more 9 can be made, remainder 4 (NCERT)

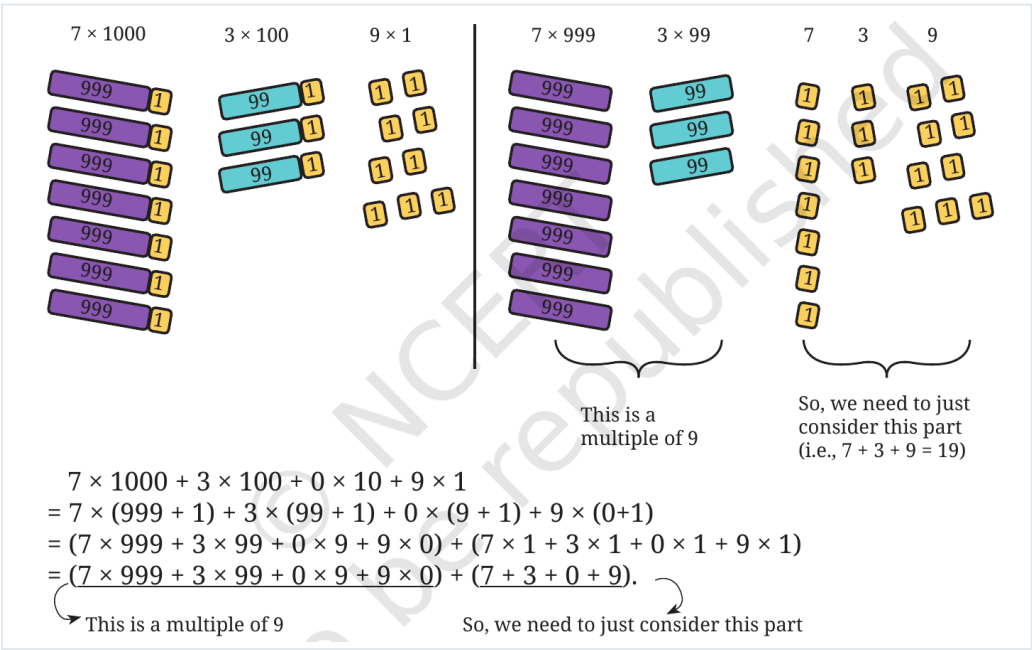
Therefore, 427 ÷ 9 leaves a remainder of 4 (check: 427 = 9 × 47 + 4).

QUESTION 47 *A Shortcut for Divisibility by 9 · p. 125*

Will this work with bigger numbers?

ANSWER

Yes. 1 = 0 + 1, 10 = 9 + 1, 100 = 99 + 1, 1000 = 999 + 1, 10000 = 9999 + 1, ... Every place value is a multiple of 9 plus 1, so each digit contributes its own value as remainder.



7309 split into multiples of 9 and the digits 7, 3, 0, 9 (NCERT)

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7309 = 7 × 1000 + 3 × 100 + 0 × 10 + 9 × 1 = 7 × (999 + 1) + 3 × (99 + 1) + 0 × (9 + 1) + 9 × (0 + 1) = (7 × 999 + 3 × 99 + 0 × 9 + 9 × 0) + (7 + 3 + 0 + 9). The first bracket is a multiple of 9, so only 7 + 3 + 0 + 9 = 19 matters.

19 → 1 + 9 = 10 → 1 + 0 = 1. So 7309 ÷ 9 leaves remainder 1 (7309 = 9 × 812 + 1).

Therefore, yes: a number is divisible by 9 if and only if the sum of its digits is divisible by 9, and repeatedly adding the digits gives the remainder.

QUESTION 48

A Shortcut for Divisibility by 9 · p. 125–126

Look at each of the following statements. Which are correct and why? (i) If a number is divisible by 9, then the sum of its digits is divisible by 9. (ii) If the sum of the digits of a number is divisible by 9, then the number is divisible by 9. (iii) If a number is not divisible by 9, then the sum of its digits is not divisible by 9. (iv) If the sum of the digits of a number is not divisible by 9, then the number is not divisible by 9.

ANSWER

The number and its digit sum leave the same remainder on division by 9. So one is divisible by 9 exactly when the other is.

Statement	Correct?	Why
(i)	**Yes**	Number divisible by 9 → remainder 0 → digit sum has remainder 0. Example: 729 → 7 + 2 + 9 = 18.
(ii)	**Yes**	Digit sum divisible by 9 → number has remainder 0. Example: 63 → 6 + 3 = 9 and 63 = 9 × 7.
(iii)	**Yes**	Same as (ii) said the other way round (if the digit sum were divisible by 9, the number would be). Example: 109 → 10, not divisible.
(iv)	**Yes**	Same as (i) said the other way round. Example: digit sum 19 (7309) → 7309 is not divisible by 9.

Therefore, all four statements are correct.

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QUESTION 49 *Figure it Out · Q.1 · p. 126*

Find, without dividing, whether the following numbers are divisible by 9. (i) 123 (ii) 405 (iii) 8888 (iv) 93547 (v) 358095

ANSWER

Number	Sum of digits	Divisible by 9?
123	1 + 2 + 3 = 6	No
405	4 + 0 + 5 = 9	**Yes**
8888	8 + 8 + 8 + 8 = 32	No
93547	9 + 3 + 5 + 4 + 7 = 28	No
358095	3 + 5 + 8 + 0 + 9 + 5 = 30	No

Therefore, only (ii) 405 is divisible by 9.

QUESTION 50 *Figure it Out · Q.2 · p. 126*

Find the smallest multiple of 9 with no odd digits.

ANSWER

The digits can only be 0, 2, 4, 6, 8, so the digit sum is even. A multiple of 9 has a digit sum that is a multiple of 9, so the digit sum must be an even multiple of 9: 18, 36, … (9 is odd, so it is impossible).

The digit sum must be at least 18. Two even digits give at most 8 + 8 = 16, so at least 3 digits are needed. The smallest 3-digit number with digit sum 18 and even digits: take the smallest possible first digit, 2, then 16 = 8 + 8 → **288**.

Check: 288 = 9 × 32.

Therefore, the smallest multiple of 9 with no odd digits is 288.

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QUESTION 51 *Figure it Out · Q.3 · p. 126*

Find the multiple of 9 that is closest to the number 6000.

ANSWER

6000 has digit sum 6, so 6000 = 9 × 666 + 6 (remainder 6).

Multiple of 9	Distance from 6000
6000 – 6 = 5994	6
6000 + 3 = 6003	3

Therefore, the multiple of 9 closest to 6000 is 6003.

QUESTION 52 *Figure it Out · Q.4 · p. 126*

How many multiples of 9 are there between the numbers 4300 and 4400?

ANSWER

4300 has digit sum 7, so the first multiple of 9 above it is 4300 + 2 = 4302. 4400 has digit sum 8, so the last multiple of 9 below it is 4400 – 8 = 4392.

Multiples: 4302, 4311, 4320, 4329, 4338, 4347, 4356, 4365, 4374, 4383, 4392.

Count = (4392 – 4302) ÷ 9 + 1 = 90 ÷ 9 + 1 = 11.

Therefore, there are 11 multiples of 9 between 4300 and 4400.

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QUESTION 53 *A Shortcut for Divisibility by 3 · p. 126*

The shortcut to find the divisibility by 3 is similar to the method for 9. A number is divisible by 3 if the sum of its digits is divisible by 3. Explore the remainders when powers of 10 are divided by 3. Explain why this method works.

ANSWER

Power of 10	1	10	100	1000	10000
Remainder ÷ 3	1	1	1	1	1

Each power of 10 is a string of 9s plus 1: 10 = 9 + 1, 100 = 99 + 1, 1000 = 999 + 1 ... and 9, 99, 999, ... are multiples of 3 (they are multiples of 9). So every place value leaves remainder 1 on division by 3.

Hence a number leaves the same remainder as the sum of its digits on division by 3. Examples: 15 → 1 + 5 = 6 (divisible), 33 → 6, 87 → 15 (divisible by 3 but not by 9).

Therefore, a number is divisible by 3 if and only if the sum of its digits is divisible by 3.

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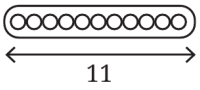
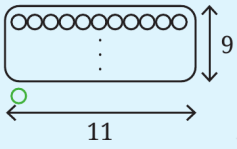
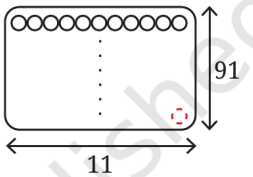
QUESTION 54 *A Shortcut for Divisibility by 11 · p. 127*

Using these observations (place values alternate as 1 more and 1 less than a multiple of 11), can you tell whether the number 462 is divisible by 11?

ANSWER

Units place (1)	$11 \times 0 = 0$ $1 = 11 \times 0 + \mathbf{1}$	1 is one more than a multiple of 11.	○
Tens place (10)	$11 \times 1 = 11$ $10 = 11 \times 1 - \mathbf{1}$	10 is one less than a multiple of 11.	
Hundreds place (100)	$11 \times 9 = 99$ $100 = 11 \times 9 + \mathbf{1}$	100 is one more than a multiple of 11.	
Thousands place (1000)	$11 \times 91 = 1001$ $1000 = 11 \times 91 - \mathbf{1}$	1000 is one less than a multiple of 11.	
...	...	...	...

Place values and multiples of 11 (NCERT)

- 4 hundreds: 400 is 4 more than a multiple of 11 (396 + 4).
- 6 tens: 60 is 6 less than a multiple of 11 (66 – 6).
- 2 units: 2 is 2 more than a multiple of 11.

Net excess = 4 – 6 + 2 = 0, so 462 is exactly a multiple of 11. Check: 462 = 11 × 42.

Therefore, yes, 462 is divisible by 11.

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QUESTION 55 *A Shortcut for Divisibility by 11 · p. 127*

What could be a general method or shortcut to check divisibility by 11?

ANSWER

The place values 1, 100, 10000, ... are 1 more than a multiple of 11; 10, 1000, 100000, ... are 1 less.

- Add the digits in the places 1, 100, 10000, ... (units, hundreds, ten-thousands ...) — the "excess".
- Add the digits in the places 10, 1000, 100000, ... (tens, thousands, ...) — the "shortage".
- Find excess – shortage. If it is 0 or a multiple of 11, the number is divisible by 11.

Therefore, a number is divisible by 11 if the difference between the sum of its digits in alternate places (starting from the units digit) is 0 or a multiple of 11.

QUESTION 56 *A Shortcut for Divisibility by 11 · p. 128*

If this difference is 11 or a multiple of 11, what does that say about the remainder obtained when the number is divided by 11?

ANSWER

The difference (excess – shortage) tells how far the number is from a multiple of 11. If it is 11, 22, –11, ... (or 0), the number is a multiple of 11 plus that multiple of 11 — i.e. still a multiple of 11. Example: 90904 → (4 + 9 + 9) – (0 + 0) = 22.

Therefore, the remainder is zero — the number is divisible by 11.

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QUESTION 57 *A Shortcut for Divisibility by 11 · p. 128*

Using this shortcut, find out whether the following numbers are divisible by 11. Further, find the remainder if the number is not divisible by 11. (i) 158 (ii) 841 (iii) 481 (iv) 5529 (v) 90904 (vi) 857076

ANSWER

Number	Excess (1, 100, …)	Shortage (10, 1000, …)	Difference	Remainder ÷ 11
158	9	5	4	4
841	9	4	5	5
481	5	8	-3	8
5529	14	7	7	7
90904	22	0	22	**0 — divisible**
857076	11	22	-11	**0 — divisible**

A negative difference such as –3 means “3 short of a multiple of 11”, i.e. remainder 11 – 3 = 8 (for 481).

Therefore: (i) 158 → remainder 4, (ii) 841 → 5, (iii) 481 → 8, (iv) 5529 → 7, (v) 90904 and (vi) 857076 are divisible by 11.

QUESTION 58 *A Shortcut for Divisibility by 11 · p. 129*

Look at the following procedure — 1. Place alternating ‘+’ and ‘–’ signs before every digit starting from the unit’s digit. 2. Evaluate the expression. 3. The result denotes the remainder obtained when the number is divided by 11. (Example for 328105: –3 + 2 – 8 + 1 – 0 + 5 = –3.) Is this method similar to or different from the method we saw just before?

ANSWER

It is the same method written more compactly. Earlier we added the excess digits (5, 1, 2 → 8) and the shortage digits (0, 8, 3 → 11) and found 8 – 11 = –3. The alternating-sign expression +5 – 0 + 1 – 8 + 2 – 3 does the same in one step and also gives –3.

Therefore, both methods are the same: –3 means 3 less than, or 8 more than, a multiple of 11.

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QUESTION 59 *A Shortcut for Divisibility by 11 · p. 129*

Fill in the following table. Find a quick way to do this? (Divisible by 2, 3, 4, 5, 6, 8, 9, 10, 11 for the numbers 128, 990, 1586, 275, 6686, 639210, 429714, 2856, 3060, 406839.)

ANSWER

Number	2	3	4	5	6	8	9	10	11
128	**Yes**	No	**Yes**	No	No	**Yes**	No	No	No
990	**Yes**	**Yes**	No	**Yes**	**Yes**	No	**Yes**	**Yes**	**Yes**
1586	**Yes**	No	No	No	No	No	No	No	No
275	No	No	No	**Yes**	No	No	No	No	**Yes**
6686	**Yes**	No	No	No	No	No	No	No	No
639210	**Yes**	**Yes**	No	**Yes**	**Yes**	No	No	**Yes**	**Yes**
429714	**Yes**	**Yes**	No	No	**Yes**	No	**Yes**	No	No
2856	**Yes**	**Yes**	**Yes**	No	**Yes**	**Yes**	No	No	No
3060	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	No	**Yes**	**Yes**	No
406839	No	**Yes**	No	No	No	No	No	No	No

a Quick way

- 2: last digit even. 5: last digit 0 or 5. 10: last digit 0.
- 4: last two digits divisible by 4. 8: last three digits divisible by 8.
- 3 and 9: sum of digits divisible by 3 / by 9. 6: divisible by both 2 and 3.
- 11: alternating sum of digits is 0 or a multiple of 11.

Note: $128 = 4 \times 32$, so for 128 the entry under 4 is Yes (the sample row printed in the book shows No there).

Therefore, the table above is complete; each entry can be found with the shortcuts without long division.

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QUESTION 60 *More on Divisibility Shortcuts · p. 129*

How can we find out if a number is divisible by 6?

ANSWER

6 = 2 × 3 and 2 and 3 have no common factor. A number divisible by both 2 and 3 is divisible by LCM (2, 3) = 6.

Therefore, a number is divisible by 6 if its units digit is even and the sum of its digits is divisible by 3.

QUESTION 61 *More on Divisibility Shortcuts · p. 129*

Will checking its divisibility by its factors 2 and 3 work? Use the shortcuts for 2 and 3 on these numbers and divide each number by 6 to verify— 38, 225, 186, 64.

ANSWER

Number	Divisible by 2?	Sum of digits (÷ 3?)	Divisible by 6 (shortcut)	Check: ÷ 6
38	Yes	11 (No)	No	6 r 2
225	No	9 (Yes)	No	37 r 3
186	Yes	15 (Yes)	**Yes**	31 r 0
64	Yes	10 (No)	No	10 r 4

Therefore, yes, checking divisibility by 2 and 3 works: only 186 is divisible by 6.

QUESTION 62 *More on Divisibility Shortcuts · p. 130*

How about checking divisibility by 24? Will checking the divisibility by its factors, 4 and 6, work? Why or why not?

ANSWER

No. 12 is divisible by both 4 and 6, but it is not divisible by 24. Similarly 36 and 60.

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To check divisibility by 24 we can instead check divisibility by 3 and by 8.

Therefore, checking 4 and 6 does not work for 24.

QUESTION 63 *More on Divisibility Shortcuts · p. 130*

Explain using prime factorisation why checking divisibility by 3 and 8 works for checking divisibility by 24, but checking divisibility by 4 and 6 is not sufficient for checking divisibility by 24.

ANSWER

Divisors	Prime factorisations	LCM (what a number divisible by both must contain)	Same as 24 = 2 × 2 × 2 × 3?
3 and 8	3; 8 = 2 × 2 × 2	2 × 2 × 2 × 3 = 24	**Yes**
4 and 6	4 = 2 × 2; 6 = 2 × 3	2 × 2 × 3 = 12	No (only one 3 and two 2s)

A number divisible by both 3 and 8 contains the prime factorisation of 3 and of 8, hence 2 × 2 × 2 × 3 = 24. A number divisible by 4 and 6 only has to contain 2 × 2 × 3 = 12.

Therefore, 3 and 8 together give the full 2 × 2 × 2 × 3, while 4 and 6 give only 2 × 2 × 3, so they are not sufficient.

QUESTION 64 *Digital Roots · p. 130*

Take a number. Add its digits repeatedly till you get a single-digit number. This is the digital root (e.g. 489710 → 29 → 11 → 2). What property do you think this digital root will have? Recall that we did this while finding the divisibility shortcut for 9.

ANSWER

Repeatedly adding the digits gives the remainder when the number is divided by 9 (and 9 if the number is exactly divisible by 9). Example: 489710 = 9 × 54412 + 2 and its digital root is 2.

Therefore, the digital root of a number is its remainder on division by 9 (it is 9 when the number is a multiple of 9).

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QUESTION 65 *Digital Roots · p. 130*

Between the numbers 600 and 700, which numbers have the digital root: (i) 5, (ii) 7, (iii) 3?

ANSWER

Digital root 5 means digit sum 5 or 14; digital root 7 means digit sum 7 or 16; digital root 3 means digit sum 3 or 12 (or 21).

Digital root	Numbers between 600 and 700
5	608, 617, 626, 635, 644, 653, 662, 671, 680, 689, 698
7	601, 610, 619, 628, 637, 646, 655, 664, 673, 682, 691
3	606, 615, 624, 633, 642, 651, 660, 669, 678, 687, 696

Therefore, there are 11 such numbers for each digital root (the numbers are 9 apart).

QUESTION 66 *Digital Roots · p. 130*

Write the digital roots of any 12 consecutive numbers. What do you observe?

ANSWER

Number	600	601	602	603	604	605	606	607	608	609	610	611
Digital root	6	7	8	9	1	2	3	4	5	6	7	8

Adding 1 to a number adds 1 to its digital root, except that after 9 the root goes back to 1. So the roots repeat the cycle 1, 2, …, 9, in order.

Therefore, digital roots of consecutive numbers go 1, 2, 3, …, 9, 1, 2, … repeating every 9 numbers.

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QUESTION 67 *Digital Roots · p. 130*

Now, find the digital roots of some consecutive multiples of (i) 3, (ii) 4, and (iii) 6.

ANSWER

Multiples of	First 12 multiples	Digital roots
3	3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36	3, 6, 9, 3, 6, 9, 3, 6, 9, 3, 6, 9
4	4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48	4, 8, 3, 7, 2, 6, 1, 5, 9, 4, 8, 3
6	6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72	6, 3, 9, 6, 3, 9, 6, 3, 9, 6, 3, 9

For 3: 3, 6, 9 repeating. For 4: 4, 8, 3, 7, 2, 6, 1, 5, 9 repeating (all nine values). For 6: 6, 3, 9 repeating.

Therefore, the digital roots of multiples of 3 (and of 6) cycle through 3, 6, 9 (in some order), while multiples of 4 cycle through all of 1 to 9.

QUESTION 68 *Digital Roots · p. 130*

What are the digital roots of numbers that are 1 more than a multiple of 6? What do you notice? Try to explain the patterns noticed.

ANSWER

Number 6k + 1	7 13 19 25 31 37 43
Digital root	7 4 1 7 4 1 7

The roots are 7, 4, 1, 7, 4, 1, … — always 1, 4 or 7.

Why: the digital root is the remainder on division by 9. Adding 6 to a number changes this remainder by 6 (equivalently, by –3): 7 → 4 → 1 → 7. Also 6k + 1 leaves remainder 1 on division by 3, and the digital roots 1, 4, 7 are exactly those that leave remainder 1 on division by 3.

Therefore, numbers that are 1 more than a multiple of 6 have digital root 1, 4 or 7.

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QUESTION 69 *Digital Roots · p. 130*

“I’m made of digits, each tiniest and odd, No shared ground with root #1—how odd! My digits count, their sum, my root— All point to one bold number’s pursuit— The largest odd single-digit I proudly claim. What’s my number? What’s my name?”

ANSWER

a Decoding the clues

- “Each tiniest and odd” → every digit is 1 (the smallest odd digit).
- “The largest odd single-digit” is 9 → my number of digits is 9, my digit sum is 9 and my digital root is 9.
- “No shared ground with root #1” → my digital root is 9, not 1, so I am not in the “#1” (digital root 1) club.

b The number

Nine digits, each 1: 111111111. Digit sum = 9 and digital root = 9. In the Indian system: 11,11,11,111.

Therefore, the number is 11,11,11,111 — eleven crore eleven lakh eleven thousand one hundred eleven.

QUESTION 70 *Figure it Out · Q.1 · p. 131*

The digital root of an 8-digit number is 5. What will be the digital root of 10 more than that number?

ANSWER

Adding 10 = 9 + 1 changes the remainder on division by 9 by 1.

Example: 40000001 has digital root 4 + 1 = 5; 40000011 has digital root 4 + 1 + 1 = 6.

Therefore, the digital root of the new number is 5 + 1 = 6.

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QUESTION 71 *Figure it Out · Q.2 · p. 131*

Write any number. Generate a sequence of numbers by repeatedly adding 11. What would be the digital roots of this sequence of numbers? Share your observations.

ANSWER

Number	10	21	32	43	54	65	76	87	98	109	120	131
Digital root	1	3	5	7	9	2	4	6	8	1	3	5

Adding 11 = 9 + 2 raises the digital root by 2 each time (going 9 → 2 after passing 9). So the roots run 1, 3, 5, 7, 9, 2, 4, 6, 8, 1, … and repeat after 9 steps, passing through every digit from 1 to 9.

Therefore, the digital roots of the sequence increase by 2 each time and repeat after 9 numbers.

QUESTION 72 *Figure it Out · Q.3 · p. 131*

What will be the digital root of the number 9a + 36b + 13?

ANSWER

$9a + 36b + 13 = 9a + 36b + 9 + 4 = 9(a + 4b + 1) + 4$.

$9(a + 4b + 1)$ is a multiple of 9, so the number leaves remainder 4 when divided by 9. Example: $a = 1$, $b = 1$ gives $58 \rightarrow 5 + 8 = 13 \rightarrow 4$.

Therefore, the digital root is 4.

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QUESTION 73 *Figure it Out · Q.4 · p. 131*

Make conjectures by examining if there are any patterns or relations between (i) the parity of a number and its digital root. (ii) the digital root of a number and the remainder obtained when the number is divided by 3 or 9.

ANSWER

a (i) Parity and digital root

Number	10	12	11	13	19
Parity	even	even	odd	odd	odd
Digital root	1	3	2	4	1

Both parities occur with every kind of digital root (10 is even with root 1; 19 is odd with root 1).
Conjecture: there is no pattern between parity and digital root.

b (ii) Digital root and remainders

Digital root	Remainder on ÷ 3	Remainder on ÷ 9
1, 4, 7	1	same as the digital root
2, 5, 8	2	same as the digital root
3, 6	0	same as the digital root
9	0	0

Therefore, the digital root gives the remainder on division by 9 (9 ↔ 0), and its remainder on division by 3 gives the remainder of the number on division by 3.

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CLASS 8 MATHEMATICS · CHAPTER 5

5.3 Digits in Disguise

Solutions · 22 questions

Directions (Questions 74–95): These are the questions, examples, activities and Figure it Out exercises that appear in Section 5.3 (pages 131–133) of the chapter “Number Play”, numbered in the order in which they appear. Where a question is open-ended, a model answer is given and you may add examples of your own.

QUESTION 74 *Digits in Disguise · p. 131*

Last year, we saw cryptarithms — puzzles where each letter stands for a digit, each digit is represented by at most one letter, and the first digit of a number is never 0. Solve the cryptarithms given below. (i) A1 + 1B = B0 (ii) AB + 37 = 6A (iii) ON + ON + ON = PO (iv) QR + QR + QR = PRR

ANSWER

a (i) A1 + 1B = B0

Units: 1 + B must end in 0, so B = 9 (1 + 9 = 10, carry 1). Tens: A + 1 + 1 (carry) = B = 9, so A = 7. Check: 71 + 19 = 90.

b (ii) AB + 37 = 6A

Tens: A + 3 + (carry) = 6. If there is no carry, A = 3 and B + 7 would have to end in 3, i.e. B + 7 = 13, which does produce a carry — a contradiction. So there is a carry: A = 2, and B + 7 = 12 gives B = 5. Check: 25 + 37 = 62.

c (iii) ON + ON + ON = PO, i.e. 3 × ON = PO

Writing 3(10·O + N) = 10·P + O gives 29·O + 3·N = 10·P. Trying O = 1, 2, 3 (larger O makes P exceed 9) gives three solutions:

O	N	P	Check
1	7	5	17 × 3 = 51
2	4	7	24 × 3 = 72

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O	N	P	Check
3	1	9	31 × 3 = 93

d (iv) QR + QR + QR = PRR, i.e. 3 × QR = PRR

Units: 3 × R ends in R, so R = 0 or 5; R = 0 would make PRR = P00, impossible with Q ≠ 0, so R = 5 (3 × 5 = 15, carry 1). Tens: 3 × Q + 1 = ends in 5 → 3Q ends in 4 → Q = 8 (24 + 1 = 25, carry 2). So P = 2. Check: 85 × 3 = 255.

Therefore: (i) A = 7, B = 9; (ii) A = 2, B = 5; (iii) (O, N, P) = (1, 7, 5), (2, 4, 7) or (3, 1, 9); (iv) Q = 8, R = 5, P = 2.

QUESTION 75 *Digits in Disguise · p. 131*

(v) PQ × 8 = RS. Guna says, “Oh, this means a 2-digit number multiplied by 8 should give another 2-digit number. I know that 10 × 8 = 80. But the units digits of 10 and 80 are the same, which we don’t want. For the same reason PQ cannot be 11 as P and Q correspond to different digits. 12 × 8 = 96 fits all the conditions”. Can PQ be 13? Think.

ANSWER

PQ	PQ × 8	Allowed?
10	80	No — Q and S would both be 0
11	88	No — P and Q must be different digits
12	**96**	**Yes** (1, 2, 9, 6 all different)
13	104	No — a 3-digit number

For every 2-digit number from 13 onwards, 8 × PQ ≥ 104 is a 3-digit number.

Therefore, PQ cannot be 13; the only solution is 12 × 8 = 96 (P = 1, Q = 2, R = 9, S = 6).

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QUESTION 76 *Digits in Disguise · p. 132*

(vi) Try this now: GH × H = 9K. This means a 2-digit number multiplied by a 1-digit number gives another 2-digit number in the 90s. Observe the letters corresponding to the units digits in this cryptarithm. Pick the solution to this question from the options given below: 11 × 9 = 99, 12 × 8 = 96, 46 × 2 = 92, 24 × 4 = 96, 47 × 2 = 94, 31 × 3 = 93, 16 × 6 = 96.

ANSWER

In GH × H the multiplier is the same letter as the units digit of GH. So the units digit of the 2-digit number must equal the multiplier; G, H, K must be different digits.

Option	Units digit of first number	Multiplier	Same?	Other conditions
11 × 9 = 99	1	9	No	
12 × 8 = 96	2	8	No	
46 × 2 = 92	6	2	No	
24 × 4 = 96	4	4	**Yes**	G = 2, H = 4, K = 6 all different ✓
47 × 2 = 94	7	2	No	
31 × 3 = 93	1	3	No	
16 × 6 = 96	6	6	Yes	K = 6 = H, but different letters need different digits ✗

Therefore, the solution is 24 × 4 = 96, i.e. G = 2, H = 4, K = 6.

QUESTION 77 *Digits in Disguise · p. 132*

(vii) Here is one more: BYE × 6 = RAY. Anshu says, “Since the product is a 3-digit number, B can’t be 2 or more. If B = 2, i.e., 2 hundreds, the product will be more than 1200. So, B = 1.”

ANSWER

a **Using Anshu’s start (B = 1)**

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With B = 1 we need 1YE × 6 = RAY, a 3-digit number, so 6 × (100 + 10Y + E) < 1000 → 10Y + E ≤ 66.

b Units digit

E × 6 ends in Y. Since Y is the units digit of a multiple of 6 it is even; and Y and E are different digits. The only choice that gives different digits for all letters is Y = 0, E = 5 (5 × 6 = 30, units 0, carry 3).

c Finish

105 × 6 = 630: R = 6, A = 3, Y = 0. All five letters B = 1, Y = 0, E = 5, R = 6, A = 3 are different digits.

Therefore, BYE = 105 and RAY = 630 (B = 1, Y = 0, E = 5, R = 6, A = 3).

QUESTION 78 *Digits in Disguise · p. 132*

What can you say about 'Y'? What digits are possible/not possible? (Anshu: "Y cannot be 7 or more because, if Y = 7, then 170 × 6 = 1020; but we want a 3-digit product. Also, Y will be even".)

ANSWER

Y	BYE	Possible?	Reason
0	10E	**Yes** (E = 5)	105 × 6 = 630 works
1	11E	No	Y would equal B
2	12E	No	E would be 2 or 7; E = 2 equals Y, and E = 7 gives 127 × 6 = 762 where R = E = 7
3, 5	13E, 15E	No	Y is the units digit of 6 × E, which is even
4	14E	No	E would be 4 or 9; 149 × 6 = 894 makes A = E = 9
6	16E	No	E would be 1 or 6: 161 × 6 = 966 makes A = Y = 6 and E = B; E = 6 equals Y
7, 8, 9	17E, 18E, 19E	No	170 × 6 = 1020 is already 4 digits

Therefore, Y must be even and less than 7, and the only value that works is Y = 0.

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QUESTION 79 *Digits in Disguise · p. 132*

We can solve cryptarithms using patterns, properties, and reasoning related to numbers and operations. Solve the following: (i) UT × 3 = PUT (ii) AB × 5 = BC (iii) L2N × 2 = 2NP (iv) XY × 4 = ZX (v) PP × QQ = PRP (vi) JK × 6 = KKK

ANSWER

	Cryptarithm	Solution(s)	Check
(i)	UT × 3 = PUT	U = 5, T = 0, P = 1	50 × 3 = 150
(ii)	AB × 5 = BC	A = 1, B = 9, C = 5	19 × 5 = 95
(iii)	L2N × 2 = 2NP	L = 1, N = 5, P = 0 or L = 1, N = 4, P = 8	125 × 2 = 250 124 × 2 = 248
(iv)	XY × 4 = ZX	X = 2, Y = 3, Z = 9	23 × 4 = 92
(v)	PP × QQ = PRP	P = 2, Q = 1, R = 4 or P = 3, Q = 1, R = 6 or P = 4, Q = 1, R = 8	22 × 11 = 242 33 × 11 = 363 44 × 11 = 484
(vi)	JK × 6 = KKK	J = 7, K = 4	74 × 6 = 444

a Key reasoning

- (i) 3 × UT = 100P + UT gives 2 × UT = 100P → UT = 50P; so UT = 50, P = 1.
- (ii) 5 × (10A + B) = 10B + C gives 50A = 5B + C. As 5B + C ≤ 54, A = 1 and 5B + C = 50, so B = 9, C = 5 (19 × 5 = 95).
- (iii) 2 × L2N = 2NP: L = 1 (product starts with 2), then 2 × (120 + N) = 240 + 2N = 200 + 10N + P gives P = 40 – 8N, so N = 4, P = 8 or N = 5, P = 0.
- (iv) Z X = 4 × XY, so X = 2 (4 × XY < 100) and 4 × 23 = 92 gives Y = 3, Z = 9.
- (v) PP × QQ = 11P × 11Q = 121PQ = PRP: Q = 1 gives 121P = P(2P)P, i.e. R = 2P, valid for P = 2, 3, 4 (P = 1 would make P = Q).
- (vi) JK × 6 = 111K: 6 × (10J + K) = 111K → 60J = 105K → 4J = 7K → J = 7, K = 4.

Therefore, the solutions are as in the table; (iii) and (v) have more than one solution.

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QUESTION 80 *Figure it Out · Q.1 · p. 132*

If 31z5 is a multiple of 9, where z is a digit, what is the value of z? Explain why there are two answers to this problem.

ANSWER

The digit sum is 3 + 1 + z + 5 = 9 + z. It must be a multiple of 9.

z	9 + z	Multiple of 9?	Number
0	9	**Yes**	3105 = 9 × 345
9	18	**Yes**	3195 = 9 × 355

There are two answers because both 9 + 0 = 9 and 9 + 9 = 18 are multiples of 9, and no other digit z (1 to 8) makes 9 + z a multiple of 9.

Therefore, z = 0 or z = 9.

QUESTION 81 *Figure it Out · Q.2 · p. 132*

“I take a number that leaves a remainder of 8 when divided by 12. I take another number which is 4 short of a multiple of 12. Their sum will always be a multiple of 8”, claims Snehal. Examine his claim and justify your conclusion.

ANSWER

The numbers are a = 12n + 8 and b = 12m – 4 (a multiple of 12 minus 4).

a + b = 12n + 12m + 4 = 12(n + m) + 4 = 4(3(n + m) + 1). This is a multiple of 8 only when 3(n + m) + 1 is even, i.e. when n + m is odd.

n	m	a	b	a + b	Multiple of 8?
0	1	8	8	16	**Yes**
0	2	8	20	28	No
1	1	20	8	28	No
1	2	20	20	40	**Yes**

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Therefore, Snehal's claim is not true: the sum is only sometimes a multiple of 8 (e.g. 8 + 20 = 28 is not).

QUESTION 82 *Figure it Out · Q.3 · p. 132*

When is the sum of two multiples of 3, a multiple of 6 and when is it not? Explain the different possible cases, and generalise the pattern.

ANSWER

Let the multiples of 3 be 3m and 3n. Their sum is 3m + 3n = 3(m + n).

Case	m, n	m + n	Sum 3(m + n)	Example
Both even multiples of 3 (multiples of 6)	both even	even	**multiple of 6**	6 + 12 = 18
Both odd multiples of 3	both odd	even	**multiple of 6**	3 + 9 = 12
One of each	one even, one odd	odd	not a multiple of 6	3 + 6 = 9

Therefore, the sum of two multiples of 3 is a multiple of 6 exactly when m + n is even, i.e. when both multiples are even multiples of 3 or both are odd multiples of 3.

QUESTION 83 *Figure it Out · Q.4 · p. 132*

Sreelatha says, "I have a number that is divisible by 9. If I reverse its digits, it will still be divisible by 9". (i) Examine if her conjecture is true for any multiple of 9. (ii) Are any other digit shuffles possible such that the number formed is still a multiple of 9?

ANSWER

a (i) Reversing the digits

A number is divisible by 9 exactly when its digit sum is. Reversing the digits does not change the digit sum, so the reversed number is also divisible by 9. Example: 1827 = 9 × 203 → 7281 = 9 × 809.

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b (ii) Other shuffles

Yes — every shuffle (rearrangement) of the digits keeps the digit sum the same, so the new number is still divisible by 9. Example: 1827 → 8127 (= 9 × 903), 2718 (= 9 × 302), 8721 (= 9 × 969).

Therefore, Sreelatha's conjecture is true, and any rearrangement of the digits of a multiple of 9 is again a multiple of 9.

QUESTION 84 *Figure it Out · Q.5 · p. 132*

If 48a23b is a multiple of 18, list all possible pairs of values for a and b.

ANSWER

18 = 2 × 9 and 2, 9 have no common factor, so the number must be divisible by both 2 and 9.

- Divisible by 2: b is even → b = 0, 2, 4, 6, 8.
- Divisible by 9: 4 + 8 + a + 2 + 3 + b = 17 + a + b must be a multiple of 9, i.e. 18 or 27 (the maximum is 17 + 18 = 35).

b	17 + a + b must be	a	Number
0	18 (a = 1)	1	481230
2	27 (a = 8)	8	488232
4	27 (a = 6)	6	486234
6	27 (a = 4)	4	484236
8	27 (a = 2)	2	482238

Therefore, the possible pairs (a, b) are (1, 0), (8, 2), (6, 4), (4, 6) and (2, 8).

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QUESTION 85 *Figure it Out · Q.6 · p. 133*

If 3p7q8 is divisible by 44, list all possible pairs of values for p and q.

ANSWER

44 = 4 × 11 and 4, 11 have no common factor, so the number must be divisible by both 4 and 11.

a Divisible by 4

The last two digits q8 must be divisible by 4: 08, 28, 48, 68, 88 → q = 0, 2, 4, 6, 8.

b Divisible by 11

Digits in the places of 1, 100, 10000: 8 + 7 + 3 = 18. Digits in the places of 10, 1000: q + p. Difference = 18 – (p + q) must be 0 or a multiple of 11, so p + q = 18 or p + q = 7 (p + q = 29 is too big).

- p + q = 18 would need p = q = 9, but q must be even — impossible.
- p + q = 7 with q even: (p, q) = (7, 0), (5, 2), (3, 4), (1, 6). (q = 8 would need p = –1.)

Check: 37708 = 44 × 857, 35728 = 44 × 812, 33748 = 44 × 767, 31768 = 44 × 722.

Therefore, the possible pairs (p, q) are (7, 0), (5, 2), (3, 4) and (1, 6).

QUESTION 86 *Figure it Out · Q.7 · p. 133*

Find three consecutive numbers such that the first number is a multiple of 2, the second number is a multiple of 3, and the third number is a multiple of 4. Are there more such numbers? How often do they occur?

ANSWER

Let the numbers be n, n + 1, n + 2 with n a multiple of 2, n + 1 a multiple of 3 and n + 2 a multiple of 4.

One set is **2, 3, 4**. Another is **14, 15, 16** (14 = 2 × 7, 15 = 3 × 5, 16 = 4 × 4).

The conditions n ≡ 0 (mod 2), n + 1 ≡ 0 (mod 3), n + 2 ≡ 0 (mod 4) repeat when n increases by LCM (2, 3, 4) = 12. So all such sets begin at 12k + 2: (2, 3, 4), (14, 15, 16), (26, 27, 28), (38, 39,

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40), ...

Therefore, yes — there are infinitely many such triples and they occur once in every 12 numbers.

QUESTION 87 *Figure it Out · Q.8 · p. 133*

Write five multiples of 36 between 45,000 and 47,000. Share your approach with the class.

ANSWER

Approach: 36 = 4 × 9. A number that is divisible by 4 (last two digits) and by 9 (digit sum) is a multiple of 36. Or note 45000 = 36 × 1250, so add 36 repeatedly.

Number	Last two digits ÷ 4	Digit sum (÷ 9)	Multiple of 36?
45036	36 = 4 × 9	4 + 5 + 0 + 3 + 6 = 18	**Yes** (36 × 1251)
45072	72 = 4 × 18	4 + 5 + 0 + 7 + 2 = 18	**Yes**
45108	08 = 4 × 2	4 + 5 + 1 + 0 + 8 = 18	**Yes**
45144	44 = 4 × 11	4 + 5 + 1 + 4 + 4 = 18	**Yes**
45180	80 = 4 × 20	4 + 5 + 1 + 8 + 0 = 18	**Yes**

Therefore, five multiples of 36 between 45,000 and 47,000 are 45036, 45072, 45108, 45144 and 45180.

QUESTION 88 *Figure it Out · Q.9 · p. 133*

The middle number in the sequence of 5 consecutive even numbers is 5p. Express the other four numbers in sequence in terms of p.

ANSWER

Consecutive even numbers differ by 2.

Therefore, the five numbers in sequence are 5p – 4, 5p – 2, 5p, 5p + 2 and 5p + 4.

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QUESTION 89 *Figure it Out · Q.10 · p. 133*

Write a 6-digit number that it is divisible by 15, such that when the digits are reversed, it is divisible by 6.

ANSWER

Divisible by 15 means divisible by 3 and 5. Divisible by 6 means divisible by 2 and 3.

- The number abcdef is divisible by 5, so f = 0 or 5. If f = 0, the reversed number starts with 0 and is not a 6-digit number; so f = 5.
- The reversed number 5edcba must be even, so its last digit a is even: a = 2, 4, 6 or 8.
- The digit sum must be a multiple of 3 (the same for both numbers).

Number	Digit sum	÷ 15	Reversed	Reversed ÷ 6
200025	9	200025 = 15 × 13335	520002	520002 = 6 × 86667
400005	9	400005 = 15 × 26667	500004	500004 = 6 × 83334
200055	12	200055 = 15 × 13337	550002	550002 = 6 × 91667

Therefore, one such number is 200025 (others: 400005, 200055).

QUESTION 90 *Figure it Out · Q.11 · p. 133*

Deepak claims, “There are some multiples of 11 which, when doubled, are still multiples of 11. But other multiples of 11 don’t remain multiples of 11 when doubled”. Examine if his conjecture is true; explain your conclusion.

ANSWER

Let n be any multiple of 11: n = 11k.

Doubled: 2n = 22k = 11 × (2k), which is a multiple of 11 for every k. Examples: 11 → 22, 33 → 66, 110 → 220, 407 → 814.

Therefore, Deepak’s conjecture is false: every multiple of 11, when doubled, is still a multiple of 11.

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QUESTION 91 *Figure it Out · Q.12 · p. 133*

Determine whether the statements below are ‘Always True’, ‘Sometimes True’, or ‘Never True’. Explain your reasoning. (i) The product of a multiple of 6 and a multiple of 3 is a multiple of 9. (ii) The sum of three consecutive even numbers will be divisible by 6. (iii) If abcdef is a multiple of 6, then badcef will be a multiple of 6. (iv) 8(7b – 3) – 4(11b + 1) is a multiple of 12.

ANSWER

	Verdict	Reasoning
(i)	**Always**	6x × 3y = 18xy = 9 × (2xy). Example: 12 × 9 = 108 = 9 × 12.
(ii)	**Always**	2n + (2n + 2) + (2n + 4) = 6n + 6 = 6(n + 1). Example: 4 + 6 + 8 = 18 = 6 × 3.
(iii)	**Always**	badcef has the same digits (only a↔b and c↔d swapped), so the same digit sum — divisible by 3 — and the same units digit f (even). So it is divisible by 2 and 3, hence by 6. Example: 123456 → 214356 (= 6 × 35726).
(iv)	**Never**	8(7b – 3) – 4(11b + 1) = 56b – 24 – 44b – 4 = 12b – 28 = 12(b – 3) + 8 — leaves remainder 8 on division by 12, so it is never a multiple of 12.

Therefore: (i) always true, (ii) always true, (iii) always true, (iv) never true.

QUESTION 92 *Figure it Out · Q.13 · p. 133*

Choose any 3 numbers. When is their sum divisible by 3? Explore all possible cases and generalise.

ANSWER

Every number leaves remainder 0, 1 or 2 when divided by 3, and the sum has the same remainder as the sum of the three remainders.

Remainders of the three numbers	Sum of remainders	Remainder of the sum	Divisible by 3?
0, 0, 0	0	0	**Yes**
0, 0, 1	1	1	No
0, 0, 2	2	2	No
0, 1, 1	2	2	No

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Remainders of the three numbers	Sum of remainders	Remainder of the sum	Divisible by 3?
0, 1, 2	3	0	**Yes**
0, 2, 2	4	1	No
1, 1, 1	3	0	**Yes**
1, 1, 2	4	1	No
1, 2, 2	5	2	No
2, 2, 2	6	0	**Yes**

Therefore, the sum of three numbers is divisible by 3 exactly when the three numbers leave the same remainder (0, 0, 0 or 1, 1, 1 or 2, 2, 2) or leave three different remainders (0, 1, 2).

QUESTION 93

Figure it Out · Q.14 · p. 133

Is the product of two consecutive integers always multiple of 2? Why? What about the product of three consecutive integers? Is it always a multiple of 6? Why or why not? What can you say about the product of 4 consecutive integers? What about the product of five consecutive integers?

ANSWER

Product of	Always a multiple of	Why	Example
2 consecutive integers	2	one of the two is even	$4 \times 5 = 20$
3 consecutive integers	6	one is a multiple of 3 and at least one is even	$3 \times 4 \times 5 = 60 = 6 \times 10$
4 consecutive integers	24	one is a multiple of 3; of the two even numbers one is a multiple of 4, giving $2 \times 4 = 8$; $3 \times 8 = 24$	$2 \times 3 \times 4 \times 5 = 120 = 24 \times 5$
5 consecutive integers	120	in addition one is a multiple of 5, so $24 \times 5 = 120$	$3 \times 4 \times 5 \times 6 \times 7 = 2520 = 120 \times 21$

Examples at the start: $1 \times 2 = 2$; $1 \times 2 \times 3 = 6$; $1 \times 2 \times 3 \times 4 = 24$; $1 \times 2 \times 3 \times 4 \times 5 = 120$.

Therefore, the product of 2, 3, 4, 5 consecutive integers is always a multiple of 2, 6, 24, 120 respectively.

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QUESTION 94 *Figure it Out · Q.15 · p. 133*

Solve the cryptarithms — (i) EF × E = GGG (ii) WOW × 5 = MEOW

ANSWER

a (i) EF × E = GGG

GGG = 111 × G, and 111 = 3 × 37. So E × EF is a multiple of 111 with E a single digit: E = 3 and EF = 37 (3 × 37 = 111), giving G = 1. Check: 37 × 3 = 111, with E = 3, F = 7, G = 1.

b (ii) WOW × 5 = MEOW

5 × WOW is a 4-digit number and 5 × W ends in W, so W = 5. Then 5 × (505 + 10O) = 2525 + 50O = MEOW, and the tens digit of 2525 + 50O is (2 + 5O) mod 10, which must equal O: 4O ends in 8, so O = 2 or 7. O = 2 gives 2625 (M and O both 2, not allowed); O = 7 gives 575 × 5 = 2875: M = 2, E = 8, O = 7, W = 5.

Therefore, (i) E = 3, F = 7, G = 1 (37 × 3 = 111); (ii) W = 5, O = 7, M = 2, E = 8 (575 × 5 = 2875).

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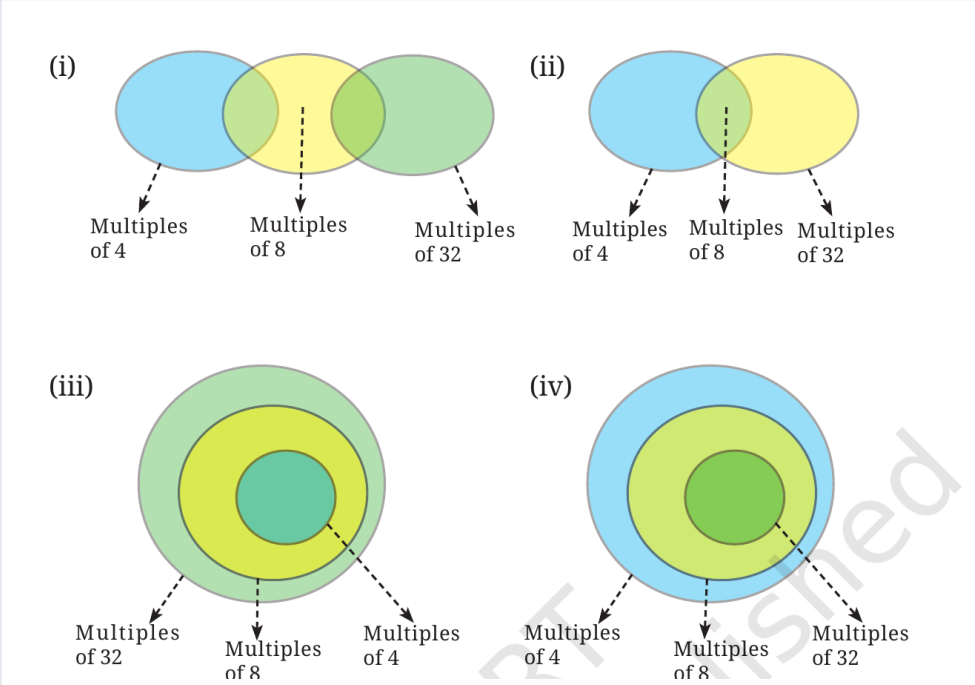
QUESTION 95 *Figure it Out · Q.16 · p. 133*

Which of the following Venn diagrams captures the relationship between the multiples of 4, 8, and 32?

ANSWER



The four Venn diagrams (NCERT)

- Every multiple of 32 is a multiple of 8 ($32 = 8 \times 4$) and of 4 ($32 = 4 \times 8$).
- Every multiple of 8 is a multiple of 4 ($8 = 4 \times 2$), but not every multiple of 4 is a multiple of 8 (e.g. 12).
- So the multiples of 32 lie inside the multiples of 8, which lie inside the multiples of 4: three circles inside one another, with 4 the biggest and 32 the smallest.

Diagram	Does it match?
(i) three overlapping circles in a row	No — multiples of 32 are not separate from multiples of 4
(ii) two overlapping circles	No — it does not show nested sets
(iii) nested circles, 32 outermost	No — the order is reversed (the largest set is the multiples of 4)
(iv) nested circles: 4 outermost, then 8, then 32	**Yes**

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Therefore, diagram (iv) captures the relationship between the multiples of 4, 8 and 32.

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